

Fall 2011
Astronomy 111

HW #9 - due Tuesday, 12/6 in class

1. By what factor does the mass of an object increase if it travels at:
 - a) 25% the speed of light?
 - b) 99.99% the speed of light?
2. Newton's Laws tell us that the kinetic energy of a moving object is $0.5mv^2$. What is the kinetic energy of a 1 kg mass moving at $v=10^4$ m/sec (about 1/3 the orbital speed of the Earth)? Compare this with the rest mass energy of the same mass, and comment.
3. The expanding universe is carrying distant objects away from each other at a rate proportional to their separations. We use the Doppler effect observed in spectra of distant galaxies and quasars to calculate recession speeds. For the most distant objects recession speeds approach c , and therefore, the relativistic Doppler shift expression must be used. We define the *redshift*, z , as the fractional change in wavelength:

$$z \equiv \frac{\Delta\lambda}{\lambda} = \left[\frac{1+v/c}{1-v/c} \right]^{1/2} - 1$$

- a) The most distant quasars known have been discovered by the Sloan Digital Sky Survey (www.sdss.org). The current record holder (www.cfht.hawaii.edu/News/Quasars07/) has a redshift of 6.43. Calculate its radial velocity in terms of v/c .
 - b) Show that in the limit of $v/c \ll 1$, the above expression for z reduces to the familiar Doppler shift equation.
4. As the binary pulsar loses energy through gravitational radiation, why do the components speed up and why does the period decrease?
 5. Suppose that the Earth were collapsed to the size of a golf ball, becoming a small black hole.
 - a) What would be the orbital period of the Moon, orbiting at a distance of 4×10^5 km?

- b) What would be the orbital period of a spacecraft orbiting at a distance of 6000 km, the current radius of the Earth?
 - c) What would be the orbital period of a mini-spacecraft orbiting at a distance of 0.1 m above the black hole? Compare the orbital speed in this case to c .
6. Why is time dilation in a gravitational field equivalent to a gravitational redshift? (*Hint: Consider the definition of frequency, and how the value of frequency is affected by a redshift.*)
 7. Calculate the Schwarzschild radius of the 1.5×10^6 solar mass black hole at the center of the Milky Way Galaxy and compare it with the radius of the sun.