# Financial Contracts Structure and Occupational Choice

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July, 2002

**Abstract**

Financial constraints and differences in entrepreneurial ability are among the key factors affecting economic performance in developing countries. Emphasizing the link between the theory microfoundations and the data, the paper considers a heterogeneous agents model of occupational choice with moral hazard under three financial contract regimes differing in their degree of financial intermediation. Using maximum likelihood estimation and statistical model comparison methods I find evidence that more advanced financial contract regimes allowing for borrowing and/or insurance provide better fit with 1997 Thailand data compared to more restrictive ones. In addition, augmenting the contracts with wealth-pooling lottery redistribution arrangements improves further the explanatory power of the model. A new numerical solution technique for incentive constrained occupational choice models based on non-linear optimization is also proposed.

**Keywords:** Economic Development, Occupational Choice, Moral Hazard, Financial Intermediation, Maximum Likelihood Estimation

**JEL Classifications:** O12, O16, G20, E2

## 1 Introduction

The recent negative economic experiences of some Asian and Latin American developing countries who have seemed to follow the 'right' macroeconomic policies have called for a reconsideration of the way policy recommendations are formed. The focus has shifted towards studying the microeconomic underpinnings of such events and identifying the key ingredients of the economic development process operating at individual or household level. A particularly important issue is the effect of these events on wealth inequality and the welfare of the poorest part of the population.

Several empirical studies[1] have found that financial constraints due to imperfect credit markets and differences in entrepreneurial ability are among the key factors affecting economic

[1] See for example Evans and Jovanovic (1989) or Paulson and Townsend (2001).

performance in developing countries. On this basis it has been argued that expanding the financial intermediation sector facilitates economic growth. Another branch of this literature[2] focuses on the degree of risk sharing observed in development economies. Basically all formal econometric tests of risk sharing reject the full insurance hypothesis. To identify the reasons for this rejection it is important to study the underlying credit and insurance contractual arrangements. The existence and functioning of formal intermediary institutions as banks and informal ones as families and kinship networks also play a significant role in this analysis. On the other hand, various theoretical models[3] have stressed the importance of incomplete information and limited enforcement in the credit markets and their implications for occupational choice in generating a developing process characterized with inequality and, in some cases, 'poverty traps', i.e. equilibria in which the economy fails to grow out of an initial subsistence or underdeveloped technological state. Unfortunately, in most cases the 'communication' between the two literatures is weak, with empirical work being purely econometric and theoretical work purely abstract.

Many important questions arise from the above empirical and theoretical findings. What are the relevant economic models explaining the observed patterns in developing countries? In particular, what degree of credit market imperfection matches the data best? Are more developed regions characterized with higher or lower degree of insurance? Are households actually constrained in their occupational choices or in the credit markets as the theoretical models stipulate? What is the time pattern of income inequality generated by the models and are re-distribution policies efficient? What are the policy predictions of these models and how can they be implemented?

The present paper attempts to answer the questions above emphasizing the link between the theory underlying the abstract models of economic development and the data. A main point of interest is the link between the structure of formal and informal financial and/or insurance contracts and institutional arrangements and the resulting consumption, investment and occupational choices. Formal, and especially informal institutions as family and local social networks which constitute the macro-level financial system, play an essential role to help support the well-being of individuals through pooling savings, arranging transfers and directing credit allocations. In dynamic aspect, the existing contractual arrangements influence the pattern of wealth distribution in the economy and may have important effects on the degree of inequality observed along the development path. With relevance to occupational choice they also have significant implications for rural-urban migration processes and entrepreneurship.

To address the above issues, I consider a model of occupational choice with moral hazard under three different financial contract regimes: savings (storage) only, borrowing/lending with limited liability, and general insurance contract (transfers to and from a financial intermediary). The main idea is, keeping the basic structure of the model unchanged, to evaluate which contractual regime[4] (i.e. what level of financial intermediation) produces outcomes that

[2]Townsend (1994), Udry (1994), Jakoby and Skoufias (1998), Rashid (1990) and Amin, Topa and Rai (1998).

[3]Banerjee and Newman (1993), Lloyd-Ellis and Bernhardt (2000), Piketty (1997), Aghion and Bolton (1997), Gine and Townsend (2002) and Townsend and Ueda (2001) are some of the most representative examples of this literature.

[4]The regime evaluation performed in the paper is similar in spirit to the work of Lehnert, Ligon and Townsend (1997) who however concentrate mostly on the theoretical implications of different regimes of financial constraints.

fit the data best. This knowledge can be then used to form predictions and policy recommendations. The model implications are evaluated using the standards suggested by economic theory on optimal allocation of risk bearing under incomplete information and neoclassical production models. The usefulness of this approach of 'meeting' the theory with the data is two-fold: on the one hand it identifies what type of theoretical models are empirically relevant and, on the other hand, it suggests directions for further improvements in our modeling of economic reality. This contribution of my research is related to the work of Paulson and Townsend (2001) with the major distinction that while they evaluate the empirical relevance of different existing occupational choice models, I concentrate on a new, more general model framework which allows a more accurate assessment of the relevance of the contractual structure. The second major contribution of my research is a methodological and technical one. So far, the vast majority of the literature on numerical solution of moral hazard models has used linear programming techniques after transforming the commodity space into a probabilistic one[5]. While having some advantages, this method suffers from heavy computational time and memory requirements. Instead, I propose a non-linear optimization method which is much faster and more accurate.

I consider an extended version of the Aghion and Bolton (1997) occupational choice model with moral hazard. Agents differ in their wealth and entrepreneurial ability and can choose one of two possible occupations (technologies) - 'entrepreneur', requiring positive amounts of investment and effort, or 'subsistence worker'[6] using only effort as an input in production. The effort levels supplied by the agents using the first technology (the entrepreneurs) are unobservable to outsiders, which creates a moral hazard problem. Agents can deposit and borrow money from a competitive financial intermediary (a bank) subject to incentive constraints and any (exogenously imposed) constraints on the structure of the financial contracts in the economy.

Given her wealth and ability endowments an agent chooses an occupation and enters in a contract with the intermediary. Due to the indivisibility of the occupational choice decision, in general there will be a non-convexity in the agent's indirect utility of wealth which suggests that pooling resources among agents and then redistributing them via lottery can be efficiency improving. The mechanism is similar to the one of rotating savings and credit associations ('roscas') observed in developing countries and thus the presence of such organizations needs to be taken into consideration as well. That is why, in each of the three financial contract regimes listed above I consider the case when such lotteries do or do not operate.

For the six resulting regimes I compute numerically the optimal contracts specifying the constrained optimal allocations of consumption, effort and investment and the occupational choices implied by them. In contrast to the existing literature, in the numerical computation of the optimal credit /insurance contracts I use a non-linear method theoretically based on the 'first order approach' (Rogerson, 1985). An important theoretical result of the paper is demonstrating the equivalence between a general insurance contract stipulating agent's consumption, effort, output and investment combinations and the implied occupational choice probabilities and a two-stage process of a lottery over wealth followed by an insurance contract *given* the optimal occupational choice implied by the wealth won in the lottery. This equivalence allows me to replace the linear programming approach usually used to solve this

[5]See for example Phelan and Townsend (1991), Prescott and Townsend (2001a,b) and Lehnert (1998).

[6]Referred to shortly as 'worker' in the rest of the paper.

type of problems with the fast and reliable non-linear method of computing the indirect utilities of wealth of choosing a given occupation and taking their convex hull if roscas are allowed by the contractual structure. This separation of the computation into two stages leads to a substantial improvement in terms of computational speed and accuracy due to the lower dimensionality of the resulting problems.

The results of the computation are used to perform both static and dynamic characterization of the model implications under the different contractual regimes and compare the welfare differences from switching between different financial structures. This corresponds to evaluating the gains or losses of altering the degree of financial regulation or expanding/contracting the degree of financial intermediation.

I find that while a higher degree of financial intermediation and insurance is welfare improving in a static sense, the low consumption variability across good and bad states in the insurance contract may lead to lower effort and/or investment for some agents, which can, in principle, have negative dynamic implications for output levels and growth. The welfare effect of lotteries is always positive as they help some ex ante poor agents to become entrepreneurs but this may be at the cost of higher degree of wealth inequality. This result relates to the numerous calls for poverty 'safety nets' by policy makers and implies that static considerations need to be carefully weighed against dynamic effects working through the wealth distribution.

In addition to the theoretical analysis described above, I use income and occupation data from Robert Townsend's Thailand economic survey to estimate statistically what mixture of credit and insurance in the financial contract matches the data best. The survey was performed in 1997 and includes 2313 households from two socioeconomically distinct regions of the country. The available data includes household wealth, occupation of head, access to various formal and informal financial institutions and education. I perform a maximum likelihood estimation of the relevant parameters in the six contractual structures, supplemented by a statistical test establishing whether or not one can reject one model specification in favor of another. The variable which is fitted is occupational choice. The econometric analysis is performed for the whole sample as well as for various sub-samples of the data chosen on regional, wealth, financial participation or educational basis. In particular, in relation to the large literature about the degree of risk sharing in developing countries, it is interesting to see if higher degree of development is consistent with a contractual structure featuring more or less insurance.

The model comparison results show that the savings only financial contract is rejected in favor of both the borrowing and lending and the insurance contracts across most data sub-samples, which provides confirmation for the presence of some degree of consumption insurance. Nevertheless, the degree of insurance present is not necessarily the maximum possible as the more limited borrowing and lending regime cannot be rejected in favor of the general insurance and transfers one. The importance of including the possibility for wealth lotteries (roscas) is demonstrated convincingly by the empirical analysis: the model specifications with lotteries provide better fit than their counterparts without lotteries. These differences in the best fitting model specifications can be used to infer theoretically and empirically backed policy recommendations. Finally, to test the dynamic implications of the three model regimes, I use a mix of calibration and maximum likelihood estimation techniques using Thai national statistics data. The results demonstrate that the model is capable to match the time paths of income growth, inequality and the fraction of entrepreneurs in the data.

The rest of the paper is organized as follows. Section two describes the model and the three basic types of financial contracts studied. It also contains a description of the numerical solution techniques used to solve the model and a basic characterization of its static and dynamic implications using simulations. In the third section I perform a maximum likelihood estimation of each of the model regimes followed by statistical comparisons between them to establish which financial contract regime fits the data best. Section four studies the time dynamics generated by the model and compares them to the data using a mix between estimation and calibration of the model parameters. Finally, section five concludes.

# 2 Model Description

The model of the paper is an extended version of the models of Aghion and Bolton (1997) and Lehnert (1998). Agents, $i \in I$ are heterogeneous in their consumption good endowments (to be referred as 'wealths'), $a_i$ and entrepreneurial ability, $\theta_i$ and have preferences given by $u(c, z)$, where $c$ is consumption and $z$ is labor effort. The utility function is assumed strictly increasing in its first argument, strictly decreasing in the second and concave. There are two available technologies in the economy through which the single consumption and investment good in the economy is produced. The first technology involves investing a positive amount, $k$ and can be written as $p^e(q|\theta, k, z)$, i.e. the probability of achieving output level $q$ given effort $z$, investment $k$ and the agent's entrepreneurial ability $\theta$. The probability $p^e$ is assumed increasing in all its arguments. We call this technology 'entrepreneurial' and the agents who use it 'entrepreneurs'. The second technology can be operated without making any investment (i.e. $k = 0$) and can be represented as $p^w(q|z)$ - the probability of achieving output $q$ given effort $z$ and no investment. As before, $p^w$ is increasing in $z$. The agents using this technology are called 'subsistence workers' or, in brief, just 'workers'. This technology is meant to be interpreted as subsistence agriculture technology, hence the assumed stochasticity of output. This is a generalization of the Aghion and Bolton (1997) and Lehnert's (1998) models as they assume that zero investment leads to zero output with certainty.

Except in their input requirements the two technologies differ in the information that they provide to outsiders, i.e. the degree of contractibility with respect to the inputs employed. In particular, I assume that the effort levels supplied by the entrepreneurs are unobservable to any outside parties, which leads to a classic moral hazard problem in all non-autarkic settings where entrepreneurs may enter contracts with other agents. It is assumed, however, that intermediaries can design contracts taking ability into consideration thus disregarding potential adverse selection complications. In contrast, workers' efforts are assumed observable[7] and thus can be stipulated directly and enforced through an appropriate contract.

Given their wealth, ability, and the possible financial contracts with the bank, agents choose the technology which would provide them with higher expected utility. Thus their maximization problem can be solved in two steps: first compute the expected utility yielded by each occupation and second choose the occupation that yields higher expected utility. Agents live for one period and leave as bequest a constant fraction $s$ of their end-of-period wealth to their single child[8] which becomes the latter's wealth endowment. Thus there is no

[7]This assumption is not very crucial for the results and can be interpreted as stemming from the fact that the subsistence technology involves performing some simple and easy to control tasks, whereas the entrepreneurial one is much more complicated.

[8]This is a standard assumption in the occupational choice literature (see Banerjee and Newman, 1993 or

population or technological growth in the model and all interesting dynamics are transitionary. This is by no means a shortcoming as the model is designed for studying developing economies which are presumably not in a steady state.

There exists a risk-neutral competitive financial intermediary (bank) in the model economy with which agents can enter into financial contracts involving borrowing, lending and/or insurance. As stated in the introduction, I am interested mainly in studying the static and dynamic effects of varying the structure or type of these financial contracts. In particular, from a static one-period perspective, I will be interested how consumption, effort and investment depend on the contractual structure. From a dynamic perspective I will be looking at the effects on inequality, income levels and growth rates, the fraction of entrepreneurs as well as the interest rate. I will be also interested in the empirical relevance of the contract structure with respect to its fit with the data.

For analytical and computational simplicity I assume that there are two possible output realizations for each occupation (two states of the world). For the entrepreneurs, output can take on two values: $q = \theta q_h$ and $q = \theta q_l$, with $q_h > q_l$, whereas for the workers output is either $w_h$ or $w_l < w_h$. I further normalize $q_l = 0$ and $w_l = 0$ which is interpreted as failure of the business project or the agricultural crop. Notice that higher entrepreneurial ability leads to higher output levels, i.e. for the same levels of investment and effort a more able entrepreneur would achieve higher expected output.

## 2.1 Types Of Financial Contracts

The paper concentrates on three alternative types of financial contracts (regimes) between the agents and the intermediary differing by their ability to serve as consumption smoothing devices. In each of the three cases the type of feasible contracts is imposed exogenously for simplicity but can be justified instead by transaction costs, lack of appropriate institutional mechanisms, government intervention, etc.

**1. Savings (Storage) Only Contract**

The first type of contract is a savings (storage) only, i.e. no borrowing is possible. Under this contract agents can only deposit (store) any amount of the consumption good with the intermediary earning a fixed gross return of $r$ in the end of the period. In the static analysis the interest rate $r$ is taken as exogenously given but this assumption is relaxed later on in the dynamic setup. Given the assumptions made above, the maximization problem of an entrepreneur with wealth $a$ and ability $\theta$ under this contractual structure looks as follows:

$$\begin{aligned} &\max_{z,k} \; p^e(z,k)u(c_h,z) + (1-p^e(z,k))u(c_l,z) \qquad (1)\\ &s.t. \;\; c_h = \theta q_h + r(a-k) \\ &\qquad c_l = r(a-k) \\ &\qquad 0 \le k \le a \end{aligned}$$

where $c_h$ and $c_l$ are the levels of consumption in the case of success or failure. The last constraint states that only saving and no borrowing is possible, thus all investment must be self-financed if only this type of financial contracts is feasible in the economy. Consumption

Piketty, 1997) which can be justified formally by assuming a 'warm-glow' preferences as in Andreoni (1989).

in each state of the world is just the realized output plus any return on savings. Despite the unobservability of effort there is no moral hazard problem under this type of contract as there is no risk for the bank.

Similarly, subsistence workers solve:

$$\begin{aligned} \max_{z} \ & p^w(z)u(c_h, z) + (1 - p^w(z))u(c_l, z) \\ s.t. \ & c_h = w_h + ra \\ & c_l = ra \end{aligned} \tag{2}$$

As there is no investment all wealth is deposited with the intermediary.

**2. Borrowing/Lending Contract**

This is a standard borrowing/lending contract between an agent and the bank: the agent either deposits some amount of money in the bank in the same way as in the savings contract above and earns $r$ or she can request a loan from the intermediary. The bank announces a repayment amount for each state of the world which in general would depend on the size of the desired loan. The agent takes the repayment schedule as given and decides how much to borrow. I assume that there is limited liability, i.e. agents' consumption cannot be negative, which means that in the case of project failure the agent would not be able to repay anything back to the bank. The latter would take this into account by setting the required repayment under failure to zero and adjusting the repayment due under success by raising the effective interest rate enough so that it breaks even on the loan (i.e. earns in expected terms at least the reservation interest rate $r$). Clearly, the repayment in case of success would then depend on the probability of success of the agent's project. A lending entrepreneur solves the problem (1) exhibited above without imposing the constraint $k \leq a$. To be consistent with lending, the optimal investment amount $k^*$ at the solution to this problem must be not bigger than agent's wealth $a$. In case that $k^* > a$ the agent would be willing to borrow some amount from the bank. Given a required repayment rate under success of $R$, the maximization problem of a borrowing entrepreneur looks as follows:

$$\begin{aligned} \max_{k,z} \ & p^e(z,k)u(c_h, z) + (1 - p^e(z,k))u(c_l, z) \\ s.t. \ & c_h = \theta q_h - R(k - a) \\ & c_l = 0 \end{aligned} \tag{3}$$

Remember that effort is unobservable to the bank so it must induce it by designing the repayment rate $R$ accordingly. We assume that in equilibrium the bank must earn at least its reservation return of $r$ on each loan, i.e. $R$ must solve:

$$R = \frac{r}{p^e(\hat{z}(R), \hat{k}(R))}$$

where $\hat{z}(R)$ and $\hat{k}(R)$ are the solutions of the entrepreneur's problem above (3) taking $R$ as given. The interpretation of the above condition is that the bank should charge an interest rate higher than $r$ in case of success, which happens with probability $p^e$, to offset the zero return it makes under failure. Clearly, such a policy would lead to suboptimal effort levels on the part of the entrepreneurs as it basically taxes success reducing the incentive of agents to supply more effort. The second-best nature of the solution stems from the moral hazard problem and the assumption of limited liability.

