Household Size and the Demand for Food: A Puzzle Resolved?

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Abstract

When household size increases at constant per-capita income, the presence of intra-household public goods should free resources that may be directed towards private, as well as public goods. In poor countries, the expectation has been that some freed resources will flow toward food, yielding a positive relationship between food expenditures and household size at constant per-capita income. Deaton-Paxson (1998) use a standard theoretical model to generate this prediction, but then find precisely the opposite pattern in a very thorough empirical analysis across both poor and rich countries. This contradiction has become known as the Deaton-Paxson puzzle. We demonstrate that the conditions upon which their theoretical prediction is based do not survive in a generalized model. Moreover, the generalized model generates a prediction completely consistent with the empirical evidence. The "puzzle" is therefore an artifact of an overly restrictive theoretical construct.

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Introduction

In an important and widely cited paper, Deaton and Paxson (1998) identify an empirical regularity in global patterns of household expenditure that appears to contradict the predictions of standard economic theory. In particular, they find that across a wide range of countries (from the very poor to the very rich), per-capita demand for food *decreases* with household size at a constant per-capita household expenditure level. This negative correlation is strongest in the poorest countries – which appears diametrically opposed to the prediction of standard theoretical models. In their words: "Such a result is paradoxical." In this paper we generalize the Deaton-Paxson model to an *N-good* world and demonstrate that the generalized model does not in-fact predict a positive relationship between food expenditures and household size under the conditions assumed by Deaton-Paxson. Most importantly, we show that the tendency for this relationship (per-capita food expenditure and household-size) to be negative increases with the initial food-share of total expenditure. This finding yields a prediction of the precise global pattern identified by Deaton-Paxson. Specifically, our model *predicts* a stronger negative relationship between household size and per-capita food expenditure (at constant per-capita income) in the poorest countries.

The puzzle identified by Deaton-Paxson (hereafter referred to as "D-P") is more than simply an academic curiosity. A clear understanding of household scale economies is a prerequisite for the accurate assessment of household welfare, for the design of effective poverty alleviation programs, and for the construction of more accurate consumer forecasting models. The expectation that per-capita food expenditures would *increase* with household size (at a constant per-capita expenditure) is motivated by the following line of reasoning: As household size increases at a constant per-capita expenditure, scale economies in (near) public goods, such as housing, generate savings that may then be directed to other uses. Whether pure (or near pure) private goods are the target of these freed resources depends on the magnitudes of their income and substitution effects. On the one hand, the scale economies reduce the effective price of the (near) public goods, causing a substitution from private to effectively cheaper public goods. On the other hand, the income effect will tend to increase purchases of normal private goods. D-P argue that since food is normal and should have a low price relative to income elasticity, it should be the beneficiary of the resources freed by scale economies, especially in poor countries. To support this line of reasoning D-P develop a model with one strongly public good (housing) and a second strongly private good (food). They then demonstrate that if food is a pure private good a necessary and sufficient condition for per-capita expenditure to rise with household size is that the income elasticity of food exceeds the absolute value of its own-price elasticity. This, they argue, should be true for food in poor countries since food does not have close substitutes and should also be strongly normal for the poor.

The D-P paradox is that their very careful empirical work reveals the reverse outcome of the theoretical prediction outlined above. Indeed D-P demonstrate convincingly that the strongest negative correlation between household size and per-capita food expenditure is in the poorest countries, where they believe the income elasticity of food should be the greatest. It is important to note at this point, that we do not dispute the results of the exceptionally thorough empirical analysis performed by D-P. We believe the empirical regularity they uncover is both robust and important. Moreover, we agree with their treatment (and ultimate rejection) of a number of alternative explanations (such as scale economies in food purchasing etc.) that could account for this phenomenon. We do however offer a new explanation for the empirical regularity that left D-P to conclude: "…our results remain a puzzle."

