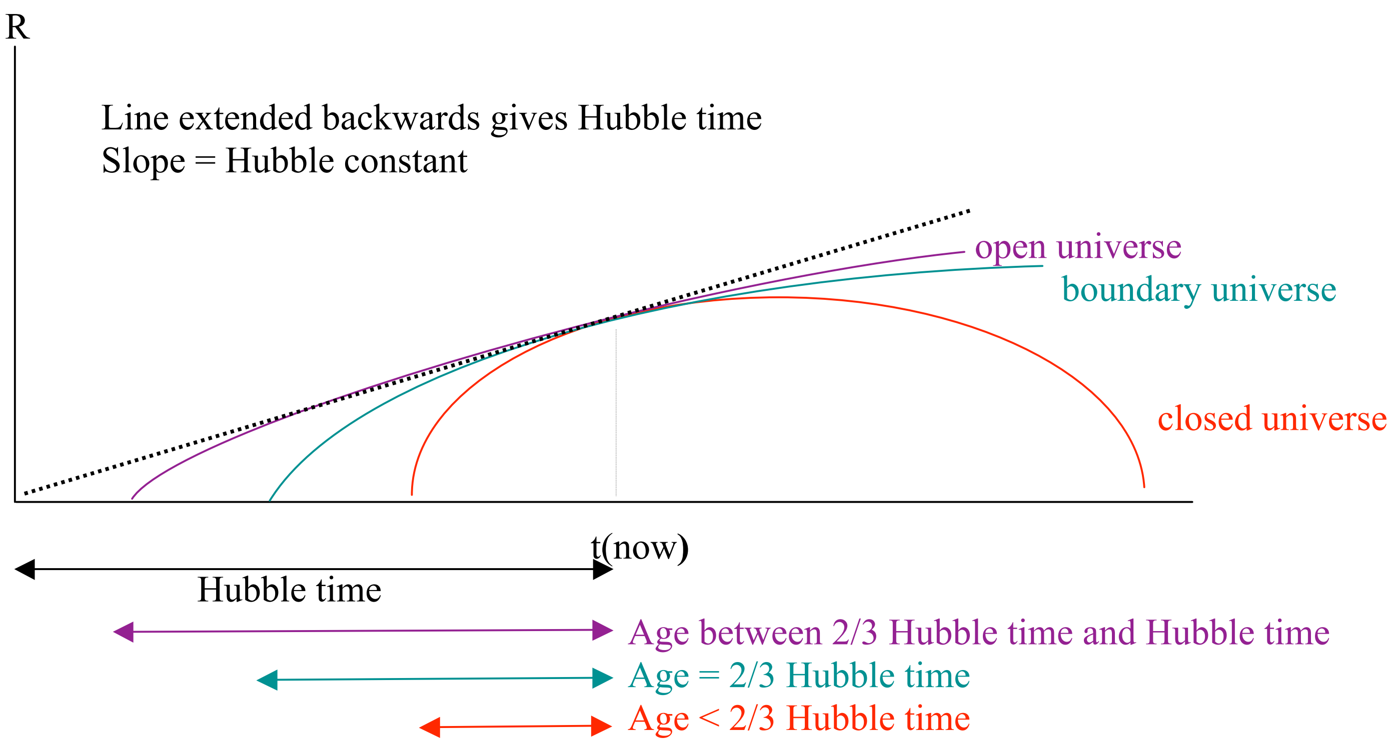
The Age of the Universe

Given the Friedmann Equation of the standard model: $H^2 - \frac{8}{3}\pi G\rho = -\frac{kc^2}{R^2}$,
what can we say about the ages of the three possible outcomes (k<0, k=0, k>0)?

We can calculate R(t) exactly for k=0 (flat geometry). In that case, $R \propto t^{2/3}$.
Now we can calculate the Hubble time (=1/H) from the Hubble constant, which is just $\frac{\dot{R}}{R}$.

For $R \propto t^{2/3}$, we have $\dot{R} = \frac{2}{3}t^{-1/3}$ so that:

$H = \frac{\dot{R}}{R} = \frac{\frac{2}{3}t^{-1/3}}{t^{2/3}} = \frac{2}{3t}$, which, solving for **t**, gives: $t = \frac{2}{3H} = \frac{2}{3}\left(\frac{1}{H}\right)$, where $\left(\frac{1}{H}\right)$ = the *Hubble time*. So, finally, we see that for a **flat universe**, the age is equal to 2/3 times the Hubble time. But what about universes with non-zero curvature, k≠0? From the graph below we can see the relative ages compared with the age of the flat universe:



A closed universe is younger than an open universe with the same observed Hubble constant (expansion rate) because the higher density in the closed universe has caused more rapid deceleration. This means that the closed universe has been expanding for a shorter time. The open universe, which has a lower density to act against the expansion, has taken longer to decelerate to the observed expansion rate.