The workers solve exactly the same problem as before, (2) as they do not invest and thus do not need to borrow from the intermediary. Obviously the borrowing/lending contract provides more opportunities for consumption smoothing for the agents who choose the entrepreneurial occupation compared to the savings only contract and as such it (weakly) Pareto dominates the savings contract. However, it is still quite restrictive as it provides no insurance transfers to the agents in case of project failure.

**3. General Insurance/Transfers Contract**

The last financial contract type allows all possible types of state contingent transfers between the agent and the intermediary to achieve maximum consumption smoothing. Under full information it is well known that, since the agent is risk-averse and the intermediary is risk-neutral, the optimal contract will be the one providing equal consumption to the agent in both states of the world (success or failure). This is achieved by the agent making a transfer to the intermediary in case of success and the intermediary making a transfer to the agent in case of failure providing full insurance. This is exactly the case with the contracts between the workers and the bank. Since entrepreneurial effort is unobservable, however, full consumption smoothing is not possible as it is not incentive compatible. Indeed if the agent is promised the same consumption under success and failure she will always choose the lowest possible level of effort (zero) as effort is costly and output will be always zero. What would happen instead is that the bank would provide less than full insurance which however still Pareto dominates the borrowing/lending contract described above.

Under this type of financial contract I assume that, due to competition, bank's profits are going to be zero in equilibrium and thus we can think of it as maximizing the expected utility of its customers as a function of their wealth and ability subject to breaking even and incentive compatibility (where applicable). What the bank does in fact is set $k, c_h$ and $c_l$ and recommend an effort level $z$ which maximize the expected utility of the agent. If the bank finds optimal to set $k > 0$ the agent will be an entrepreneur, while if $k = 0$ she will be a worker. Potentially, the occupation assignment inherent in the choice of $k$ can be in the form of a lottery, i.e. the bank may find optimal to assign $k = 0$ with some positive probability and some level of $k > 0$ with the residual probability. In general there might be similar lottery assignments of effort or consumption levels for given occupation and state of the world, although it is easy to see that the assumed concavity of the utility function would make such lotteries degenerate at the optimum.

The usual way of writing[9] the general insurance/transfer contract described above involves introducing new variables in the form of the probabilities that a particular consumption, $c$, output, $q$, effort, $z$ and investment, $k$ allocation is assigned as a function of the agent's wealth, $a$ and ability, $\theta : \pi(c, q, z, k|a, \theta)$. The consumption level in such an assignment will be in general a function of the output level due to the moral hazard problem. Notice that these probabilities can be also interpreted as lotteries over different consumption, output, investment and effort allocations that are being offered to the agent. Following Lehnert (1998) let us call this type of lotteries 'input lotteries'. Introducing the $\pi's$ allows us to write the bank's problem as the

[9]See Prescott and Townsend (1984a and 1984b), Phelan and Townsend (1991), Lehnert (1998), Paulson and Townsend (2001), and Karaivanov (2001).

following linear program[10] in $\pi(c,q,z,k|a,\theta)$:

$$\max_{\pi(c,q,z,k)\geq 0} \sum_{c,q,z,k} \pi(c,q,z,k)u(c,z) \tag{4}$$

s.t.

$$\sum_{c} \pi(c,\bar{q},\bar{z},\bar{k}) = p(\bar{q}|\bar{z},\bar{k}) \sum_{c,q} \pi(c,q,\bar{z},\bar{k}) \quad \text{for all } \bar{q},\bar{z},\bar{k} \tag{5}$$

$$\sum_{c,q,z,k} \pi(c,q,z,k)(c-q) = r \sum_{c,q,z,k} \pi(c,q,z,k)(a-k) \tag{6}$$

$$\sum_{c,q} \pi(c,q,z,k)u(c,z) \geqq \sum_{c,q} \pi(c,q,z,k) \frac{p(q|z',k)}{p(q|z,k)} u(c,z') \quad \text{for all } k>0, z, z', \tag{7}$$

$$\sum_{c,q,z,k} \pi(c,q,z,k) = 1 \tag{8}$$

where

$$p(q|z,k) = \begin{cases} p^e(z,k) & \text{if } k>0 \\ p^w(z) & \text{if } k=0 \end{cases}. \tag{9}$$

Let us describe the problem above in a more detailed way. The objective is simply the expected utility that an agent would get at the assigned combinations $(c,q,z,k)$. The first constraint, (5) ensures that the probabilities constituting the contract, $\pi(c,q,z,k)$ are consistent in Bayes sense with the production technology $p(q|z,k)$. The second constraint is the break-even (zero profit) condition for the bank, stating that on average all outgoing transfers for the bank must equal all incoming transfers. The following constraint, (7) is the incentive compatibility constraint which ensures that the recommended effort level will be indeed implemented by the agent. Basically it states that the expected utility of implementing the recommended level of $z$ (the left hand side) must be bigger or equal to the expected utility of deviating to some alternative effort level $z'$. Finally, the last constraint, (8) ensures that the probabilities sum to one.

## 2.2 Solution Techniques

In this section I describe the techniques used to solve the optimization problems for the three types of financial contracts stated above. All problems were solved numerically as in general closed form solutions cannot be obtained. Since the purpose of the paper is comparing the implications of the financial contract regimes and taking them to the data, I have tried to use more general and flexible parametrizations and functional forms for the fundamentals of the models, instead of aiming for analytic simplicity and risking to obtain only restricted conclusions. The functional forms used in the computed solutions are as follows:

$$u(c,z) = \frac{c^{1-\gamma_1}}{1-\gamma_1} - \lambda \frac{z^{\gamma_2}}{\gamma_2} \tag{10}$$

[10]This linear program is known in the literature as a 'moral hazard problem with lotteries'.

$$p^e(z,k) = \frac{k^\alpha z^{1-\alpha}}{1+k^\alpha z^{1-\alpha}} \qquad p^w(z) = \frac{z}{1+z}$$

The utility function[11], $u(c,z)$ displays constant relative risk aversion in consumption represented by the parameter, $\gamma_1 \geq 0$[12]. This is a generalization of the functional forms used in Aghion and Bolton (1997) and Lehnert (1998) who impose risk-neutrality ($\gamma_1 = 0$). Allowing for risk aversion has the important consequence of making the agents demand insurance in case of failure which not all of the financial contracts described above are able to provide and thus their implications can be distinguished more easily. The other two parameters, $\lambda > 0$ and $\gamma_2 \geq 0$ determine respectively the relative disutility of effort and the degree of aversion to variations of effort and also represent a generalization of the quadratic effort cost used in Aghion and Bolton or Lehnert.

The production (probability of success) functions differ from the ones used in Paulson and Townsend (2001) by allowing $k$ and $z$ to be chosen on the whole positive ray of the real line, $[0,\infty)$ instead of restricting them to lie on $[0,1]$. The advantage of this approach is that one need not worry about corner solutions for effort and investment which can make the interpretation of the results complicated and which have no economic meaning endogenous to the model. The parameter $\alpha \geq 0$ provides further flexibility with respect to the relative importance of investment and effort for achieving the high output level.

Given the above functional forms the optimization problems for each occupation in the savings only and the borrowing/lending setups are solved employing standard non-linear techniques based on the quadratic programming approach. The relative simplicity of the problems allows substituting the constraints in the objective and transforming them to standard unconstrained optimization problems and simple non-linear equations[13].

### 2.2.1 Wealth Lotteries

After solving the utility maximization problem for the workers and entrepreneurs we can derive their indirect utility functions, $v^E(a,\theta)$ and $v^W(a)$. In this section I will assume that ability is fixed at some value and thus interpret the indirect utility functions of wealth only. Since consumption depends directly on wealth in a linear way and since $u(.)$ is concave in $c$ it is clear that, by the envelope theorem, $v^E$ and $v^W$ are concave in $a$ for both the savings only and the borrowing/lending setups. Since, given her wealth level, $a$ an agent would choose the occupation that provides her with higher indirect utility, the utility realization she would obtain is the outer envelope of $v^E(a)$ and $v^W(a)$, i.e. $v(a) \equiv \max\{v^E(a), v^W(a)\}$. Clearly, even though $v^E$ and $v^W$ are concave in wealth, $v(a)$ will not be concave in general[14]. The reason for the non-concavity is the discreteness inherent in the occupational choice problem - an agent is assumed to be able to hold only one occupation at a time, i.e. she cannot split her time endowment between the two occupations. Clearly then an ex-ante lottery in wealth can

[11] The form of the utility function used here is basically the same as the one in Paulson and Townsend (2001) except for small notational differences.

[12] The case $\gamma_1 = 1$ is interpreted as $\ln c$.

[13] More details on the numerical computation methods used, including the actual Matlab codes used to compute the solutions, are available from the author upon request.

[14] Unless one of the occupations provides higher utility for all wealth levels.

restore concavity[15]. Basically what such a lottery does for an agent with wealth $a$ that puts her in the non-concave region of $v(a)$ is offer her, with some probability, a wealth level, $a_1 > a$ at which she would be on the right concave region of $v$ and, with the residual probability, a wealth of $a_2 < a$ at which she would be at the left concave region of $v$ (see fig. 1).

Obviously the above arrangement would provide agents with higher expected utility. A natural counterpart for such wealth lotteries exists in various developing countries in the form of the so-called 'rotating savings and credit associations', or 'roscas', which allow individuals to pool their wealths and then assign the pooled wealth by a lottery to one of them to buy a durable good or implement certain investment project. Roscas have been studied extensively in the development literature both theoretically and empirically[16] . In our setup we can think of them as having several individuals with given wealth level, $a$ at which $v(a)$ is convex, pool their wealths and then some fraction of them being assigned via lottery to a higher wealth level and the rest to a lower wealth level such that the total pooled wealth is exhausted. Notice that the wealth lotteries create inequality, i.e. if we take a number of agents with equal wealths and they play the lottery, as a result they are split into two groups ('losers' and 'winners') with potentially very different wealth levels. This shows that sometimes welfare improving arrangements can lead to increased inequality.

In the following sections I will look at the implications of the models described above when such ex-ante wealth lotteries ('roscas') are or are not possible. The no-lottery contracts simply state that the agent chooses the occupation that provides her with higher utility given her wealth but she does not have the opportunity to make this wealth higher by entering a lottery. Thus, I will be interested to see how allowing for the agents to participate in wealth pooling schemes can help alleviate the constraints imposed by the structure of the feasible financial contracts with the intermediary. The welfare effect of the lotteries will also be studied.

### 2.2.2 Equivalence of Input and Wealth Lotteries

Let us now describe in more detail the technique used to solve the general insurance/transfers problem, since, as mentioned in the introduction, it represents a new approach for solving moral hazard problems with lotteries. The commonly used in the literature method of solving these types of problems numerically is to choose discrete grids for the possible values that $c, z$ and $k$ can take and solve the resulting constrained linear program with respect to the probabilities $\pi(c, q, z, k)$. Although this method is very general and does not rely on almost any assumptions about the functional forms used, it has several major drawbacks. First, even with very modest grid sizes, e.g. 10 points each, the dimension of the problem expressed in the number of variables and constraints is very high[17]. This 'dimensionality curse' requires a lot of computer memory and time especially when one wishes to use denser grids. If too coarse grids are used the quality of the solution deteriorates and the results may be unreliable. A second shortcoming of the linear programming approach which is related to the first is that discretizing the problem using grids may introduce the so-called 'grid-lotteries' which arise when the real solution is between two of the grid points. These lotteries have no economic

[15] Among the previous literature that has noted this property of discrete choice models are Rosen (2002), Lehnert (1998).

[16] See Besley, Coate and Loury (1993), Besley and Levenson (1996), Calomiris and Rajaraman (1998).

[17] With 10 grid points each for $c, z$ and $k$ and 2 output levels the number of variables in the program is 2000 and there are 2002 constraints.

interpretation and may 'contaminate' the solution. Finally, if the end grid points turn out to be chosen incorrectly it is possible to obtain a corner solution when in fact the true one is not. Because of these reasons I have opted to use the particular structure of the problem and propose a new method of solving such moral hazard problems with lotteries.

The main innovation in my approach is that it does not rely on linear programming solution methods but instead transforms the original problem into an equivalent one involving solving two non-linear constrained optimization problems, one for each occupation. The basic idea is to use a result due to Cole and Prescott (1993) which shows that solving the linear program with lotteries (4) is equivalent to solving the optimization problems for the two occupations separately and allowing a lottery over wealth, $a$ only, as described above.

Under the general insurance/transfers contract the problem of the bank when dealing with an entrepreneur looks as follows:

$$\max_{k,z,c_h,c_l} \; u^*(z,k) \equiv p^e(z,k)u(c_h,z) + (1-p^e(z,k))u(c_l,z) \tag{11}$$

$$s.t. \qquad z \in \arg\max_z \; u^*(.,k) \tag{ICC}$$

$$p^e(z,k)c_h + (1-p^e(z,k))c_l = r(a-k) + p^e(z,k)\theta q_h \tag{BE1}$$

The interpretation is that the bank sets $k, c_h$ and $c_l$ and recommends an effort level $z$ which maximize the expected utility of the entrepreneur subject to two constraints: first, the recommended effort level must be indeed the optimal one to be chosen by the agent given $k, c_h$ and $c_l$, and second, the bank must break even, i.e. the expected outlays (the left hand side of (BE1)) must be equal to the expected income (the right hand side). Intuitively what happens is that the agent commits to giving all her wealth and output to the bank[18] and in exchange obtains consumption transfers of $c_h$ or $c_l$ depending on the state of the world.

The problem of the intermediary when dealing with a worker is similar, except that there is no need of the incentive compatibility constraint (ICC) as effort is fully observable. Due to the concavity of the utility function it is optimal to set consumption and effort equal in the two states. Thus we can write the problem of the bank simply as[19]:

$$\max_{z,c} \; u(c,z) \tag{12}$$

$$s.t. \; c = p^w(z)w_h + ra \tag{BE2}$$

The second constraint is simply (BE1) written for this problem since now $c_h = c_l$ and $k = 0$.

The following result provides the basis of the solution technique used to solve for the optimal contracts in the general insurance setup.

**Proposition 1 (Equivalence between Input and Wealth Lotteries)**

*The optimal effort, investment and consumption levels*[20] *corresponding to the solution of the linear moral hazard program, (4) (input lottery problem) coincide with the solutions of the optimization problems (11) and (12) combined with an ex-ante lottery over wealth only (wealth lottery problem).*

[18] They are observable and it is assumed that it is not possible to hide any wealth or output.

[19] Note that we could have written the problem in its general form as in (11) and then show that the solution satisfies $c_h = c_l$. We take a short-cut instead and impose the latter condition to make the problem simpler.

[20] I.e. the optimal contracts between the agents and the intermediary.

The above proposition states that under our assumptions we can replicate the solution to the problem with input lotteries by using only a wealth lottery among the agents with a given $a$. This implies that the only contribution of the input lotteries is to allow agents to engage in implicit lotteries over wealth. The agents find it optimal to participate in such wealth lotteries as their indirect utility of wealth has convex regions induced by the indivisibility of occupational choice.

Showing the equivalence between the solutions to the input lottery and the wealth lottery problems as described above is just the first step of the computational algorithm to solve for the optimal general insurance/transfers contract. The main purpose of this step is to reduce the dimensionality of the problem by making possible to solve the problems of agents of different occupations separately. Moreover, it was shown that only wealth lotteries are sufficient to reproduce the optimal contracts, i.e. we can disregard any other types of possible input lotteries.

The next question is how do we solve the problems (11) and (12) and how do we compute numerically the wealth lottery which is equivalent to convexifying the $v(a)$ function. One possibility is to use the linear programming approach but this has all the disadvantages discussed above. Instead, I solve the problems in their original, non-linear form. The worker's problem (12) is a standard non-linear maximization program and can be solved by conventional nonlinear optimization methods. To solve the entrepreneur's problem (11), however, we need to transform the incentive compatibility constraint (ICC) into a more manageable form. The standard way to do this is to replace it by the first order condition of the maximization problem with respect to effort. This is known as the 'first order approach' and, as demonstrated by the literature[21], is by no means universally valid but requires some restrictive properties to be satisfied by the probability function determining output as a function of effort. The next result shows that under our assumptions the first order approach is valid, i.e. the solution obtained by replacing the maximization problem in (ICC) by its first order condition is a maximum of the objective function of (11).

**Proposition 2 (Validity of the First Order Approach)**

*The production function $p^e(q|z,k)$ satisfies the monotone likelihood ratio property (MLRP) and the convexity of the distribution function condition (CDFC) implying that the first order approach is valid.*

The proof of the sufficiency of the MLRP and CDFC for the validity of the first order approach is due to Rogerson (1985). In general, in a setting with more than two output levels the two conditions become quite restrictive but in our case they are easily satisfied. The numerical solution method proposed in the paper does not necessarily require the use of the first order approach although if the latter is valid the computation is sped up considerably.