Our answer to the D-P puzzle is essentially that their two-good model precludes important cross-product inter-relationships. Generalization of their model, to even three goods, reveals a structure that reconciles theory and their empirics. We demonstrate that in an N-good world an income elasticity greater than own-price is neither necessary or sufficient for a negative relationship between per-capita expenditure and household size. In particular, the model that predicts a positive correlation between per-capita food expenditure and household size (ceteris paribus) is based on the assumption that the scale economies enjoyed by larger households (in items such as shelter) generate savings that are channeled to food expenditures – particularly in very poor households near the subsistence level of food consumption .

The remainder of the paper is organized as follows: Section II outlines the basic model and its generalization. Section III discusses the model results and implications. Section IV summarizes and concludes.

II. The Model

We begin by sketching a generalized version of the Barten (1964) model presented by D-P and by Deaton and Muellbauer (1980, p. 196). Let q_i denote the *household* purchases of good "*i*," p_i its unit price, x total household income, and let there be Z distinct goods: $i = \{1, 2, . . . ,Z\}$. A household contains n members and is egalitarian in intra-household distribution. Though in the typical consumer model the distinction between per-capita consumption and expenditure is purely semantic, the intra-household "publicness" of some goods creates a wedge between per-capita expenditure and per-capita consumption. For a pure public good, the value of per-capita consumption is $p_i\, q_i$

while per-capita expenditure is $p_i\, q_i$ /n. Stated alternatively, for the pure public good the effective per-capita unit price is p_i/n, while for a pure private good the effective per-capita unit price is p_i. The per-capita household budget constraint is:

(1) $$\sum_{i=1}^{Z} \frac{p_i q_i}{n} = \frac{x}{n}$$

All goods may potentially yield scale economies within the household and these economies are captured by the function $\phi_i(n) \in (1, n)$, where $q_i/\phi_i(n)$ is effective per-capita consumption of good i. If good i is a pure intra-household public good $\phi_i(n)= 1$, and each household member "consumes" the total household purchase. At the other extreme, if good i is a pure private good $\phi_i(n)= n$, and each household member consumes $1/n$ of the total household purchase. As in D-P, we define the good-specific economy of scale measure as:

(2) $$\sigma_i = 1 - \frac{\partial \ln \phi_i(n)}{\partial \ln n}.$$

Household utility is:

(3) $$u = nv\left(\frac{q_1}{\phi_1(n)}, \quad \cdots \quad, \frac{q_Z}{\phi_Z(n)}\right),$$

and maximization of *(3)* subject to *(1)* (see D-P and/or Deaton and Muellbauer (1980,p. 200)) yields per-capita demand functions of the form :

(4) $$\frac{q_i}{n} = \frac{\phi_i(n)}{n} g_i\left(\frac{p_1\phi_1(n)}{n}, \quad \cdots \quad, \frac{p_Z\phi_Z(n)}{n}, \frac{x}{n}\right).$$

The effective per-capita price of good i is therefore $P_i(n) = p_i\phi_i(n)/n$ and the per-capita adding-up restriction is:

(5) $$\sum_{i=1}^{Z} P_i(n) g_i\left(\frac{p_1\phi_1(n)}{n}, \quad \cdots \quad , \frac{p_Z\phi_Z(n)}{n}, \frac{x}{n}\right) = \frac{x}{n}.$$

Recalling that the relevant thought experiment is how expenditure changes with n at a constant per-capita income level, differentiation of *(5)* with respect to n and some manipulation yields:

(6) $$-\sum_{i=1}^{Z} S_i\left(\sigma_i + \sum_{j=1}^{Z}\varepsilon_{ij}\sigma_j\right) = 0,$$

where $S_i = p_i\, q_i/x$ is the expenditure share of good i and ε_{ij} are price elasticities of demand (including own-price and cross-price). The change in per-capita expenditure for good i is:

(7) $$\frac{\partial p_i q_i / n}{\partial n} = -S_i\left(\sigma_i + \sum_{j=1}^{Z}\varepsilon_{ij}\sigma_j\right).$$

Clearly, since the sum of (7) over all i is zero, if it is positive for some i it must also be negative for some other good to satisfy the adding-up restriction at constant per-capita income. This is where the distinction between per-capita consumption and expenditure becomes substantive. Specifically, at constant per-capita income any reallocation of expenditure across goods associated with changing household size must cause a reduction in per-capita expenditure for some good. However, per-capita consumption can increase for all goods with increasing household size at constant per-capita income due to publicness. We now use this generalized model to examine the D-P theoretical prediction.

