The Cosmological Constant

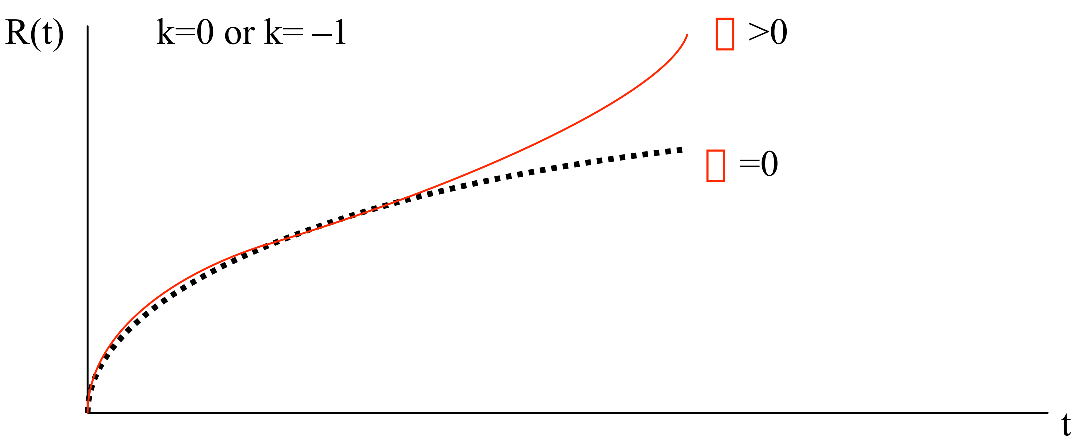
What happens to the Friedmann Equation if we include a cosmological constant? It looks like this:

$$H^2 = \frac{8}{3}\pi G\rho + \Lambda/3 - \frac{kc^2}{R^2}.$$

The parameter, Λ, the *cosmological constant*, was originally included by Einstein in the equations of GR in order to make the otherwise dynamic universe, static. After Hubble's observations led to the notion of an expanding universe, Λ dropped out of favor as unnecessary. But it's baaaaaaack! Λ may be the manifestation of **vacuum energy density**, a prediction of quantum theory requiring even empty space to be full of virtual particles popping into and out of existence. Λ would behave gravitationally like matter and energy, but exert a *negative pressure* which acts like a repulsive force. There is one important difference between Λ and gravity: while the effect of gravity decreases as the universe expands and matter is spread more thinly, the effect of Λ does not change as the universe expands.

What effect does a non-zero Λ have on our world models? A Λ>0 works **against** gravity and acts to expand the universe. (A Λ<0 **supplements** gravity, causing all models, even those with k=0 and k= –1, to collapse.)

For k=0 and k= –1, the results are similar when we have Λ>0. The R(t) graph below shows that the expansion accelerates (this will happen sooner for k= –1 than for k=0):



The case k=+1 is even more interesting. There is a critical value of Λ, below which the recollapse of the closed model is merely delayed. But at, or slightly above the critical value of Λ, the universe will be *open, even though k=+1!* There can be an extended *hovering period*, during which the scale factor remains nearly constant, but after this, the universe accelerates its expansion. Such a model is called a **LeMaître Model**.

time.