Having shown that the first order approach is valid, we can replace the ICC by the first order condition $\frac{\partial u^*}{\partial z} = 0$ which is a non-linear equality constraint in $z$, $c$ and $k$ and use non-linear optimization methods to solve the problem. The only remaining issue is the wealth lottery, which is equivalent to convexifying $v(a) = \max\{v^E(a), v^W(a)\}$, i.e. taking its upper convex hull, $v^C(a)$. This step is performed by choosing a dense discrete grid on wealth $a$, computing the value of $v(a)$ at the grid points by solving directly the non-linear problems

[21]See for example Mirlees (1975) and Rogerson (1985).

(11) or (12), and computing the upper convex hull of the points with coordinates $(a_j, v(a_j))$ where $a_j$, $j = 1,..n$ are the points in the grid. After the convex hull is computed we can evaluate it at any value of $a$ by using a cubic spline approximation[22]. Knowing the convex hull of $v$, however gives us only the indirect utility value obtained by an agent with a given wealth $a$. To get the optimal contract between the agent and the bank we also need to know at which interval for $a$ (if any) wealth lotteries will be used at optimum. This amounts to solving for the points $A$ and $B$ on fig. 1. In the numerical algorithm these two points are found by comparing the values of the $v^C(a)$ with $v(a)$ and taking the first and last grid point at which they differ. Once $A$ and $B$ are known, all agents with $w \in (w_A, w_B)$ will optimally participate in a wealth lottery with payoffs the wealth levels $w_A$ and $w_B$, with the probabilities of getting each level depending on the relative distance between $w$ and $w_A$ and $w_B$. The rest of the agents do not participate in wealth lotteries as the indirect utility function is concave at their wealth endowments.

The non-linear method of solving for the optimal insurance/transfer contract described above has the following advantages over the standard linear programming method used in the previous literature. First, no grids are used which on the one hand reduces the memory and computational time requirements[23] and on the other hand improves the solution precision as the optimization is done on continuous as opposed to discrete sets of values. Second, lotteries are used only and exactly when they are needed, in contrast to the input lottery $\pi$-formulation in which grid lotteries are prevalent and can affect the results. Finally, the results of the optimization are not in the form of the artificial objects $\pi(c, q, z, k)$ which are hard to interpret from an economic point of view but instead we directly obtain the assigned consumption, investment and effort levels.

As in the savings only and the borrowing/lending regimes, it is also possible to solve the problem above when no wealth lotteries are allowed (i.e. when the agents or the intermediary cannot implement rosca-type arrangements). In the input lottery interpretation this is equivalent to restricting the possible input lotteries to values of one and zero only[24]. Thus, in total, there are six possible financial contract regimes which are studied in the paper[25]:

1. Savings only contract without wealth lotteries (SNL)
2. Savings only contract with wealth lotteries (SL)
3. Borrowing/Lending contract without wealth lotteries (BNL)
4. Borrowing/Lending contract with wealth lotteries (BL)
5. Insurance/Transfers contract without wealth lotteries (INL)
6. Insurance/Transfers contract with wealth lotteries (IL)

[22] The full details of the numerical algorithm plus the program codes that implement it are available from the author upon request.

[23] The relative performance in terms of computational speed is about five to ten times higher for the non-linear approach proposed here compared to the linear programming approach using standard non-commercial maximization routines and average grid sizes.

[24] For more details see the discussion in Lehnert (1998) who follows this approach as he uses the linear programming solution method.

[25] The abbreviations in the brackets refer to the regime names used in the tables.

## 2.3 Model Implications

In this section I describe the results of the numerical simulation of the model. All three types of financial contracts are considered, with and without including wealth lotteries. I start with some static properties of the model solution and then turn to the model's dynamic implications under the six different contractual regimes from above. This section is intended just as an illustration of the model implications and as such bears no relevance to the data. Nevertheless, the benchmark parameters are chosen to match the value ranges implied by the data. A maximum likelihood estimation of the model with respect to data and a calibration of its dynamics are performed in the next sections of the paper.

Table 1 lists the benchmark parameters at which the model is simulated for each of the three contract types. Ability, $\theta$ is normalized to 1 as it plays no role in the analysis in this section. The savings rate, $s$ is set to .34 which is also in the ballpark of the levels observed in reality in Thailand.

First, let us look at the static (per period) implications of the model. Figures 2a, 2b and 3 characterize the model solution for the three different types of contracts. In the first row of panels in fig. 2a we see the indirect utility (value) functions for workers and entrepreneurs in each of the three setups. They are all concave with the exception of the one for the borrowing entrepreneurs. The non-convexity there occurs due to the fact that the limiting liability constraint binds at low wealth levels. The savings only graph illustrates clearly the case for Pareto improving wealth lotteries as described above and in fig. 1. Notice also that the insurance contract provides strictly higher indirect utility for both occupations (especially at low wealth levels) due to the consumption smoothing it provides. The borrowing/lending contract for the entrepreneurs also dominates the savings only one with this effect being strongest at low wealth levels as agents are allowed to borrow.

Consumption is depicted in the second and third rows of graphs in fig. 2a. We see that, under the insurance contract, the gap between consumption in the high and low states is smallest[26]. Under the borrowing/lending (B/L) and savings contracts consumption is zero for small values of $a$ in the low state as no insurance is present. Notice also the jump in consumption under success in the B/L contract when the limited liability stops binding and the effective interest rate drops from $R$ to $r$. On fig. 2b we see the behavior of effort, investment, borrowed amount and the probability of success as functions of wealth. In general, effort is decreasing in wealth except for low levels of $a$ in the B/L and savings setups. The intuition is that agents are both poor and financially constrained in that wealth interval and thus their only way to obtain consumption is to work hard. Investment is slightly increasing in wealth since it becomes a cheaper substitute for effort as the agent becomes richer. Once again, notice the sharp drop in effort and investment when the limited liability constraint ceases to bind. Across the models notice that entrepreneurial effort is on average lowest in the insurance/transfers setting. This is due to the fact that, given the consumption smoothing provided by the intermediary, the agents do not have to work very hard to avoid the bad state of the world. The probability of success is generally increasing in wealth as more investment is possible. By construction no (positive) borrowing is possible in the savings only setup, whereas agents with low wealths are being lent money by the intermediary under the insurance and the B/L contracts and can potentially become entrepreneurs.

Finally, figure 3 depicts the differences in the indirect utilities achieved under the three

[26]It is actually zero for the workers as no moral hazard is present and full insurance is possible.

financial contract regimes and also the cases when wealth lotteries are and are not allowed. We see that the biggest differences occur for the low wealth agents who are financially constrained in the savings and borrowing/lending settings. As expected, allowing for wealth lotteries improves the utility only of the agents with wealths in the non-concave portion of the upper envelope of the indirect utility functions for the two occupations.

To measure in real terms the magnitude of the differences in utility obtained under the different contractual structures at the given parameters, I compute the utility equalizing consumption supplements (see Table 2) which would make an agent indifferent under two different setups. For example, we see that banning wealth lotteries in the borrowing/lending setup would require a .74% raise in consumption on average (over all wealth levels) to make an agent indifferent with the case when lotteries are allowed. We see that the direct welfare improving effect of lotteries is not very strong at these particular parameter values but it can potentially be much stronger if risk aversion is higher. The comparison across the three types of financial contracts reveals much larger utility differentials. On average, to move an agent from a borrowing and lending to a savings only contract would require a consumption supplement of 9.5%, whereas the same shift but from the insurance contract would require nearly 70% additional consumption! These numbers imply that allowing for more sophisticated financial contracts has the potential to lead to big welfare gains at least in a static sense.

Let us now move to the dynamic implications of the model. To characterize the dynamics I have simulated the model economy under the six different setups listed in the previous section. In the simulation the interest rate, $r$ is endogenous i.e. it is chosen to make borrowing equal to lending in each of the contracts. In the savings only contract there is no borrowing so the interest rate is one as credit supply (total savings) exceeds credit demand by construction. All agents (500 in the simulation) start with zero wealth.

Looking at figures 4a-4c we see that the model is capable of delivering output growth and an increase in the number of entrepreneurs matching qualitatively the patterns observed in developing countries. In the savings only setup allowing wealth lotteries makes a crucial difference to the result - without them output converges to a much lower level and no agents ever become entrepreneurs. This implies that, apart from their static welfare effects, wealth lotteries can be also welfare improving in a dynamic sense as shown in the average utility panel on fig. 4a. This however, comes at the cost of a much higher inequality level. Comparing the Gini coefficients in the lottery and no-lottery economies we see that in the former inequality is increasing, attaining a value of .7, whereas without income lotteries inequality actually declines reaching a value around .45. The intuition for this result is that lotteries 'create' inequality in the model by shifting agents with equal wealths to different wealth levels.

In the borrowing/lending and insurance setups we observe rising income, investment, consumption and number of entrepreneurs over time while the interest rates are falling as credit becomes cheaper. Inequality exhibits a typical Kuznets curve, first rising and then falling as the economy develops. The wealth lotteries do not seem to have a dramatic effect on the results under these two financial contracts perhaps because few agents fall into the relevant wealth range. Nevertheless, the lottery economies performs slightly better than the no-lottery ones, especially in the B/L case.

Comparing across the contractual structures we see that, once again, highest output and utility levels are achieved under the most sophisticated financial contract (insurance) and lowest under the savings only regime. The difference in the level of output achieved is around 3 times with lotteries and 8 times without lotteries. We conclude that enriching the contractual structure is unambiguously welfare improving in the dynamic sense as well. Notice, however, that average effort under the insurance setup is much lower compared to the other two as agents' incentives to work hard are mitigated by the consumption smoothing provided by the intermediary. Entrepreneurship is strongly affected by the form of the financial contracts available in the economy with virtually all agents becoming entrepreneurs by period 3 in the insurance contract, whereas this number peaks respectively at 300 and 65 in the borrowing/lending and the savings only setups. The potential of the model to generate differences in its implications with respect to occupational choice under the three of financial contracts regimes will be utilized more extensively in the next section where I perform a maximum likelihood estimation of the three model setups with and without lotteries aiming to match the occupational choice patterns generated from them with those observed in the Thai data.

# 3 Structural Maximum Likelihood Estimation

To broaden our understanding of the questions posed in the introduction, namely how differences in the structure of the available financial contracts affect the occupational choices that agents make I perform an empirical evaluation of the goodness of fit of the six model setups presented above with respect to data on village households in Thailand. First, I perform a structural maximum likelihood estimation of each model setup and obtain estimates for its parameters. The estimation is done for the whole sample and also for various sub-samples of households available in the data. The parameter estimates are then analyzed in view of the implied relationship between the model and the data. Next, the six different model structures are compared pairwise in order to establish the regime that come closest to the data generating process.

## 3.1 Data and Estimation Methodology

To perform the maximum likelihood estimation of the model I use data from Robert M. Townsend's socioeconomic survey of Thailand villages[27]. The survey was fielded in 1997 in four villages in different regions of Thailand. The sample I use consists of 2313 households, about 14% of which run their own businesses. To obtain this sample I dropped all non-positive wealth observations as well as the observations from the top wealth percentile as outliers.

The households in the data are from two distinct regions of the country: the rural and semi-urban central region close to the capital and the much poorer and more traditionally rural northeastern region. The survey data includes information on household wealth, occupational history, access and use of various formal and informal financial institutions and detailed demographic and educational data. Economic theory suggests that household characteristics of this type are important determinants of the household's decisions regarding supply and demand of credit and occupational choice. Thus, I will use the variability in household characteristics to test the relative performance of the different model regimes, anticipating that different model setups will perform best in the different data stratifications.

Table 3 presents a brief statistical summary of the data, including separate sections for the central and North-East regions. We see that the distribution of household wealth is

[27]For a more detailed description of the data see Paulson and Townsend (2001), Binford, Lee and Townsend (2001), and also Robert Townsend's website: http://www.src.uchicago.edu/users/robt

highly skewed to the right with the median much lower than the mean. Thus the sample is characterized with relatively few very rich households and many relatively poor ones. The range of wealth is very large with the poorest household having a wealth level almost a million times lower than that of the richest one. The fraction of entrepreneurs is 14% on average and it is crucially depending on education with only 9% of the low education[28] agents running a business, while this number among the ones with more than 4 years of education is 21%. It turns out that entrepreneurship does not depend that much on the wealth of the household - the difference in the fraction of entrepreneurs from the above or below median wealth households is only 2 percentage points. The mean wealth of agents with financial access is about two times higher than that of agents without financial access which can be treated as evidence for borrowing being dependent on wealth and/or for the existence of borrowing constraints for poorer households. Finally, entrepreneurs are on average two times richer than workers.

As already mentioned, the North-East region is on average much poorer than the central one (the mean wealth is 3.5 times lower) and the skewness of the wealth distribution is very high there. There are much less entrepreneurs in the North-East compared to the central region (9% versus 19%). In both regions entrepreneurship is strictly increasing in the level of education and access to credit.

Figure 5 shows the relationship between household wealth and the fraction of entrepreneurs in the data. The horizontal axis shows the ten wealth deciles, while the fraction of entrepreneurs in each decile is on the vertical axis. We see that, after an initial rise, the fraction of entrepreneurs declines in wealth from the second to the seventh decile and then rises sharply for the richest households. Thus, to explain the occupational choice pattern in the data, the model must be able to predict a non-monotonic relationship between wealth and the probability of becoming entrepreneur. The model in the paper has the necessary ingredients to provide such a relationship. On the one hand, an increase in wealth implies relaxing of the financial constraints making it easier for the agent to borrow which allows higher investment and makes entrepreneurship more attractive. On the other hand, however, higher wealth means that the agent can consume out of her interest income and does not have to put high effort so she may choose to be a rentier "worker" instead (wealth effect). Depending on which of the two effects dominates, entrepreneurship can potentially decrease or increase with income.

Another source of the relationship between income and entrepreneurship may originate from the agents' entrepreneurial ability. Indeed, most probably the steep rise in entrepreneurship between the 7th and 10th deciles is due to the fact that the richest agents are also the most educated and able ones. In the model we control for entrepreneurial ability through the parameter $\theta$ which was held constant so far but which will be let to vary in the estimation procedure below. In particular, I assume that talent is unobserved to us as econometricians but follows some distribution characterized by parameters which are to be estimated. Thus I allow the data to 'drive' the ability distribution in the right direction with regards to the needed correlation with wealth and the observed occupational choice behavior.

Formally, the talent variable, $\theta$ is assumed to be continuously distributed on the unit interval $[\kappa, \kappa+1]$ with a probability density function $\eta(\theta) = 2m(\theta-\kappa)+1-m$, where $\kappa \geq 0$ and $m \in [-1, 1]$ are parameters which will be endogenously determined during the estimation.

[28]Less than 4 years of schooling.

The parameter $m$ determines the shape of the talent distribution. When $m$ is equal to zero talent is uniformly distributed. When $m$ is one more mass is put on high ability agents, whereas for $m$ equal to -1 most of the mass is put on low-talent agents.

I test the empirical relevance of the model by performing a maximum likelihood estimation of the probability of being entrepreneur generated by the model and its counterpart from the data. The log-likelihood function can be written as:

$$L(\phi) \equiv \frac{1}{N} \sum_{i=1}^{N} E_i \ln H(A_i|\phi) + (1 - E_i) \ln(1 - H(A_i|\phi))$$

where $N$ is the number of observations, $E_i$ is a binary variable, which takes the value of 1 if agent $i$ is an entrepreneur in the data and 0 otherwise, $A_i$ is the wealth of agent $i$ in the data, $\phi$ is a vector of model parameters as described below, and $H(A_i|\phi)$ is the (expected) probability of being entrepreneur generated by the model for an agent with wealth $A_i$ integrated over the unobserved talent variable.

The model-generated probability of being entrepreneur, $H(A_i|\phi)$ is a function of the following parameters[29]:

$\gamma_1$ - the coefficient of risk aversion;

$\gamma_2$ - a curvature parameter of the disutility of effort;

$\lambda$ - a multiplicative constant governing the relative weight of utility derived from consumption or leisure;

$q$ - a parameter determining the higher value of output for the entrepreneurs;

$\alpha$ - the investment share in the probability of success function;

$m$ - a parameter governing the shape of the talent distribution;

$\kappa$ - a parameter determining the support of the talent distribution;

$w$ - a parameter determining the higher value of output for the workers;

$r$ - the interest rate;

The first eight parameters are estimated meaning that the log-likelihood function is maximized with respect to them. Following Paulson and Townsend (2001) I set the interest rate to 1.25 as found in the Thai data. The likelihood maximization is performed separately for each of the six different model setups described above as the implicit function $H(A_i|\phi)$ differs across them.

Given a wealth level $a = A_i$, the model generates a value for the probability that a person with such wealth is an entrepreneur. Maximum likelihood estimation is then used to recover the model parameters which provide the best match between these generated probabilities and the actual ones observed in the data. To ensure identification the wealth levels from the Thai data were normalized to lie on the interval $(0, 1]$.

The actual numerical procedure to solve the problem described above takes several steps. First, for any parameter vector, $\phi$ and any given values for $\theta$ and $A$ we need to solve the relevant non-linear optimization program and compute the probability of being entrepreneur, $h(A|\phi, \theta)$. In this step I use extensively the results of Propositions 1 and 2 which optimize the numerical solution technique employed as described above. Second, since $\theta$ is assumed unobservable by the econometrician, we need to compute the expected value of $h(A|\phi, \theta)$ integrating over $\theta$, i.e. $h^e(A|\phi) \equiv \int_{\kappa}^{1+\kappa} h(A|\phi, \theta)\eta(\theta)d\theta$. The method used is Gauss-Legendre

[29] See also the description of the model above.

quadrature with 5 nodes for $\theta$ (see Judd, 1998). It represents a standard numerical integration technique performed by evaluating the integrand at suitably selected nodes in the interval of integration. The method was chosen because it minimizes the number of function evaluations and also because of its nice asymptotic properties.