Suppose there are only two goods, food (*f*) and housing (*h*), and consider the sign of per-capita food expenditure as *n* increases. Using *(8)*, and that demand is homogeneous of degree zero (which implies $\varepsilon_{ff} + \varepsilon_{fh} + \varepsilon_{fx} = 0$) we obtain:

$$(9) \qquad \frac{\partial p_f q_f / n}{\partial n} = S_f\left(\sigma_h(\varepsilon_{fx} + \varepsilon_{ff}) - \sigma_f(1 + \varepsilon_{ff})\right),$$

where ε_{fx} is income elasticity of food. D-P then reason that if food is a pure private good ($\sigma_f = 0$), $\varepsilon_{fx} > |\varepsilon_{ff}|$ is a necessary and sufficient condition for *(9)* to be positive. They further assert that this condition ($\varepsilon_{fx} > |\varepsilon_{ff}|$) should be satisfied in most settings, particularly in poorer households since there are few substitutes for food, and food should be strongly normal.

De-emphasized in the D-P narrative is the accompanying requirement that $\varepsilon_{fh} < 0$. That is, food and housing *must* be complements. Are food and housing complements or substitutes? We suspect that if the question was posed in this manner, rather than whether it is reasonable to assume food price elasticity is very low relative to income elasticity in poor countries, there would less immediate consensus. Yet it is this question, *not* the relationship between the absolute value of own-price and income elasticity, that survives in a more general model. We address this question further in Section III.

To illustrate this we first add a third good, which we will refer to as good "*c*." Again using homogeneity, the change in per-capita food expenditures with household size becomes:

(10) $$\frac{\partial p_f q_f / n}{\partial n} = S_f\left(\sigma_h(\varepsilon_{fx} + \varepsilon_{ff} + \varepsilon_{fc}) - \sigma_c \varepsilon_{fc} - \sigma_f(1+\varepsilon_{ff})\right),$$

and it is immediate that even if both food and good *c* are pure private goods ($\sigma_f = \sigma_c = 0$) the D-P condition (($\varepsilon_{fx} > |\varepsilon_{ff}|$) is neither necessary nor sufficient to yield a positive equation (10). Rather, what remains critical is that food and housing are complements ($\varepsilon_{fh}<0$). Indeed, even in a *Z*-good world with one intra-household public good, the sign of *(7)* is hinges on a complementarity restriction not the relationship between own price and income elasticity. In a world with many good of varying degrees of intra-household publicness, the reasonableness of $\varepsilon_{fx} > |\varepsilon_{ff}|$ becomes even more irrelevant to the sign of the change in per-capita food expenditures as family size changes.

Since the key to changes in food expenditure with household size hinges of substitutability relationships, it is natural to consider the implication of other restrictions on the cross-price effects in a many good world. In particular, the symmetry of the substitution (or Slutsky) matrix yields, after some manipulation, yields the following cross-price restrictions in elasticity terms:

(11) $$\varepsilon_{ij}\frac{1}{S_j} + \varepsilon_{ix} = \varepsilon_{ji}\frac{1}{S_i} + \varepsilon_{jx}.$$

By homogeneity $\varepsilon_{ix} = -\sum_{j=1}^{Z} \varepsilon_{ij}$, and using this to substitute out for the income elasticity term we obtain:

(12) $$\varepsilon_{ij}\left(\frac{1-S_j}{S_j}\right) - \sum_{k \neq j}^{Z} \varepsilon_{ik} = \varepsilon_{ji}\frac{1}{S_i} + \varepsilon_{jx}.$$

For $j \neq i$ equation *(12)* defines a system of *Z-1* equations, embodying the symmetry restrictions of the substitution matrix. Together with the equation *(7)* we have *ZxZ* system that combines the change in per-capita consumption with household size at a constant per-capita income, and the symmetry restrictions of the substitution matrix.

