Astronomy 111 **Fall 2011**

HW #5 - due Tuesday 11/1 in class

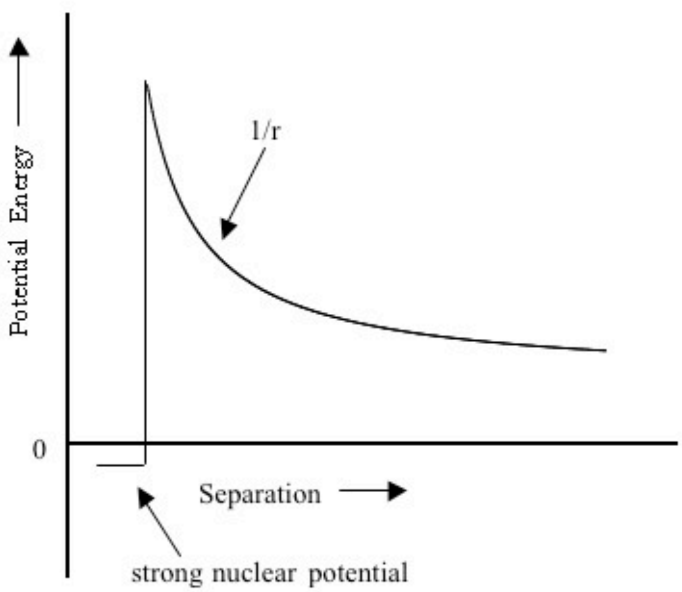
Kutner
Ch. 9: Problems 6-10

Also these problems:

1. For the calculations you will do below, you might appreciate knowing that $e=1.6 \times 10^{-19}$ in SI units, that $k=1.38 \times 10^{-23}$J/K, and that $h=6.62 \times 10^{-34}$ J-sec. In part A, you can ignore the statement about Z_1 and Z_2 both equaling 1 – the right-hand side of the equation for $T_{classical}$ actually includes the product of $Z_1^2Z_2^2$, necessary for calculations involving heavier elements, but irrelevant here.

We covered the basic concept in class about quantum mechanical tunneling being necessary in order for fusion to occur in the sun and other stars. Here you will actually calculate that this is true.

Consider two protons in the core of the sun. The height of the potential energy barrier, as shown below, goes as 1/r for large separation (Coulomb repulsion), and becomes negative (attractive) at small separations, as the strong nuclear force dominates over the Coulomb repulsion.



If we assume that the energy required to overcome the Coulomb barrier is provided by the thermal energy of the gas, and that all protons are non-relativistic, then we can estimate the temperature, $T_{classical}$ as follows. If we equate the average kinetic energy of an incoming proton to the energy of the Coulomb barrier at r, we find: $\frac{3}{2}kT_{classical} = \frac{\alpha e^2}{r}$, where α is $1/4\pi\varepsilon_0$, ($=9\times10^9$ in SI units) and r is the separation. We then find that $T_{classical} = \frac{2\alpha e^2}{3kr}$.

PART A. **For 2 protons, $Z_1=Z_2=1$. Assume a separation equal to the radius of a proton (10^{-15} m). Calculate $T_{classical}$ and compare it with the actual central temperature of the Sun. Comment?**

Now, if we assume that for quantum tunneling to occur a proton must be within one deBroglie wavelength ($\lambda=h/p$, where p is the momentum and h is Planck's constant) of the target. We can rewrite the kinetic energy in terms of momentum: $\frac{1}{2}mv^2 = \frac{p^2}{2m}$. If we set the distance of closest approach equal to one proton wavelength, and let the barrier height equal the original kinetic energy, we find $\frac{\alpha e^2}{\lambda} = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m}$.

PART B. **Solve the above equation for λ and substitute $r = \lambda$ in the equation for $T_{classical}$, which now becomes $T_{quantum}$. Solve for $T_{quantum}$ and compare it with the actual central temperature of the Sun. Comment?**

2. (a) The Earth's atmosphere is (very nearly) in hydrostatic equilibrium. What does this imply about the pressure as a function of altitude?

(b) The pressure at sea level is 1 atmosphere = 10^5 Pascals. (The Pascal is the SI unit of pressure, equal to 1 newton/square meter.) Use the equation of hydrostatic equilibrium ($dP/dR = -g\rho$), to calculate dP (the change in atmospheric pressure) as one ascends to Denver, whose altitude is 1600 m. You will need to know that the mass density, ρ, of the atmosphere at sea level is roughly 1 kg/m^3, and that g is the local acceleration of gravity at sea level, which you can look up. What percentage of sea-level pressure is Denver's atmospheric pressure?

(c) Do the same calculations for Mauna Kea, whose altitude is 4300 m. Are you surprised that oxygen is sometimes needed for observers at the summit?