To save on computation time we cannot afford to compute $h^e(A|\phi)$ at all data points for $A$ (more than 2000 in the Thai data), so we construct a 20-point non-uniformly spaced grid[30] on [0,1] and compute $h^e(A|\phi)$ only at the grid points. In order to be able to compute the probability for all wealth points in the data, I use a cubic spline interpolation on the grid, which generates the probability of being entrepreneur predicted by the model for given wealth level $A$, $H(A|\phi)$. The actual maximization of the log-likelihood function $L(\phi)$ is done as follows:

- in order to ensure that a global maximum is reached, I perform an extensive grid search over the eight parameters and pick the parameter configuration which maximizes $L$ as the vector of initial parameter values for the actual optimization procedure.

- given the above initial guess, I solve the non-linear optimization problem of maximizing $L$ to obtain the maximum likelihood estimate $\phi^*$. The solution procedure represents a generalization of the polytope method using the Nelder-Mead simplex algorithm. It was chosen because of its high reliability, relative insensitivity to different initial values, and good performance with low-curvature objective functions such as those that usually appear in multivariate likelihood maximization problems.

Finally, I compute standard errors for the estimated parameters taking an approximation of the parameter covariance matrix using the outer product of the estimated score vectors, $M = SS'/n$, where $S$ denotes the $n \times 8$ matrix of score vectors with respect to the estimated parameters, i.e. $S_{.j} = \dfrac{\partial L(\phi)}{\partial \phi_j}$, $j = 1..8$. The standard errors of the estimated parameters are then the square roots of the main diagonal elements of the matrix $M^{-1}$. The score vectors themselves are computed using one-sided numerical differentiation of $L$ around the maximizing parameter values with tolerance of $10^{-3}$, i.e. the actual derivative $\dfrac{\partial L(\phi)}{\partial \phi_j}$ is approximated by $\dfrac{L(\phi') - L(\phi)}{h}$, where $h = 10^{-3}$ and $\phi' = (\phi_1, ..\phi_j + h, ..\phi_8)$.

## 3.2 Results

The results of the structural maximum likelihood estimation of the six model setups are presented in tables 4-6, each consisting of two parts corresponding to the cases with or without wealth lotteries. I have chosen to report results for the ten most important data stratifications: by region, wealth, access to credit and education, although numerous others were also estimated as a robustness check.

Looking at the parameter estimates, we see that, in the cases in which the standard errors are low, risk aversion, $\gamma_1$ tends to be lower for high-wealth stratifications in all model setups: it is in general higher for agents with low (below median) wealth and agents from the relatively poorer North-East region. Risk aversion also tends to decrease with education (as it is correlated with wealth) although there are some exceptions. In most of the cases the risk aversion parameter is fairly accurately estimated implying that the changes in risk aversion

[30] The unequal spacing between the grid points is needed since the wealth data is heavily right-skewed.

across the data stratifications are statistically significant. With the exception of the savings only with lotteries case, the estimated risk aversion for the whole sample is relatively low - around .10. It is much higher, however, for the low wealth and North-East stratifications. My results are thus consistent with those of Paulson and Townsend (2001) who have similar findings.

In five of the six model regimes the effort disutility curvature parameter, $\gamma_2$ is significantly higher for high wealth individuals relative to their low wealth counterparts indicating higher aversion to changes in the effort level. In the cases in which the parameter is accurately estimated the same result is confirmed across the two regions. Interestingly, the estimated value for $\gamma_2$ in the whole sample increases as we move towards financial contracts providing more insurance and credit: it is around .25 in the savings only contracts, .9 in the borrowing and lending ones and 1.5 in the insurance/transfer setup. Allowing for wealth lotteries does not seem to affect much the curvature parameter estimates.

The effort disutility parameter, $\lambda$ tends to increase on average with education although the direct effect of wealth on it is not very significant. Effort disutility is estimated to be higher in the insurance model which is intuitive as the agents can afford to work less given the amount of consumption smoothing provided by the intermediary. Notice that this confirms our theoretical results from the previous section.

The above results demonstrate that the preference parameters of the model vary across the different data stratifications and thus wealth effects can alter the demand for credit and consequently the occupational choice decisions of the agents. This can generate in principle the regions of declining and rising probability of becoming an entrepreneur as a function of wealth as observed in the data.

The production function parameter, $\alpha$ is, in most cases, slightly higher for higher wealth agents showing that they use to a higher extent the relatively cheaper for them investment input rather than effort. Across the three models the investment share in production increases from .19 in the savings to .45 in the B/L and to .8 in the insurance setup which is consistent with the fact that credit and thus investment is becoming easier to obtain as the financial contract limitations are gradually relaxed. This result is also true when we look at the parameter estimates for the stratifications with and without access to credit.

In most cases the ratio between the high output for entrepreneurs, $q$ and subsistence workers, $w$ is estimated at around or above 2 which is consistent with the data statistics regarding the mean wealths of entrepreneurs and non-entrepreneurs exhibited in table 3.

Finally, there are two parameters in the model, $m$ and $\kappa$ which govern the support and shape of the talent distribution. For the whole sample in the B/L and the savings only with lotteries setups $m$ is close to zero implying a virtually uniform talent distribution. In contrast, in the remaining cases $m$ is negative implying that more mass is put on low talent agents. However, the standard errors for $m$ are quite high so these conclusions are not statistically significant. Looking at the results for $\kappa$ we see some confirmation of the intuitive trend for higher parameter estimates as education increases, although in the no lottery specifications the middle education group has higher estimate for $\kappa$ than the high education one. In most cases the parameter is also increasing in wealth as one would expect.

Looking at the likelihood values we see that all models achieve best fit in the North-East region and among the low education households, while worst fit is achieved in the central and the high education stratifications. This is a direct confirmation of the results of Paulson and Townsend (2001) and is mostly due to statistical reasons as the model performs better in stratifications with fewer entrepreneurs due to the lower degree of variation in the data. To summarize, we see that the model produces estimates which are intuitively consistent and thus it has explanatory power to account for the occupational choice patterns observed in the data.

## 3.3 Model Comparisons

In this subsection I perform a formal statistical test of how well the six model specifications fit the occupational choice pattern in several stratifications of the data. The model regimes are compared in pairs one against another using the Vuong likelihood ratio test for non-nested models (see Vuong, 1989). The null hypothesis of the test is that the two compared models are equally likely to have generated the observed data. The Vuong method involves computing a likelihood ratio test statistic which, under certain conditions that we assume to hold and after a suitable normalization, is distributed as a standard normal random variable.

The results of the model comparisons are exhibited in tables 7a and 7b. The first table reports the maximized likelihood values for the six model specifications and for 19 data stratifications. I have added a few more categories to the ten discussed above, dealing mostly with access to different types of formal and informal financing. The purpose is to try to distinguish better the implications of the different model setups about how occupational choice varies with wealth, education and access to different forms of credit. Simply looking at the numbers, we see that the savings only setup with no wealth lotteries performs significantly worse than all others, providing evidence that the extreme form of financial restrictions imposed in it is far from the data generating process.

Table 7b contains the results of nine bilateral comparisons between the six model specifications. First I compare all no lottery and all lottery models among each other, which is followed by a test of the effect of including wealth lotteries within each of the three basic financial contract regimes. We see that the savings only with no lotteries setup (SNL) is rejected to be equally likely to have generated the data when compared to the borrowing and insurance regimes across basically all data stratifications. This is an evidence that households in the data have some ability to borrow or obtain transfers from financial intermediaries unlike the assumption of the savings only setup. A similar result is true for the saving only contract with lotteries although for fewer data categories.

In general, I cannot reject the null hypothesis that the borrowing/lending and the insurance/transfers contracts are equally likely to have generated the occupational choice pattern observed in the data in neither of the data stratifications. Thus, both types of contracts seem to have similar ability to provide agents with credit and insurance at least measured by the likelihood of the implied occupational choice pattern. This implies that my results differ somewhat from those of Paulson and Townsend (2001) who find that limited liability constraints (as in the borrowing model) are more important (i.e. provide better fit) at low wealth levels while constraints due to moral hazard (as in the insurance contract) are more important at higher wealth levels. Of course, the parallel between the results in the two papers is not direct as Paulson and Townsend compare two models with completely different structure in many other aspects (the Evans and Jovanovic, 1989 and the Aghion and Bolton, 1997 and Lehnert, 1998 models), while the models compared in the present paper are much close to each other and both feature asymmetric information unlike the Evans and Jovanovic model.

Notice, however, that the savings only contract is not rejected in the low wealth stratifications (Low Wealth, North-East, Education $<$ 4 years and North-East, access to BAAC[31]) which can be interpreted as indirect evidence for the existence of more restrictive financial constraints operating for these groups of households. This is some weak confirmation of the Paulson and Townsend's findings as the savings only setup can be treated as an extreme form of the Evans and Jovanovic model in which agents can borrow up to some fixed fraction of their wealth (zero in our case). This result can be also interpreted as an evidence that a lower degree of development as in the North-East region is more likely to be characterized with a lower degree of insurance and less sophisticated and more restricted financial contracts.

The last three comparisons in table 7b concentrate on the effect of adding lotteries to each of the three types of financial contracts. From the theoretical results above we know that allowing such rosca-type wealth lotteries has the potential to change substantially the model's implication about occupational choice at different wealth levels. In particular, agents with wealths at which the upper envelope of the occupational value functions is convex can be assigned each of the two occupations with some positive probability in a lottery setup, whereas without lotteries they will have to choose one of the occupations with probability one. Some of the effects of adding such wealth lotteries to the available financial contracts in the economy were studied by Lehnert (1998) in a context simpler than the one in the present paper. He concentrates on the dynamic effects of lotteries and finds that while allowing them can lead to higher aggregate output levels, this happens at the cost of a substantial increase in inequality. However, his analysis remains purely theoretical, while the present paper goes one step further and looks at the empirical relevance of adding wealth lotteries to the feasible financial contracts in the economy.

We see from the table that there is considerable evidence that contracts with lotteries provide better fit with the data compared to their counterparts without lotteries. The result is most evident for the savings only contract and, across all three types of contracts, for the whole sample. Although quite a few of the Vuong likelihood ratio statistics for the borrowing and insurance setups are not significant at the 10% level, their signs are negative, indicating that lottery contracts achieve better fit with the data. The only exception is the group of households from the North-East with access to BAAC financing for which the borrowing contract with no wealth lotteries performs better. The reason for this may be the specific lending practices employed by the BAAC in this region as discussed in Paulson and Townsend (2001).

Finally, we can see some difference between the results for the households from the Central and North-East regions with access to BAAC financing: while the restrictive SNL contract is rejected in the Central region, it cannot be rejected in the North-East, providing once again some limited evidence that stronger financial constraints are more relevant for the poorer households of the North-East.

# 4 Model Dynamics and Calibration

The maximum likelihood estimation performed in the previous section provides estimates just for the static parameters of the model - the preference parameters, $\gamma_1, \gamma_2$ and $\lambda$, the production parameters $\alpha, q$ and $w$, and the talent distribution parameters $m$ and $\kappa$. In order to study the

[31]The Bank for Agriculture and Agricultural Cooperatives (BAAC) is one of the dominant lenders in Thai villages.

capability of the different model regimes to match the dynamics of the Thai economy, we also need to pin down certain dynamic parameters of the model. In particular, in this section I assume that $q$ grows at a rate of $g$ and look for the values of $g$ and the savings rate $s$ at which each of the three basic models (with wealth lotteries) generates dynamics as close as possible to those in the Thai data. The method used to determine these two parameters is calibration, i.e. I search for the best $(g, s)$ combination which, according to some suitably chosen metric, comes closest to the data[32].

In the calibration process I match the following three time series generated by each of the model setups and present in the data: the growth rate of income[33], the Gini coefficient of wealth inequality, and the fraction of entrepreneurs in the economy. Due to computational reasons, I restrict attention to time series of five periods generated by each of the model regimes which are matched with Thai National Statistics data from the period 1978-1982. Thus one period in the model is assumed to be equivalent to one year in the data. The data for the income growth rate, Gini coefficient and fraction of entrepreneurs in Thailand were taken from successive rounds of the national income and expenditures Socio-Economic Survey (SES) conducted by the Thai government. A more detailed description of this data set can be found in Gine and Townsend (2002) or Jeong (1999).

To perform the calibration I use as a metric the normalized sum of period by period squared deviations of the three time series predicted by the model from the actual Thai data. The normalization is done by dividing the deviations by their means from the data. Formally, I use the following metric, proposed by Gine and Townsend (2002):

$$\frac{1}{3}\sum_{j=1}^{3}\sum_{t=1978}^{1982}\left(\frac{z_{st}^{sim} - z_{st}^{data}}{\mu_{z_s^{data}}}\right)^2$$

where $z_{st}$ denotes the variable $s$ (one of the three matched time-series) at time $t$ and $\mu_{z_s^{data}}$ is the mean of the variable $z_s$ from the data.

Apart from the calibration metric, in order to be able to simulate the model, we also need data for the initial wealth distribution. This data comes from the 1976 round of the SES dataset, with a sample size of 10,613 observations. Some descriptive statistics of the data are exhibited in table 8. We see that they are qualitatively similar to those of the Townsend dataset. The wealth figures are lower in nominal terms since the data is for 1976 rather than 1997 as in the Townsend data.

Finally, to be consistent, we also need to re-estimate using maximum likelihood the eight 'static' parameters listed above, from the 1976 SES data. These parameters are needed to generate the simulated time series for income growth, inequality and the fraction of entrepreneurs. The results of the estimation are reported in table 9a. A significant difference from our previous results is the good fit achieved by the savings only model specification which even turns out to dominate the borrowing and insurance contracts at the 5% level in the Vuong test. Of course, we should not forget that this data refers to 1976 when the financial intermediation sector in Thailand was much less developed compared to 1997 so perhaps this result should not come as a surprise.

[32] The approach is basically the same as that of Gine and Townsend, 2002.

[33] (Aggregate) income is defined in the model as the difference between the sums of initial wealths of two successive generations of agents.

The parameter values from table 9a are used together with the initial wealth distribution in 1976 to simulate the three lottery model specifications, SL, BL and IL for six periods which are taken to correspond to the years 1977-1982. In the actual calibration process I drop the first simulated period due to the deficiency of the model to predict implausibly high initial growth rates which hampers the calibration procedure. The results of the calibration can be found in table 9b and figure 6. In terms of the chosen metric the best fit is achieved once again by the savings only model specification, followed closely by the insurance one. Both of them manage to match relatively well the pattern of the income growth rate but underestimate the degree of inequality. The SL model predicts zero entrepreneurs at the estimated and calibrated parameters which is below the actual fraction in the data, whereas the IL model over-predicts the fraction of entrepreneurs by 5 to 15 percent. The BL model performs worst from a dynamic perspective, hugely over-predicting the fraction of entrepreneurs and generating implausibly low initial growth rates of income. However, notice that it is the only model capable to generate enough inequality (at least initially) to match the numbers in the data. The parameter estimates for the savings rate range between .27 and .33 which is close, although slightly higher than the actual savings rate in Thailand for the period of 22%. The parameter $g$ has no direct counterpart in the data but can be interpreted as a rate of return on business investment projects. The calibrated values of 6 to 15% seem quite realistic given that interpretation.

To summarize, in this and the previous section I have demonstrated the ability of the model to account for not only the static occupational choice patterns in the data but also for the dynamic behavior of key economic indicators. This feature of the model in each of its six financial contract regimes can therefore be used to infer relevant theoretically and empirically backed policy recommendations.

# 5 Conclusions

Financial constraints, wealth inequality, and differences in entrepreneurial are some of the most important factors influencing economic development. In this paper I used a general model of occupational choice under moral hazard featuring agents heterogeneous in wealth and ability to analyze the implications of differences in the degree of financial intermediation available in the economy on consumption, investment, effort, income growth, inequality and entrepreneurship. In particular, three types of financial contract regimes were studied and compared: a savings only regime, a borrowing/lending regime and a general insurance/transfers regime. The three regimes differ in the level of complexity of the financial contracts that can be written by the agents and financial intermediaries and thus in their ability to serve as consumption smoothing devices.

Agents in the model can choose between two occupations which amounts to operating one of two different technologies. The indivisibility inherent in the discreteness of the occupational choice generates a non-convexity in the indirect utility function which makes pooling wealth among agents and then redistributing it via lottery efficiency improving. The mechanism behind these wealth lotteries is similar to the one of the rotating savings and credit associations (roscas) operating in many development economies. To account for the existence of such institutions, I have considered the cases when wealth lotteries are or are not present for each of the three regimes listed above.

I analyze the implications of differences in financial contracts structure on the model economy from both a theoretical and an empirical point of view using advanced numerical solution techniques. The model simulations show that while the insurance regime achieves highest utility in any given time period, it is also characterized by lower levels of effort which can potentially have negative dynamic effects on output. I also demonstrate that it is possible to achieve huge gains in income and welfare by switching to more advanced financial contracts. This suggests that institutional improvements in the financial intermediation sector of developing countries can have dramatic effects. Allowing for wealth lotteries also leads to welfare improvements in both static and dynamic sense, however, this may be at the cost of much higher wealth inequality level.