Returning to our three good example, and allowing the possibility of *some* intra-household publicness for all goods ($\sigma_i \neq 0$), the *3x3* system described above takes the following form:

(13) $$\begin{bmatrix} -\sigma_f & -\sigma_c & -\sigma_h \\ -1 & \dfrac{1-S_c}{S_c} & -1 \\ -1 & -1 & \dfrac{1-S_h}{S_h} \end{bmatrix} \begin{bmatrix} \varepsilon_{ff} \\ \varepsilon_{fc} \\ \varepsilon_{fh} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial p_f q_f / n}{\partial n} + \sigma_f \\ \dfrac{\varepsilon_{cf}}{S_f} + \varepsilon_{cx} \\ \dfrac{\varepsilon_{hf}}{S_f} + \varepsilon_{hx} \end{bmatrix}.$$

Solving this system we obtain from the first element of the solution vector after some tedious but straightforward manipulation:

$$(14)\qquad \frac{\partial p_f q_f / n}{\partial n} = -\sigma_f - \varepsilon_{fh}(\sigma_h - \sigma_c) - \varepsilon_{ff}\left(\frac{\sigma_c}{S_f}(1-S_f) + \sigma_f\right) + \varepsilon_{fx}\left(\frac{1}{S_f} - S_h(\sigma_h - \sigma_c)\right).$$

This derivative is the sum of four terms, with the first being non-positive, but perhaps close to zero. The second term is negative when food and housing are substitutes and housing the most public good. The third and fourth terms are unambiguously positive (so long as food is normal), and also unambiguously decreasing in S_f. Since the two negative terms are independent of S_f, *(14)* is decreasing in S_f under these conditions.

Now reconsider the “Deaton-Paxson puzzle.” In their 1998 *JPE* article D-P develop a theoretical model that predicts a positive relationship between per-capita food expenditures and household size (at constant per-capita income). In their model, if food is a private good and housing has some publicness, a food income elasticity greater than its own-price elasticity is sufficient for the positive relationship. Moreover, it seems clear that this relationship between own-price and income elasticity will *strongest in the poorest countries*. The often cited “D-P puzzle” is that empirical work shows precisely the opposite pattern.

In this paper I show that the D-P puzzle is based on a miss-leading theoretical construct. While in many settings, a two-good model of consumer choice generates predictions that hold in a more generalized setting, such is not the case here. Equation *(14)* generates a prediction contrary to D-P, and in complete harmony with the empirical regularity. That is, as the food share increases the positive terms of *(14)* are diminished. The implication is that the poorest countries, where the food shares are highest, will have the greatest tendency to exhibit a negative relationship between per-capita food expenditure and household size (at constant per-capita expenditure (PCE)).

III. Discussion and Further Analysis

Do Larger Households Reap Scale Economies?

The D-P expectation of declining per-capita food expenditure with household size is really dependent on two distinct hypotheses. The first premise is that the intra-household publicness of housing generates savings for larger households at constant per-capita income. The second premise is that these freed resources will be channeled towards food, particularly in poor countries. It is possible for this chain of events to break at either stage. Clearly, a failure of the first premise renders the second moot. In seeking to understand the failure of the second premise, it is therefore essential to first establish the validity of the prior.

D-P do in fact present convincing evidence of reduced per-capita housing expenditure as household size grows (see D-P, Table 5), which strongly suggests intra-household scale economies in housing. The failure of the D-P expectation, must therefore occur at the second stage – the disposition of the saving generated by housing scale economies. As demonstrated previously, in a world with multiple private goods, a positive relationship between a pure private good and household size depends ultimately upon the existence of a complementary relationship between the private good and some public good. Alternatively, if the private good in question is a substitute for the intra-household public good(s), the correlation between the private good and household size must be negative regardless the relationship between the private good's own price and income elasticity. In the current context, the issue reduces to whether food and housing are complements or substitutes, and whether this relationship is likely to change across high and low income countries.

When considering the substitute-complement relationship of food and housing it is important to distinguish *functional* substitutes-complements and *expenditure* substitutes-complements.[1] Below some nutritional threshold, food (as an attribute bundle) may literally have no substitutes – in either a functional or expenditure sense. Within this "starvation set," to use Sen's terminology, standard economic choice models (with interior solutions) are likely inappropriate frameworks. The question then becomes; Are the poor countries (or more specifically, the surveyed individuals) in the D-P sample at a binding nutritional constraint (so that the choice model approach of both D-P and this paper are inappropriate), or are they operating in the realm where food has expenditure substitutes. Although many households in the D-P sample from Pakistan, South Africa, and Thailand are very poor by virtually any standard, a household can be poor without actually experiencing malnutrition, or having a binding food constraint. According to the UNDP Human Development Report (1995), Thailand, South Africa, and Pakistan were respectively high, medium and borderline medium/low human development countries during the surveys used by D-P.[2] Per-capita caloric intake was again respectively 2443, 2705, and 2316 – which are adequate to avoid malnutrition by most standards. Consequently, the D-P narrative, which emphasizes the lack of

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[1] It is natural to think of substitutes and complements in a functional sense: the substitutes Coke and Pepsi, or the complements tennis balls and rackets. For the theory in question, however, a focus on functional relationships may be misleading. Expenditure substitutes-complements may have no discernable functional relationship, even as attributes broadly defined.

[2] The 1995 UNDP reports primarily 1992 data.

substitutes for food may focus unduly on the functional substitutability of food, rather than the expenditure substitutability.

If the standard choice model is the appropriate framework, we have arrived at the question of complementary-substitute relationship (in an expenditure sense) between food and other goods. Some evidence on this question may be found in Deaton (1997, p.319). He estimates cross-price elasticities between the major food groups (e.g., wheat, rice, dairy, meat, etc.) and nonfoods in Pakistan. The findings are mixed with regard to complementary versus substitute relationship between food and nonfoods. Significant positive cross-price elasticities (indicating substitutes) are found between rice and nonfood, and between oils and nonfood. A significant negative cross-price elasticities (indicating complements) is found between dairy and nonfood. Cross-price elasticities between major food groups and nonfood in the Indian state of Maharashtra (Deaton 1997, p. 322) also show a mixture of substitute and complementary relationship. While neither set of estimates include a cross-price elasticity between food (as an aggregate) and housing, or other intra-household public goods, they do speak to our central point. Namely, that it is difficult to generalize, even in poor countries such as Pakistan or India, as to complementary versus substitute relationship between food and nonfoods. Yet, it is this question which dictates the theoretical prediction of the sign of the per-capita food expenditure and household size.

In addition to the lack of substitutes, D-P also presume that income (or expenditure) elasticity of food must be high for the poor. While this seems reasonable, the empirical evidence is not so clear – particularly when one focuses directly on nutrition intake. Bouis and Haddad (1992) and Behrman and Deolalikar (1987), for example, find evidence of expenditure elasticities significantly below conventional expectations, and indeed not far from zero (see also Strauss and Thomas 1995, and Deaton 1997, p. 212). This pattern can also be observed at the macro-level where one of the most robust regularities in development is the growth in the expenditure share of services at the expense of food (shares), even in relatively poor countries.

Whither the Savings?

If per-capita food expenditure is decreasing at constant PCE, some other expenditure category must be increasing. Though we have argued that the D-P samples are unlikely to be dominated by those at a binding food constraint, these households' are indeed poor, and it would natural to conjecture that other necessities would be the target of scale savings. In the D-P samples a clear beneficiary of the scale windfall appears to be clothing (see D-P, Table 5). The regression coefficient of clothing expenditures on the log of household size is positive and strongly significant in all countries – from the rich to the poor. Entertainment also exhibits positive coefficients in a number of countries, thought the effect is weaker and less uniform.[3] In the remaining narrative, we will focus on clothing as the third good – our good "c."