The main theoretical result of the paper is demonstrating the equivalence of general lotteries on consumption, investment, effort and output in the insurance/transfers regime to an ex-ante lottery over wealth followed by a non-probabilistic insurance contract *given* the optimal occupational choice implied by the wealth won in the lottery. This equivalence allows for a significant simplification of the numerical solution method for incentive constrained occupational choice problems over the commonly used in the literature linear programming approach. In addition it can be easily applied to any type of discrete choice problems.

Entrepreneurship, viewed as an engines of economic growth is one of the main points of interest in the present paper. The model simulations show that entrepreneurship is strongly affected by the form of financial contracts available in the economy. More restrictive financial regimes limit seriously the fraction of agents that become entrepreneurs because of borrowing constraints. The potential of the model to generate differences in its implications with respect to occupational choice under the three types of financial contract regimes is used extensively in the empirical part of the paper where I perform a maximum likelihood estimation of the different model setups matching the generated occupational choice patterns with those observed in the Thai data. One of the main empirical results in the paper is the rejection of the savings only model in favor of the more general borrowing and lending and insurance models. This represents evidence for the existence of consumption smoothing contractual arrangements in the data although not necessarily at the incentive constrained Pareto optimal level as the formal likelihood ratio test cannot reject the null hypothesis that the more limited borrowing/lending regime is equally likely to have generated the data as the more general insurance/transfers one.

Testing formally for the empirical relevance of including versus not including wealth lotteries in the financial contracts structure represents a contribution to the existing literature. The paper demonstrates that augmenting each of the three contract regimes with rosca-type wealth lotteries improves the fit with the data. The effect is especially strong for the savings only contract, providing some indirect evidence for the existence and importance of asset pooling arrangements in developing countries such as Thailand.

There exist several potential directions in which the results presented above can be extended. First, I have considered just three of the numerous possible types of financial contracts. Although they were chosen because of their similarity to many financial arrangements observed in developing countries, other types of contracts do exist and deserve attention. Second, the type of financial contract present in the model economy at any given time was exogenously set in advance. Certainly, a more general framework allowing for endogenous changes in the level of financial intermediation over time as in Townsend and Ueda (2002) would move the analysis closer to reality. Third, in the empirical part of the paper I have used only wealth and occupational choice data to perform the structural maximum likelihood estimation of the model. Incorporating data on business income and wages can undoubtedly enrich the current results. Finally, in the present model I assumed for simplicity that entrepreneurial talent is unobservable to the econometrician and thus I let the estimation procedure determine if any correlation between wealth and talent exists. An alternative methodology would be to impose some structure on this relationship and use the implied wealth-talent feedback in the estimation.

To summarize, in this paper I analyzed the implications of different financial contracts for the process of economic development and evaluated which contractual structures are best suited to explain the patterns observed in the data. This 'marriage' between theory and data provides meaningful answers to the questions posed in the introduction and should be able to enhances the ability of economists to make better policy recommendations based on the microeconomic underpinnings of the economic processes involved.

# 6 Appendix

**Proof of Proposition 1**

The idea for the proof is based on Proposition 5 of Cole and Prescott (1993). Basically we need to show that for any given wealth level, $a$ the contract $(c, q, z, k)$ resulting from the solution of the input lottery program can be mapped into the solutions of the two optimization problems (11) and (12) combined with an ex-ante lottery over wealth. The unique (because of concavity) solution to the former problem consists of the set of probabilities[34] $\{\pi^*(c, q, z, k|a) > 0\}$[35] satisfying the constraints in (4), whereas the solution to the latter (wealth lottery) problem can be written as $(c_j^*(a_1), z^*(a_1), k^*(a_1))$, $j = l, h$ and $(c_j^*(a_2), z^*(a_2), k^*(a_2))$, $j = l, h$ together with a probability $\mu^*(a)$ such that

$$a_1\mu^*(a) + a_2(1 - \mu^*(a)) = a \tag{13}$$

and where $c_j^*$, $k^*$ and $z^*$ are the solutions to (11) and (12). It is also clear that $q$ in $\pi^*(c, q, z, k|a)$ can take only the values $0, \theta q_h$ and $w_h$ due to technological feasibility.

First, notice that given the model's assumptions about the utility and production functions the problems (11) and (12) have unique solutions in terms of $c, z, k$ for any given value of $a$. Also, the indirect utility functions $v^E(a)$ and $v^W(a)$ defined as before are concave, thus when wealth lotteries are used for convexification the losers and the winners of the lottery would have different occupations. Suppose that without loss of generality an agent with wealth $a_1$ would optimally choose to be an entrepreneur (i.e. $v^W(a_1) > v^E(a_1)$) whereas an agent with wealth $a_2$ would optimally choose subsistence work. Let us denote $\Pi_1 \equiv \{\pi^*(c, q, z, k|a) \mid k > 0\}$, i.e. the set of contracts under which the agent would be an entrepreneur[36] and $\Pi_2 \equiv \{\pi^*(c, q, z, k|a) \mid k = 0\}$, i.e. the contracts under which the agent is a worker.

Suppose that there exist two optimal contracts $\pi_1^*(c_1, q_1, z_1, k_1)$ and $\pi_2^*(c_2, q_2, z_2, k_2)$ in $\Pi_1$ such that their corresponding effort/investment assignments are not the same, i.e. $(z_1, k_1) \neq (z_2, k_2)$. As $u$ and $p$ are concave in $z$ and $k$ this would imply that a linear combination

[34] I will refer to this set as the set of optimal contracts.

[35] I disregard zero probability contracts as they have no economic interpretation.

[36] It can never be optimal to assign $k = 0$ for an entrepreneur as this implies zero output at any effort level whereas if the agent were assigned to be a worker at the same effort level she would produce positive output. Thus all agents assigned entrepreneurship will have $k > 0$.

of the two would achieve higher utility for the entrepreneur and still be feasible which is a contradiction. Thus it must be the case that $z_1 = z_2 = z^1$ and $k_1 = k_2 = k^1$ implying that there are only two elements in $\Pi_1$, $\pi^*_{11}(c^1_h, \theta q_h, z^1, k^1)$ and $\pi^*_{12}(c^1_l, 0, z^1, k^1)$. Similarly, there are only two elements in $\Pi_2 : \pi^*_{21}(c^2_h, w_h, z^2, 0)$ and $\pi^*_{22}(c^2_l, 0, z^2, 0)$.

Now we only have to show that $(c^i_j, z^i, k^i) = (c^*_j(a_i), z^*(a_i), k^*(a_i))$ for $j = l, h$, $i = 1, 2$ to finish the proof. Define $\hat{\pi}_{11} = \mu^*(a)p(\theta q_h|z^*(a_1), k^*(a_1))$, $\hat{\pi}_{12} = \mu^*(a)p(0|z^*(a_1), k^*(a_1))$, $\hat{\pi}_{21} = (1-\mu^*(a))p(w_h|z^*(a_2), 0)$ and $\hat{\pi}_{22} = (1-\mu^*(a))p(0|z^*(a_2), 0)$. It can be seen immediately from (BE1), (BE2) and (ICC) together with (13) that the vector $(\hat{\pi}_{11}, \hat{\pi}_{12}, \hat{\pi}_{21}, \hat{\pi}_{22})$ satisfies (6) and (7). It also satisfies (5) and (8) by construction thus it is feasible for the linear program (4). Conversely, (5) implies that $\dfrac{\pi^*_{11}}{p(\theta q_h|z^1, k^1)} = \dfrac{\pi^*_{12}}{p(0|z^1, k^1)} \equiv \mu_1$ and $\dfrac{\pi^*_{21}}{p(w_h|z^2, k^2)} = \dfrac{\pi^*_{22}}{p(0|z^2, k^2)} \equiv \mu_2$. Then from (8) we have that $\mu_1 + \mu_2 = 1$, i.e. it is clear that $(z^i, k^i)$, the implied $c^i_j$, and $\mu_1$ and $1 - \mu_1$ satisfy the constraints of (11) and (12) together with (13) and thus are feasible for the wealth lottery problem. Finally, suppose that $(c^*_j(a_i), z^*(a_i), k^*(a_i))$ together with $\mu_1$ and $1-\mu_1$ is not maximizing for the wealth lottery problem, i.e. some agent can achieve higher utility. But then the mapping of $(c^*_j(a_i), z^*(a_i), k^*(a_i))$ and $\mu^*$ into the $\pi$-contracts described above would produce an allocation in which all agents are at least as better off as in $(\pi^*_{11}, \pi^*_{12}, \pi^*_{21}, \pi^*_{22})$ which is a contradiction.■

**Proof of Proposition 2**

Let us first verify the MLRP which means that we need to prove that $\dfrac{\partial p^e(q|z,k)}{\partial z}\dfrac{1}{p^e(q|z,k)}$ is non-decreasing in $q$. In our setup since there are only 2 possible levels that $q$ can take we simply need to show that:

$$\frac{\partial(1 - \frac{k^\alpha z^{1-\alpha}}{1+k^\alpha z^{1-\alpha}})}{\partial z}\frac{1}{1 - \frac{k^\alpha z^{1-\alpha}}{1+k^\alpha z^{1-\alpha}}} \leq \frac{\partial(\frac{k^\alpha z^{1-\alpha}}{1+k^\alpha z^{1-\alpha}})}{\partial z}\frac{1}{\frac{k^\alpha z^{1-\alpha}}{1+k^\alpha z^{1-\alpha}}}$$

which is obviously true as the left hand side is negative and the right hand side is positive.

Now let us verify the CDFC which is equivalent to showing that $\dfrac{\partial^2 p^e(q_1|z,k)}{\partial z^2}$ and $\dfrac{\partial^2 p^e(q_1|z,k)}{\partial z^2} + \dfrac{\partial^2 p^e(q_2|z,k)}{\partial z^2}$ are non-negative where $q_1$ and $q_2$ are the two possible output levels and $q_1 < q_2$. The first expression is equivalent to:

$$\frac{\partial(-\dfrac{(1-\alpha)k^\alpha z^{-\alpha}}{(1+k^\alpha z^{1-\alpha})^2})}{\partial z} = \frac{\alpha(1-\alpha)k^\alpha z^{-\alpha-1}(1+k^\alpha z^{1-\alpha}) + 2(1-\alpha)^2 k^{2\alpha} z^{-2\alpha}}{(1+k^\alpha z^{1-\alpha})^3} > 0$$

We also have:

$$\frac{\partial^2 p^e(q_2|z,k)}{\partial z^2} = \frac{\partial(\dfrac{(1-\alpha)k^\alpha z^{-\alpha}}{(1+k^\alpha z^{1-\alpha})^2})}{\partial z} = -\frac{\partial^2 p^e(q_1|z,k)}{\partial z^2}$$

thus the second expression is non-negative as well. Finally, Proposition 1 in Rogerson (1985) implies immediately that the first order approach is valid in our setting.■

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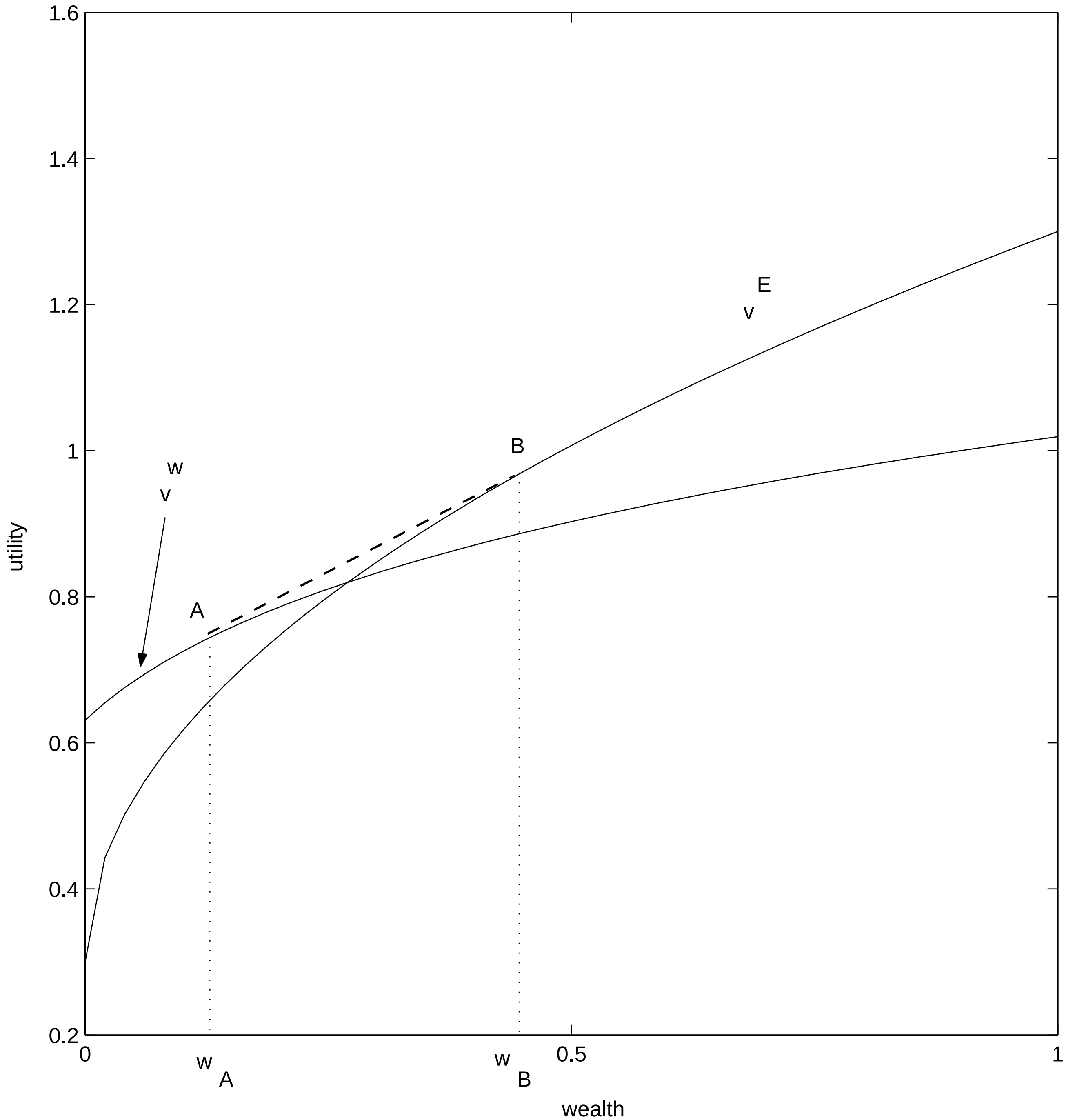
Fig. 1 − Wealth Lotteries



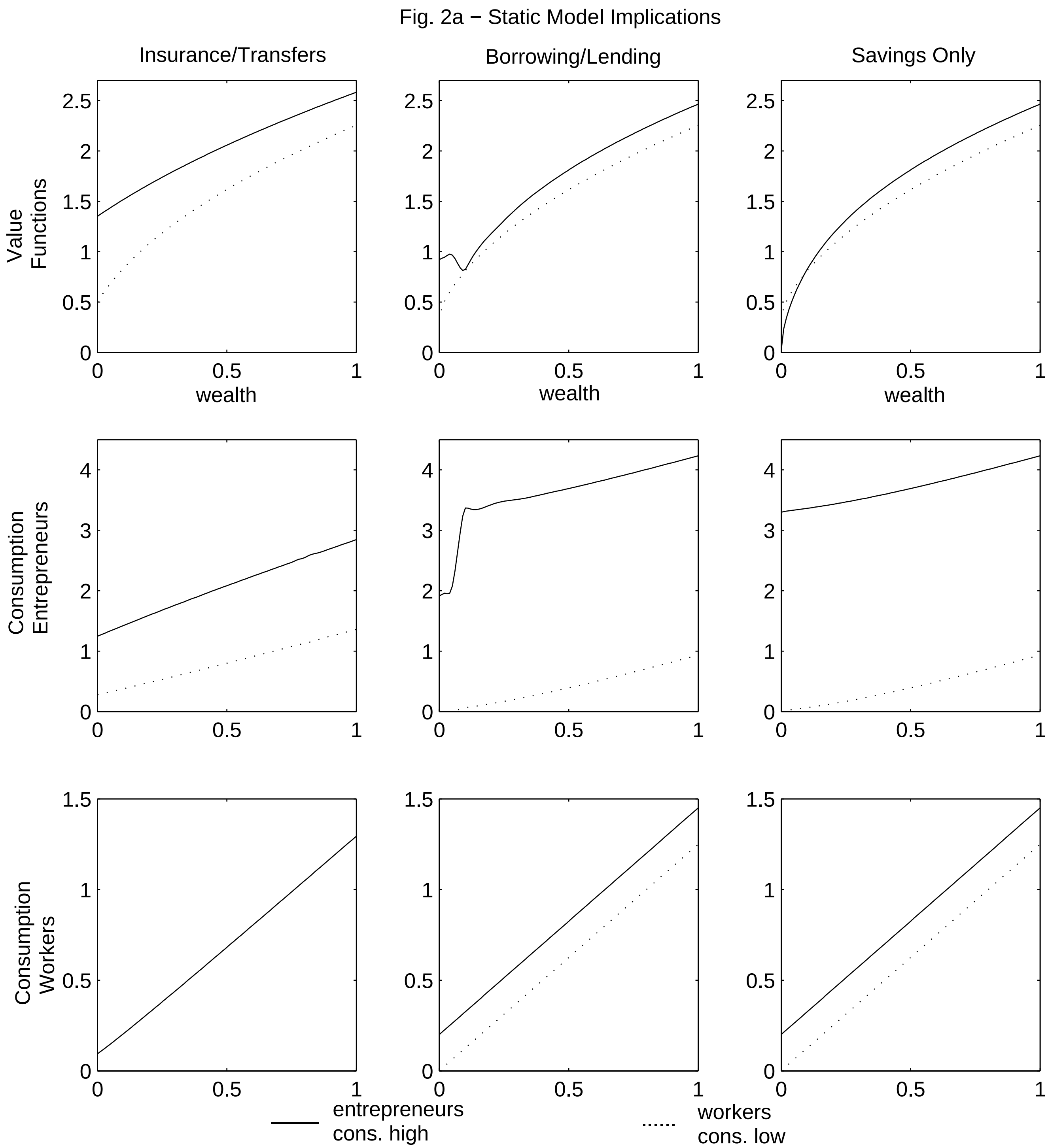


Fig. 2a − Static Model Implications
Insurance/Transfers
Borrowing/Lending
Savings Only
Value Functions
Consumption Entrepreneurs
Consumption Workers
wealth
entrepreneurs cons. high
workers cons. low

Fig. 2b − Static Model Implications

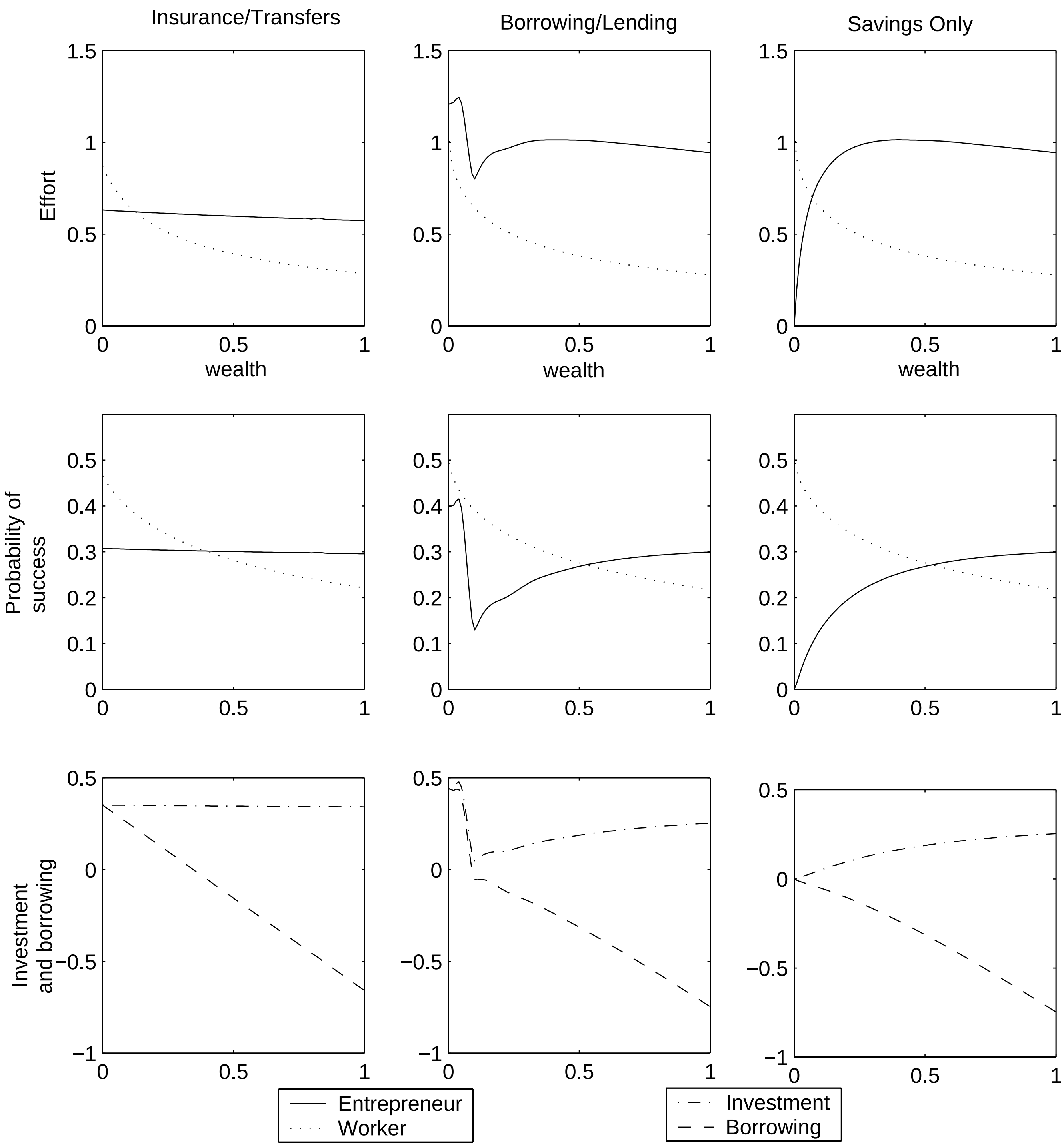


Fig. 3 Utility Differences

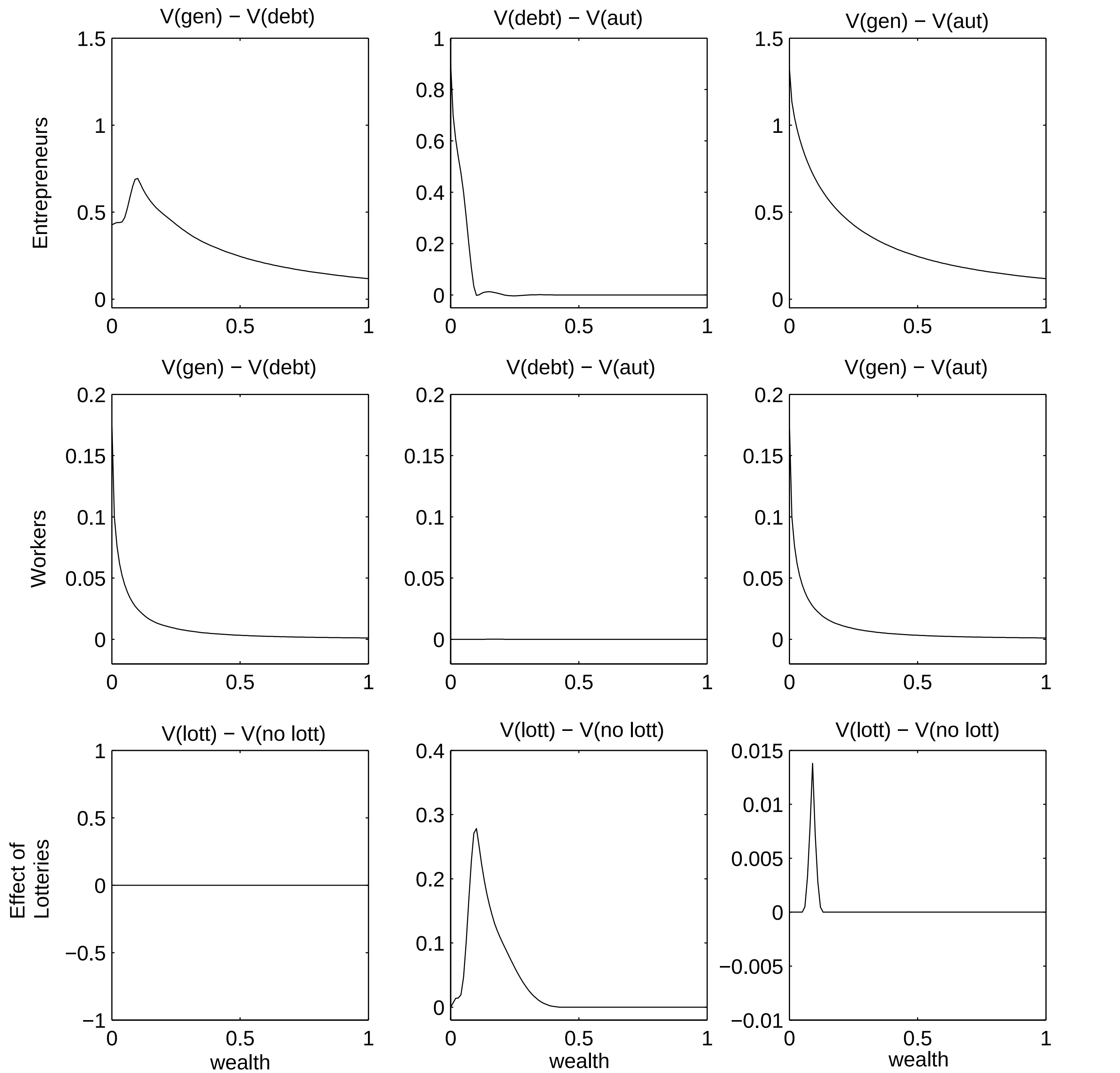


Fig. 4a − Model Dynamics, Savings Only

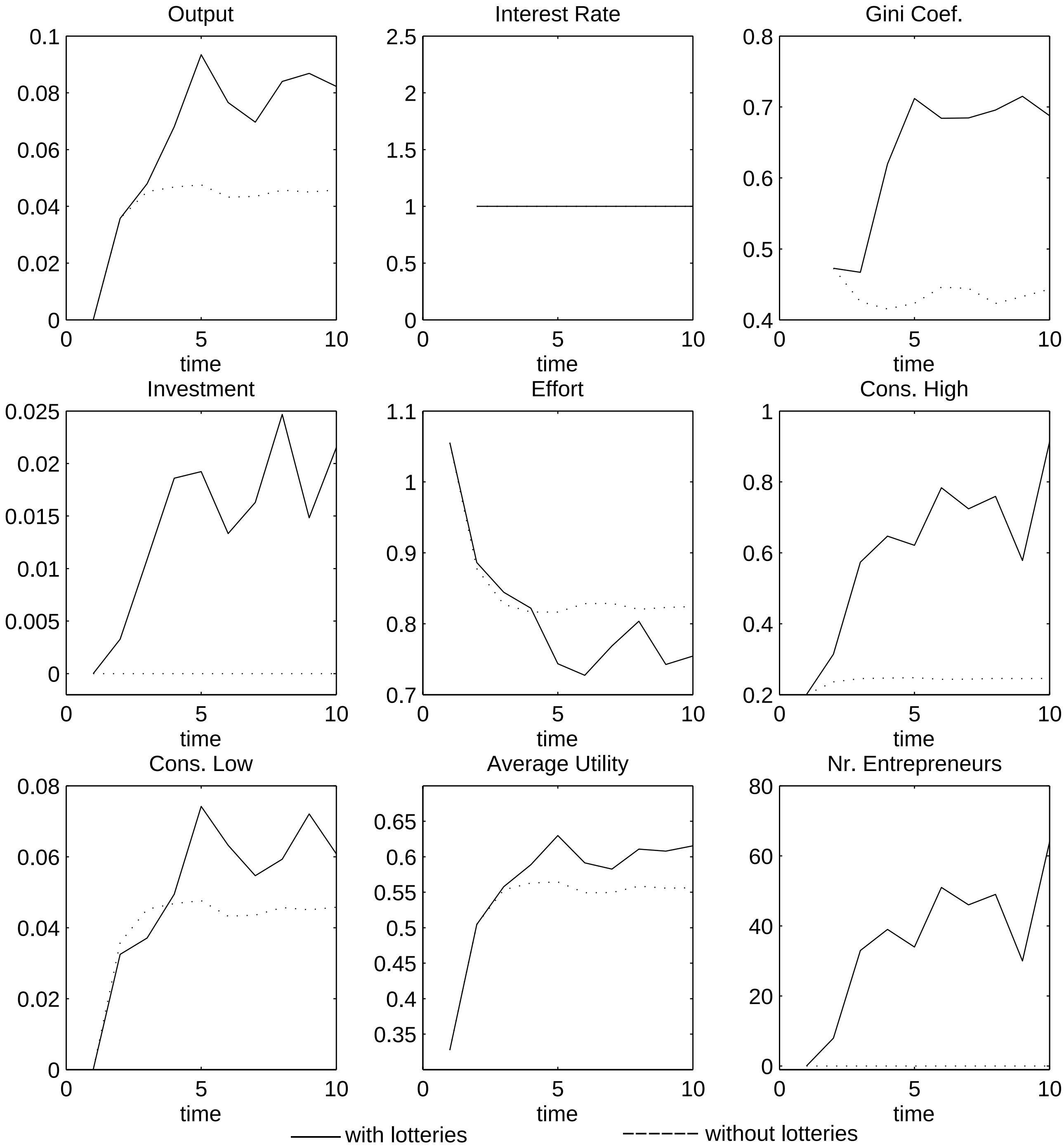


Fig. 4b − Model Dynamics, Borrowing/Lending

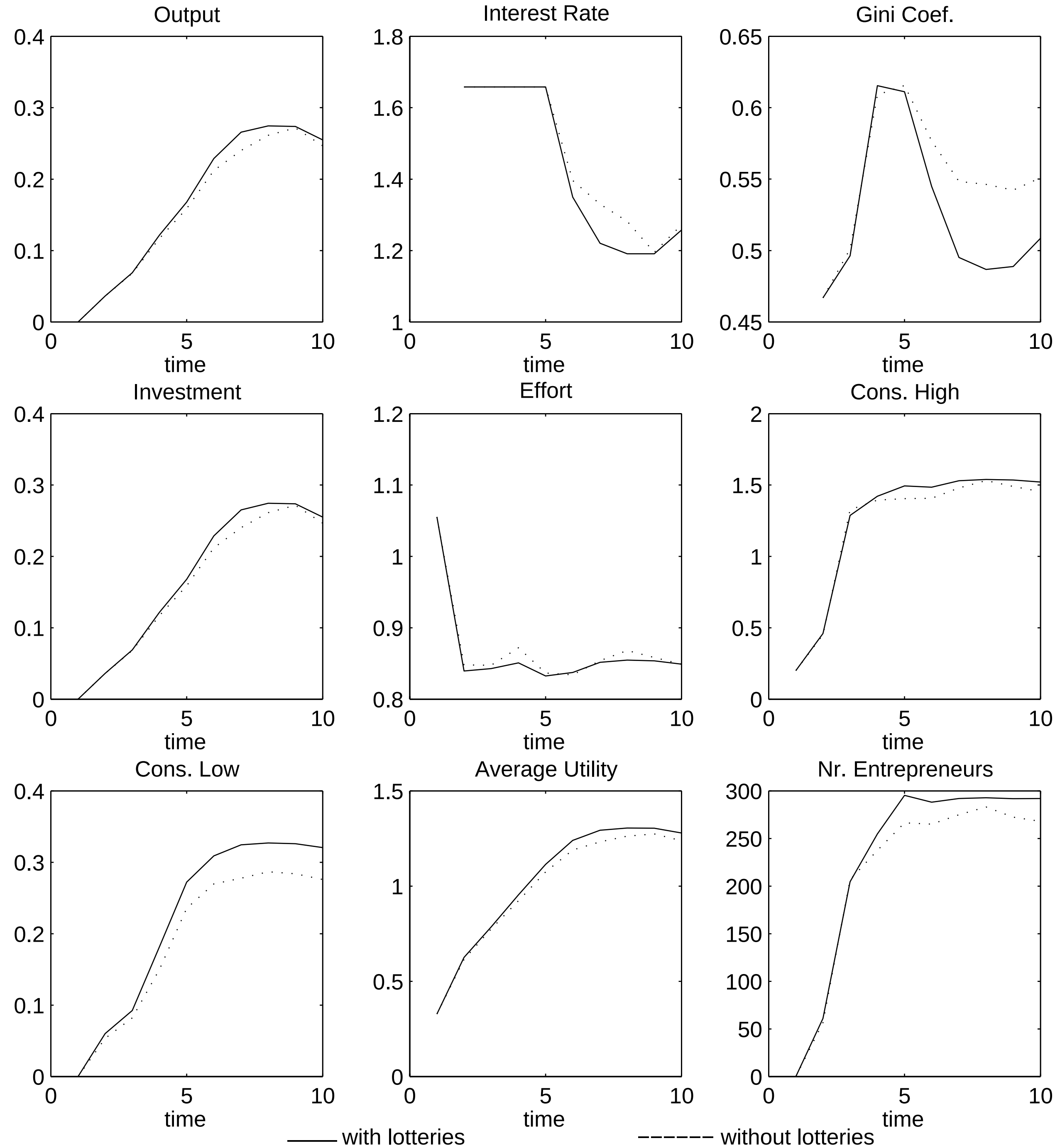


Fig. 4c − Model Dynamics, Insurance/Transfers

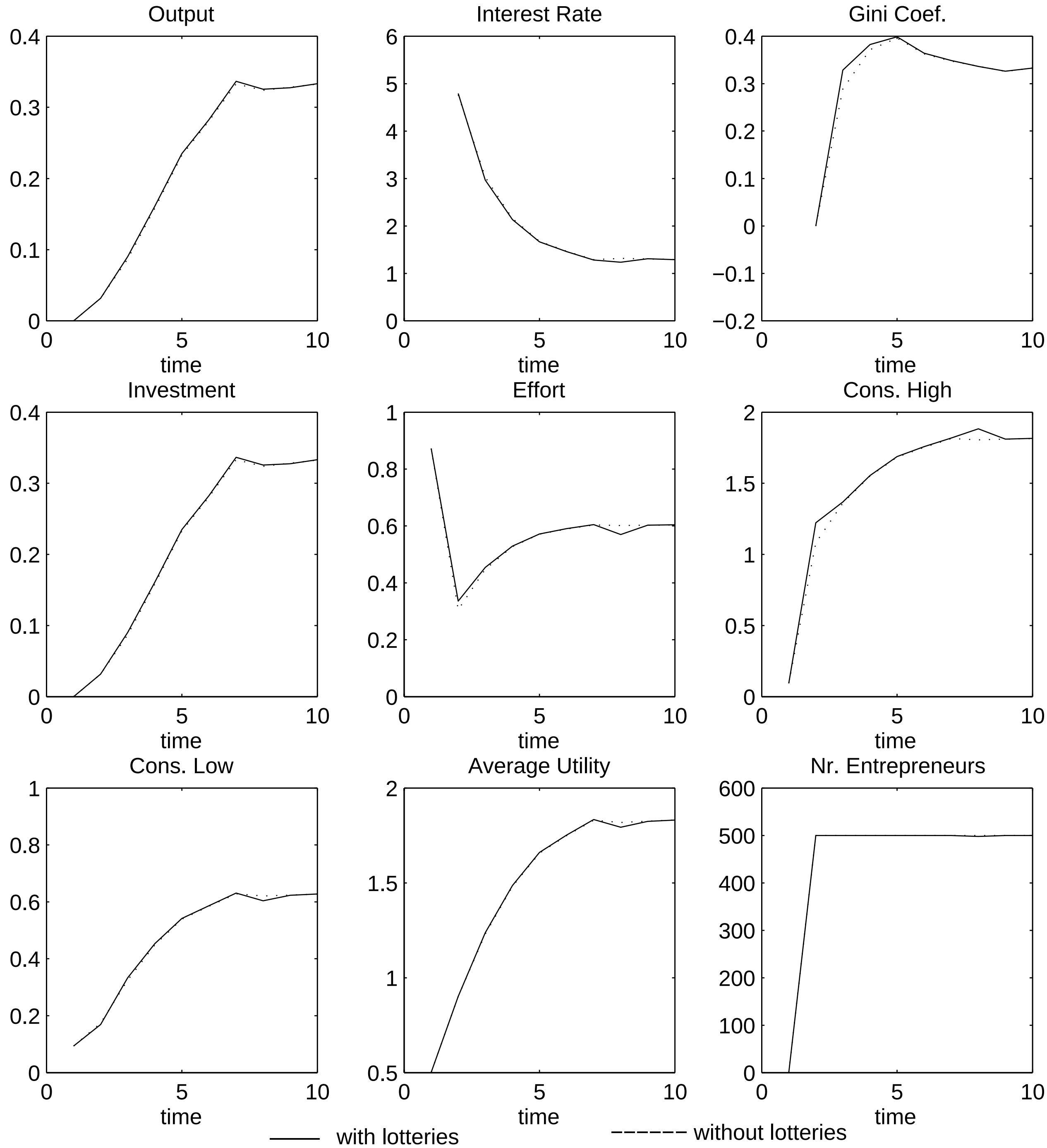


Fig. 5 − Fraction of Entrepreneurs in the Data

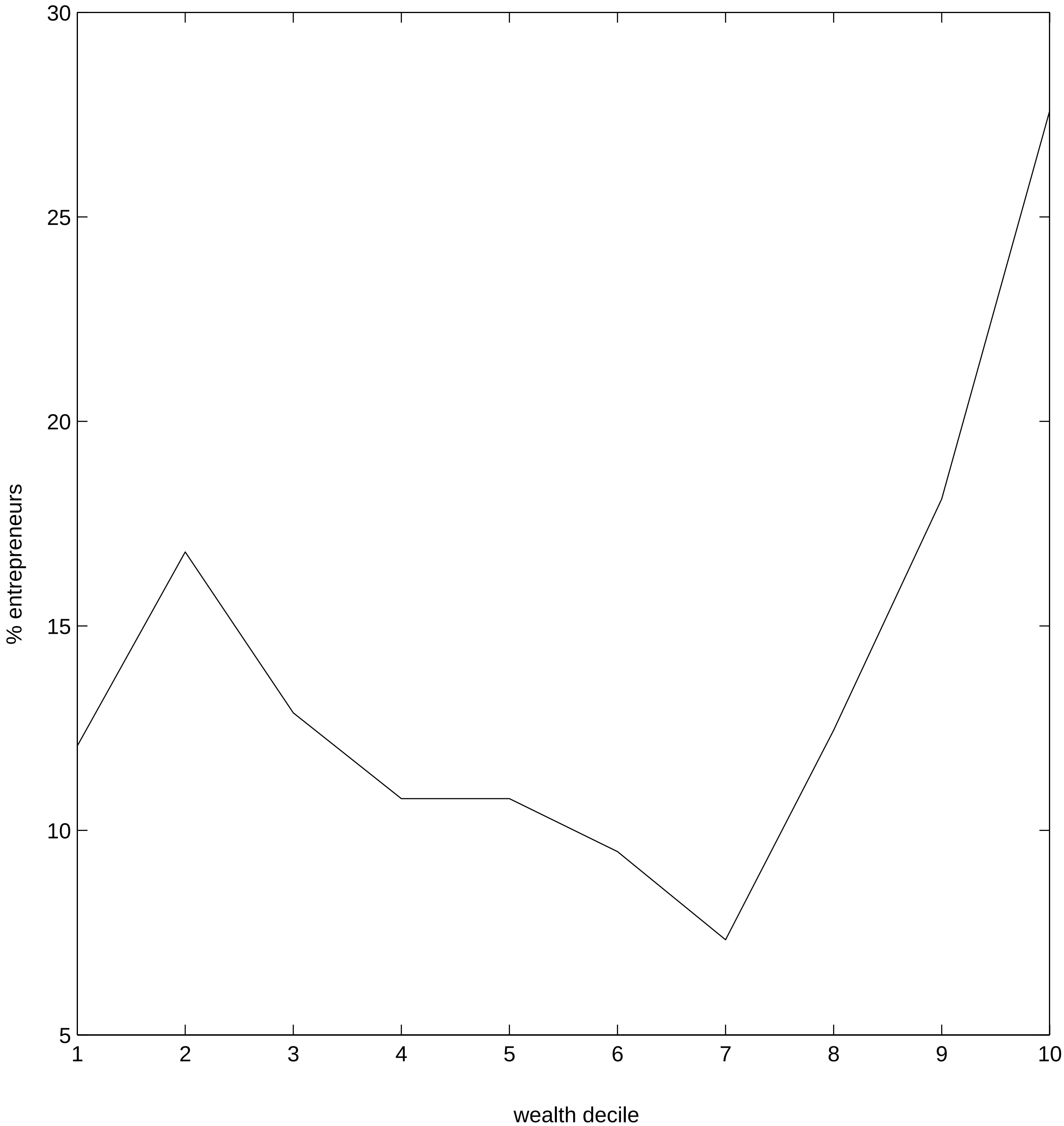
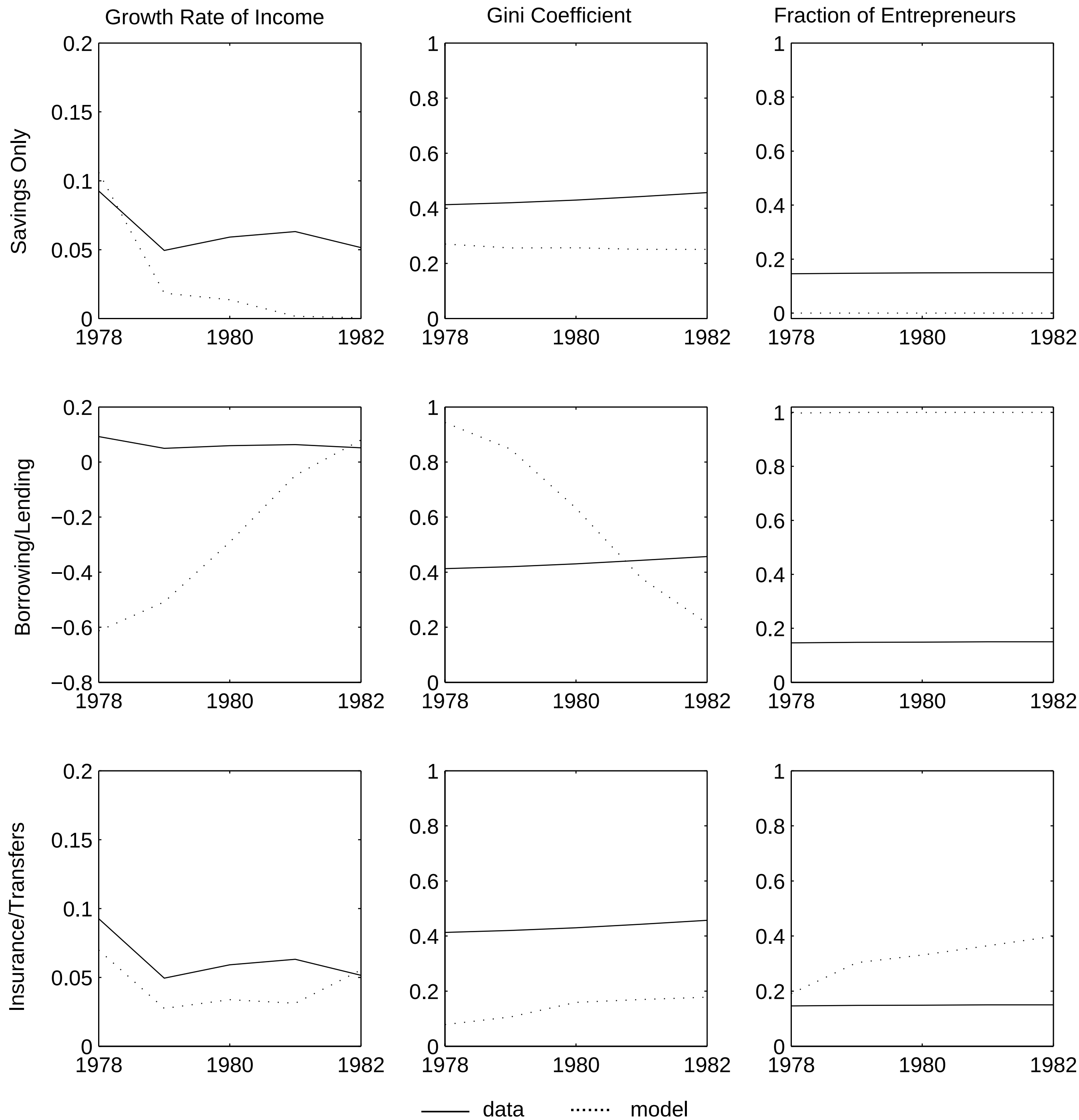


Fig. 6 − Model Calibration (Best Overall Fit)



## Table 1 - Benchmark Parameters

| | |
|---|---|
| $\gamma_1$ | 0.50 |
| $\gamma_2$ | 1.50 |
| $\lambda$ | 0.20 |
| $\alpha$ | 0.60 |
| q | 3.30 |
| $\theta$ | 1.00 |
| w | 0.20 |
| s | 0.34 |
| r | 1.25 |

## Table 2 - Utility Equalizing Average Consumption Supplements

| **No Lottery vs. Lotttery** | **%** |
|---|---|
| Insurance | 0.00% |
| Borrowing/Lending | 0.74% |
| Savings Only | 0.26% |
| **Contract Comparison** | |
| Savings vs. B/L | 9.47% |
| Savings vs. Insurance | 69.76% |
| B/L vs. Insurance | 46.45% |

## Table 3 - Descriptive Statistics, Townsend Data

**ALL SAMPLE**

| | | | |
|---|---|---|---|
| Observations | 2,313 | Low Education entrepreneurs (% of all) | 9% |
| Mean Wealth (Baht) | 765,313 | Med. Education entrepreneurs | 14% |
| Median Wealth | 204,863 | High Education entrepreneurs | 21% |
| St. Dev. Wealth | 1,727,022 | Mean wealth for agents with fin. access | 1,054,983 |
| Max Wealth | 16,400,000 | Mean wealth for agents without fin. access | 515,091 |
| Min Wealth | 20 | % of entrepreneurs for agents with fin. access | 16% |
| Wealth skewness | 4.8154 | % of entrepreneurs for agents without fin. access | 12% |
| Fraction of Entrepreneurs | 14% | % of entrepreneurs for agents with wealth above median | 15% |
| Mean Wealth (entrepreneurs) | 1,465,582 | % of entrepreneurs for agents with wealth below median | 13% |
| Mean Wealth (workers) | 652,876 | | |

| NORTH-EAST | | CENTRAL | |
|---|---|---|---|
| Observations | 1,222 | Observations | 1,091 |
| Mean Wealth (Baht) | 386,359 | Mean Wealth (Baht) | 1,189,768 |
| Median Wealth | 180,000 | Median Wealth | 252,715 |
| St. Dev. Wealth | 667,794 | St. Dev. Wealth | 2,342,122 |
| Max Wealth | 12,600,000 | Max Wealth | 16,400,000 |
| Min Wealth | 20 | Min Wealth | 520 |
| Wealth skewness | 7.1911 | Wealth skewness | 3.4084 |
| Fraction of Entrepreneurs | 9% | Fraction of Entrepreneurs | 19% |
| Mean Wealth (entrepreneurs) | 451,251 | Mean Wealth (entrepreneurs) | 2,004,294 |
| Mean Wealth (workers) | 379,876 | Mean Wealth (workers) | 996,757 |
| Low Education entrepreneurs (% of all) | 5% | Low Education entrepreneurs (% of all) | 13% |
| Med. Education entrepreneurs | 9% | Med. Education entrepreneurs | 20% |
| High Education entrepreneurs | 16% | High Education entrepreneurs | 25% |