How does the privateness of clothing compare to food? We believe a case can made that its privateness exceeds that of food. As D-P note, intra-household economies of scale for food may exist despite its apparent privateness. Sources of intra-household food economies include bulk purchases, economies in preparations, and reduced wastage.

[3] Entertainment might seem curios beneficiary of scale savings in poor countries. Deaton (1997, p. 206), however, notes that it is not unusual to find expenditure diverted to categories such as tobacco and entertainment in poor countries -- even among those near food subsistence levels.

Even food taken away from home is likely to have some intra-household public component in the sense that those who receive the extra out-of-household meals may contribute more caloric effort to home-tasks. Clothing, on the other hand, likely provides few such spillovers. Though it is true that clothing is often passed down across children as they age, it is less usual that clothing is shared contemporaneously. Indeed, gender specific clothing conventions impose a severe restriction on apparel sharing opportunities within the household. Though this argument is certainly not conclusive evidence of the relative privateness of clothing vis-à-vis food, let us for expositional purposes assume that clothing is indeed a pure private good ($\sigma_c = 0$) and that food may exhibit some (albeit, perhaps small) intra-household publicness. Then using *(14)* the necessary and sufficient condition for per-capita food expenditure to *fall* with household size, as in the D-P empirics is:

$$(15) \qquad S_f > \frac{\varepsilon_{fx}}{\sigma_f\left(1+\varepsilon_{ff}\right)+\sigma_h\varepsilon_{fh}+S_h\sigma_h\varepsilon_{fx}}.$$

If food and housing are substitutes, all the right-hand-side denominator terms are positive. Then the stronger the publicness of food, and the stronger the substitute relationship between food and housing, the more likely we are to observe the D-P result. If, as D-P implicitly assume, food and housing are complements, the right-hand-side rises, but *(15)* is still likely to satisfied as the food share becomes large.

IV. Summary and Conclusions

In a widely cited paper, Deaton and Paxson (1998) expose a seeming contradiction between standard micro-theory and empirical regularity. Namely, *at a constant per-capita income* household per-capita food expenditures fall, rather than rise (as their theory predicts), with household size. The theoretical expectation of a positive relationship between household size and food expenditure stems from intra-household economies of scale in household size. We believe the D-P empirical work is exceptionally thorough, and we are convinced of its veracity. The contradiction must therefore have its origin in faulty theory. This paper generalizes the Deaton-Paxson model and reconciles theory and empirical results. We demonstrate that in a "*Z*" good world, the condition that generates the D-P prediction – that food income elasticity exceeds own price elasticity – is neither necessary nor sufficient for the positive relationship between household size and per-capita food expenditures that D-P anticipate. In the generalized model, the relationship predicted by D-P is shown to depend ultimately on a complementary relationship between food and housing (in an expenditure sense). We draw upon prior empirical work to argue that there is no compelling evidence to suggest the required complementary relationship, and that clothing (not food) appears a significant beneficiary of the household scale windfall.

Beyond resolving the theoretical and empirical discord regarding the sign of the relationship between household size and food expenditure, our generalized model also speaks to the global pattern in the strength of the observed negative relationship. Specifically, our model predicts a stronger negative relationship between household size and per-capita food expenditure as the food share rises. This is precisely the global pattern the D-P empirical work reveals.

Having resolved the puzzle of dissonant theory and empirics, much work remains in the realm of household scale economies, and their implications. Accurate measures of household welfare are a prerequisite to assessing the progress of development, and to the design of effective and efficient poverty alleviation programs. Though the objective of this paper has been to reconcile theory and empirics, a finer accounting of the expenditure targets of household scale economies can only assist in efficient program design.

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