| Mean wealth for agents with fin. access | 479,109 | Mean wealth for agents with fin. access | 1,618,106 |
| Mean wealth for agents without fin. access | 315,323 | Mean wealth for agents without fin. access | 766,893 |
| % of entrepreneurs for agents with fin. access | 11% | % of entrepreneurs for agents with fin. access | 22% |
| % of entrepreneurs for agents without fin. access | 8% | % of entrepreneurs for agents without fin. access | 17% |

## Table 4a - Savings Only, no Wealth Lotteries

| Case # | Stratification | logL | $\gamma_1$ | $\gamma_2$ | $\lambda$ | $\alpha$ | q | m | $\kappa$ | w |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Parameter Estimates | | | | | | | |
| 1 | All Sample | **-0.4121** | 0.0914 | 0.2443 | 0.1258 | 0.1836 | 0.8801 | -0.7156 | 0.8706 | 0.8894 |
| | | | *(0.1150)* | *(0.6102)* | *(0.0437)* | *(0.0337)* | *(0.0411)* | *(3.8494)* | *(0.0861)* | *(0.7473)* |
| 2 | Central | **-0.4884** | 0.0981 | 0.2712 | 0.0962 | 0.1555 | 0.9443 | -0.6990 | 0.7820 | 1.0161 |
| | | | *(1.0034)* | *(1.0447)* | *(0.0176)* | *(0.0792)* | *(0.7711)* | *(6.9960)* | *(0.6591)* | *(0.7729)* |
| 3 | North-East | **-0.3193** | 1.3243 | 0.3010 | 0.0939 | 0.1996 | 1.0164 | -0.0002 | 0.2073 | 0.2018 |
| | | | *(0.1480)* | *(0.3542)* | *(0.1190)* | *(0.6036)* | *(3.7039)* | *(5.2612)* | *(4.5476)* | *(0.7301)* |
| 4 | Low Wealth | **-0.3792** | 1.2414 | 0.2593 | 0.0930 | 0.1953 | 1.0688 | 0.0013 | 0.1829 | 0.2374 |
| | | | *(1.8385)* | *(4.4304)* | *(5.3878)* | *(0.9546)* | *(3.5113)* | *(0.2351)* | *(2.0330)* | *(1.3569)* |
| 5 | High Wealth | **-0.4112** | 0.1586 | 1.5929 | 0.7435 | 0.1717 | 1.0338 | -0.0097 | 0.2682 | 0.5771 |
| | | | *(0.4203)* | *(5.1638)* | *(3.8865)* | *(1.8281)* | *(1.5612)* | *(6.4267)* | *(0.2273)* | *(1.8281)* |
| 6 | access to credit | **-0.4476** | 0.0624 | 0.3260 | 0.0847 | 0.1814 | 0.9259 | -0.7893 | 0.7700 | 0.9417 |
| | | | *(1.2257)* | *(1.0230)* | *(0.1135)* | *(0.1002)* | *(0.8880)* | *(5.7846)* | *(0.8312)* | *(0.7754)* |
| 7 | no access to credit | **-0.3754** | 0.0982 | 0.2641 | 0.0844 | 0.1953 | 0.8672 | -0.8763 | 0.7429 | 0.8676 |
| | | | *(3.5309)* | *(0.7480)* | *(0.0816)* | *(0.2874)* | *(3.6855)* | *(2.0127)* | *(3.6392)* | *(0.2468)* |
| 8 | Education < 4 yrs | **-0.3008** | 1.3765 | 0.3194 | 0.0939 | 0.1820 | 1.1175 | -0.3152 | 0.2200 | 0.2257 |
| | | | *(1.3939)* | *(0.9239)* | *(0.9863)* | *(4.4161)* | *(1.3898)* | *(3.1294)* | *(1.2436)* | *(4.4161)* |
| 9 | Education = 4 yrs | **-0.4144** | 0.0888 | 0.2441 | 0.1260 | 0.1835 | 0.8785 | -0.7255 | 0.8786 | 0.8875 |
| | | | *(0.0503)* | *(0.1162)* | *(0.0028)* | *(0.0485)* | *(0.2096)* | *(3.1944)* | *(0.2055)* | *(0.3965)* |
| 10 | Education > 4 yrs | **-0.5066** | 0.0990 | 0.2887 | 0.0995 | 0.1855 | 1.0321 | -0.5573 | 0.7898 | 1.0456 |
| | | | *(1.4153)* | *(1.5962)* | *(0.1404)* | *(0.0124)* | *(0.2968)* | *(2.6304)* | *(0.2879)* | *(1.0055)* |

## Table 4b - Savings Only, with Wealth Lotteries

| Case # | Stratification | logL | $\gamma_1$ | $\gamma_2$ | $\lambda$ | $\alpha$ | q | m | $\kappa$ | w |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Parameter Estimates | | | | | | | |
| 1 | All Sample | **-0.3966** | 1.2907 | 0.3142 | 0.1008 | 0.1933 | 1.1596 | 0.0016 | 1.0998 | 0.1944 |
| | | | *(2.5358)* | *(5.7088)* | *(5.2592)* | *(3.2730)* | *(0.0012)* | *(2.9532)* | *(0.0012)* | *(0.0015)* |
| 2 | Central | **-0.4808** | 1.2525 | 0.3111 | 0.0996 | 0.2014 | 0.9584 | 0.5609 | 1.4963 | 0.2008 |
| | | | *(5.7644)* | *(1.1575)* | *(0.2478)* | *(1.5599)* | *(2.7771)* | *(5.8927)* | *(2.1628)* | *(0.0012)* |
| 3 | North-East | **-0.3083** | 0.0665 | 0.8062 | 0.5262 | 0.2084 | 2.5566 | 0.0049 | 1.4529 | 0.1327 |
| | | | *(0.8762)* | *(2.1782)* | *(1.0101)* | *(4.0368)* | *(3.3581)* | *(3.0368)* | *(2.4909)* | *(0.0023)* |
| 4 | Low Wealth | **-0.3793** | 0.0470 | 1.0706 | 0.1923 | 0.1942 | 0.4001 | 0.0090 | 0.0134 | 0.0827 |
| | | | *(1.4667)* | *(3.1810)* | *(2.6582)* | *(4.2974)* | *(1.7480)* | *(4.3435)* | *(4.1603)* | *(0.0034)* |
| 5 | High Wealth | **-0.4029** | 0.1007 | 1.5373 | 0.0971 | 0.5014 | 1.0084 | 0.5121 | 0.2010 | 0.1977 |
| | | | *(5.3300)* | *(0.0000)* | *(5.3300)* | *(0.5518)* | *(1.2159)* | *(4.0793)* | *(1.2261)* | *(0.5518)* |
| 6 | access to credit | **-0.4347** | 1.3451 | 0.2899 | 0.1001 | 0.1980 | 1.2501 | 0.3098 | 1.2281 | 0.1984 |
| | | | *(0.1245)* | *(0.1137)* | *(0.1140)* | *(0.0034)* | *(3.7555)* | *(3.7333)* | *(4.6012)* | *(0.0964)* |
| 7 | no access to credit | **-0.3571** | 0.1157 | 0.5680 | 0.0936 | 0.2260 | 2.0383 | -0.0092 | 0.4476 | 0.9370 |
| | | | *(0.0029)* | *(0.0024)* | *(0.0000)* | *(0.0024)* | *(0.0400)* | *(3.3180)* | *(0.0024)* | *(0.0015)* |
| 8 | Education < 4 yrs | **-0.2988** | 1.3622 | 0.3203 | 0.1117 | 0.1782 | 1.3626 | -0.2981 | 1.4606 | 0.1913 |
| | | | *(0.0037)* | *(0.0028)* | *(0.2178)* | *(0.2181)* | *(0.0000)* | *(4.8708)* | *(0.0000)* | *(0.0019)* |
| 9 | Education = 4 yrs | **-0.3963** | 1.2907 | 0.3142 | 0.1008 | 0.1933 | 1.1596 | 0.0016 | 1.0998 | 0.1944 |
| | | | *(2.1791)* | *(4.8326)* | *(4.5198)* | *(2.8125)* | *(0.0015)* | *(2.9731)* | *(0.0000)* | *(0.0018)* |
| 10 | Education > 4 yrs | **-0.5010** | 0.0993 | 0.8157 | 0.0994 | 0.2019 | 0.5006 | 0.4885 | 1.5321 | 0.2026 |
| | | | *(3.0791)* | *(2.6152)* | *(0.2192)* | *(0.1178)* | *(2.1583)* | *(4.6636)* | *(2.5958)* | *(0.9723)* |

Note: Standard errors in the parentheses.

## Table 5a - Borrowing/Lending, no Wealth Lotteries

| Case # | Stratification | logL | $\gamma_1$ | $\gamma_2$ | $\lambda$ | $\alpha$ | q | m | $\kappa$ | w |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Parameter Estimates | | | | | | | |
| 1 | All Sample | **-0.3953** | 0.0633 | 0.8433 | 0.1065 | 0.4466 | 1.1894 | 0.0025 | 0.5466 | 0.5818 |
| | | | *(0.0795)* | *(0.0795)* | *(0.0795)* | *(0.0795)* | *(0.1610)* | *(2.9690)* | *(0.1610)* | *(0.0795)* |
| 2 | Central | **-0.4810** | 0.1006 | 1.4614 | 0.0995 | 0.4937 | 1.0082 | 0.5186 | 0.1973 | 0.2055 |
| | | | *(0.0010)* | *(0.0020)* | *(5.0904)* | *(3.1352)* | *(0.0274)* | *(4.1833)* | *(0.0286)* | *(0.0023)* |
| 3 | North-East | **-0.3067** | 0.5012 | 0.3727 | 0.1995 | 0.8518 | 3.3794 | -0.0109 | 0.6175 | 1.2507 |
| | | | *(0.0013)* | *(0.0000)* | *(0.0000)* | *(0.0013)* | *(0.0013)* | *(2.3519)* | *(0.0659)* | *(0.0013)* |
| 4 | Low Wealth | **-0.3792** | 0.5630 | 0.3264 | 0.1052 | 0.2072 | 2.9216 | -0.0016 | 0.7573 | 0.5617 |
| | | | *(0.4617)* | *(0.4617)* | *(0.4617)* | *(0.4617)* | *(0.0012)* | *(4.2389)* | *(0.0013)* | *(0.4248)* |
| 5 | High Wealth | **-0.4112** | 0.1418 | 0.8628 | 0.0718 | 0.5487 | 1.0196 | -0.0026 | 0.7774 | 0.4064 |
| | | | *(3.6722)* | *(2.2824)* | *(0.2837)* | *(0.0013)* | *(0.0952)* | *(4.3381)* | *(0.2837)* | *(2.9783)* |
| 6 | access to credit | **-0.4351** | 0.1249 | 1.0830 | 0.1307 | 0.4441 | 0.9280 | 0.2938 | 0.7760 | 0.4707 |
| | | | *(0.2309)* | *(0.0000)* | *(0.2309)* | *(0.1135)* | *(1.2451)* | *(0.0033)* | *(2.6039)* | *(0.2309)* |
| 7 | no access to credit | **-0.3545** | 0.1067 | 2.0852 | 0.1936 | 0.5356 | 0.5283 | -0.0029 | 1.3629 | 0.2009 |
| | | | *(0.1367)* | *(0.0011)* | *(0.9004)* | *(0.1367)* | *(0.0233)* | *(4.6556)* | *(0.0335)* | *(0.1367)* |
| 8 | Education < 4 yrs | **-0.2987** | 0.1267 | 1.0881 | 0.1306 | 0.4425 | 0.9258 | -0.2887 | 0.7755 | 0.4698 |
| | | | *(0.0000)* | *(0.0000)* | *(0.3919)* | *(0.3919)* | *(2.6639)* | *(2.6892)* | *(4.2457)* | *(2.2011)* |
| 9 | Education = 4 yrs | **-0.3922** | 0.1008 | 1.4767 | 0.4030 | 0.4334 | 0.5012 | 0.0012 | 1.2292 | 0.2243 |
| | | | *(0.0172)* | *(0.0992)* | *(0.0000)* | *(0.0172)* | *(0.0032)* | *(5.5154)* | *(0.0019)* | *(0.0172)* |
| 10 | Education > 4 yrs | **-0.5025** | 1.3088 | 0.7841 | 0.4042 | 0.4957 | 1.0064 | -0.5046 | 0.1998 | 0.2021 |
| | | | *(0.0172)* | *(0.0115)* | *(0.0172)* | *(0.0172)* | *(0.0172)* | *(3.0583)* | *(0.0114)* | *(0.0172)* |

## Table 5b - Borrowing/Lending, with Wealth Lotteries

| Case # | Stratification | logL | $\gamma_1$ | $\gamma_2$ | $\lambda$ | $\alpha$ | q | m | $\kappa$ | w |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Parameter Estimates | | | | | | | |
| 1 | All Sample | **-0.3921** | 0.1109 | 0.9432 | 0.0763 | 0.4600 | 0.9804 | 0.0010 | 0.6679 | 0.4729 |
| | | | *(0.0054)* | *(0.0000)* | *(0.0033)* | *(0.0033)* | *(0.0045)* | *(2.9842)* | *(0.0000)* | *(0.0033)* |
| 2 | Central | **-0.4734** | 0.1141 | 0.8290 | 0.1024 | 0.4608 | 0.9850 | 0.4485 | 0.8058 | 0.5274 |
| | | | *(0.0012)* | *(0.0012)* | *(0.0014)* | *(0.0386)* | *(0.0000)* | *(3.6158)* | *(0.0000)* | *(0.0386)* |
| 3 | North-East | **-0.3047** | 0.0990 | 1.4801 | 0.4116 | 0.4963 | 0.5016 | -0.4995 | 1.5291 | 0.1991 |
| | | | *(0.0011)* | *(1.2963)* | *(1.2963)* | *(1.2963)* | *(0.3836)* | *(3.3524)* | *(1.2963)* | *(0.0012)* |
| 4 | Low Wealth | **-0.3793** | 0.6569 | 0.2990 | 0.1094 | 0.8125 | 3.7422 | 0.0005 | 0.5697 | 1.0869 |
| | | | *(0.0623)* | *(0.0623)* | *(0.0623)* | *(0.0623)* | *(0.0213)* | *(3.0020)* | *(0.0623)* | *(0.0012)* |
| 5 | High Wealth | **-0.4028** | 0.1014 | 0.7738 | 0.4066 | 0.8090 | 2.9794 | 0.4981 | 0.2007 | 1.0145 |
| | | | *(0.0000)* | *(0.0000)* | *(0.0013)* | *(0.0013)* | *(1.0708)* | *(3.1610)* | *(1.0492)* | *(0.0013)* |
| 6 | access to credit | **-0.4321** | 0.1113 | 0.8588 | 0.1137 | 0.5338 | 1.0457 | 0.2450 | 0.8446 | 0.4667 |
| | | | *(0.0012)* | *(0.0014)* | *(0.0013)* | *(0.0226)* | *(0.0000)* | *(3.4003)* | *(0.0000)* | *(0.0226)* |
| 7 | no access to credit | **-0.3549** | 0.1254 | 1.4845 | 0.4259 | 0.5150 | 0.5647 | -0.0019 | 1.4475 | 0.2167 |
| | | | *(0.0006)* | *(0.0000)* | *(0.0001)* | *(0.0001)* | *(0.0001)* | *(4.4229)* | *(0.0000)* | *(0.0006)* |
| 8 | Education < 4 yrs | **-0.2987** | 0.1061 | 0.7533 | 0.0901 | 0.4883 | 0.9349 | -0.2887 | 1.0462 | 0.6016 |
| | | | *(0.0014)* | *(0.0012)* | *(1.0842)* | *(1.9424)* | *(0.0000)* | *(2.6697)* | *(0.0001)* | *(1.0842)* |
| 9 | Education = 4 yrs | **-0.3939** | 0.1019 | 2.0407 | 0.5367 | 0.5206 | 0.4983 | -0.0069 | 1.7003 | 0.2212 |
| | | | *(0.1242)* | *(0.0000)* | *(0.0000)* | *(0.0000)* | *(0.4165)* | *(2.8435)* | *(0.5078)* | *(0.0000)* |
| 10 | Education > 4 yrs | **-0.5041** | 0.1011 | 1.4988 | 0.4043 | 0.5000 | 0.5026 | 0.4885 | 1.5134 | 0.2001 |
| | | | *(0.0445)* | *(0.0011)* | *(0.0152)* | *(0.0184)* | *(0.0014)* | *(2.6423)* | *(0.0177)* | *(0.0000)* |

Note: Standard errors in the parentheses.

## Table 6a - Insurance/Transfers, no Wealth Lotteries

| Case # | Stratification | logL | $\gamma_1$ | $\gamma_2$ | $\lambda$ | $\alpha$ | q | m | $\kappa$ | w |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Parameter Estimates | | | | | | | |
| 1 | All Sample | **-0.3950** | 0.0989 | 1.4660 | 0.4213 | 0.7974 | 1.0003 | -0.5252 | 1.4939 | 0.4972 |
| | | | *(0.0651)* | *(0.0000)* | *(0.0000)* | *(0.0000)* | *(0.1122)* | *(1.6648)* | *(0.0751)* | *(0.0651)* |
| 2 | Central | **-0.4808** | 0.0966 | 1.4102 | 0.5715 | 0.8767 | 2.8253 | -0.6443 | 0.1990 | 0.4406 |
| | | | *(0.0010)* | *(0.0013)* | *(0.0008)* | *(0.0009)* | *(0.4640)* | *(1.5925)* | *(0.1696)* | *(0.2348)* |
| 3 | North-East | **-0.3045** | 0.6042 | 0.3027 | 0.1003 | 0.2067 | 0.9905 | -0.4741 | 0.7905 | 1.0238 |
| | | | *(0.0036)* | *(0.0036)* | *(0.0036)* | *(0.0000)* | *(4.7728)* | *(6.3616)* | *(4.7728)* | *(0.0001)* |
| 4 | Low Wealth | **-0.3789** | 1.2987 | 0.8019 | 0.1013 | 0.5012 | 0.9988 | -0.5137 | 0.2005 | 0.2005 |
| | | | *(0.0972)* | *(0.0371)* | *(0.0377)* | *(3.8983)* | *(0.0247)* | *(3.8192)* | *(3.9425)* | *(0.0000)* |
| 5 | High Wealth | **-0.4142** | 0.5830 | 0.3025 | 0.1011 | 0.2049 | 0.9861 | 0.5058 | 0.8093 | 0.5061 |
| | | | *(0.0031)* | *(0.0011)* | *(0.0004)* | *(0.0017)* | *(0.0011)* | *(4.7631)* | *(0.0023)* | *(0.0001)* |
| 6 | access to credit | **-0.4350** | 0.5972 | 0.2993 | 0.1003 | 0.2006 | 1.0005 | 0.5140 | 0.7988 | 0.5015 |
| | | | *(0.0024)* | *(0.0008)* | *(0.0202)* | *(0.0024)* | *(0.0024)* | *(3.5902)* | *(0.0019)* | *(0.0003)* |
| 7 | no access to credit | **-0.3538** | 0.5992 | 0.7826 | 0.6114 | 0.8084 | 2.9439 | -0.5088 | 0.2032 | 0.5055 |
| | | | *(0.0019)* | *(0.0224)* | *(0.0138)* | *(0.0003)* | *(0.0100)* | *(2.7864)* | *(0.0033)* | *(0.0004)* |
| 8 | Education < 4 yrs | **-0.2985** | 0.5882 | 0.3016 | 0.1015 | 0.2029 | 1.0063 | -0.5070 | 0.7884 | 0.5072 |
| | | | (0.0177) | (0.0556) | (0.9139) | (0.0180) | (0.4569) | (3.1652) | (0.4598) | (0.0005) |
| 9 | Education = 4 yrs | **-0.3915** | 0.1000 | 1.5124 | 0.4044 | 0.8090 | 1.0010 | -0.5009 | 1.4967 | 0.5102 |
| | | | *(0.0041)* | *(0.0027)* | *(0.0041)* | *(0.0041)* | *(0.0041)* | *(1.5765)* | *(0.0027)* | *(0.0011)* |
| 10 | Education > 4 yrs | **-0.5092** | 0.0999 | 1.4735 | 0.5952 | 0.8091 | 2.9998 | -0.5503 | 0.1977 | 0.4822 |
| | | | *(0.0032)* | *(0.0045)* | *(0.0001)* | *(0.0023)* | *(0.6150)* | *(1.6237)* | *(0.1667)* | *(0.3188)* |

## Table 6b - Insurance/Transfers, with Wealth Lotteries

| Case # | Stratification | logL | $\gamma_1$ | $\gamma_2$ | $\lambda$ | $\alpha$ | q | m | $\kappa$ | w |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Parameter Estimates | | | | | | | |
| 1 | All Sample | **-0.3924** | 0.1012 | 1.5167 | 0.3934 | 0.8088 | 1.0017 | -0.5070 | 1.4970 | 0.5034 |
| | | | *(6.9297)* | *(3.0576)* | *(0.1661)* | *(4.6373)* | *(0.3471)* | *(1.7215)* | *(0.2323)* | *(4.0274)* |
| 2 | Central | **-0.4776** | 0.6020 | 0.8377 | 0.1002 | 0.7992 | 0.9975 | -0.5014 | 1.4958 | 0.2000 |
| | | | *(0.0676)* | *(0.2611)* | *(0.5367)* | *(0.0368)* | *(0.0671)* | *(3.4772)* | *(1.1076)* | *(0.1219)* |
| 3 | North-East | **-0.3037** | 0.5935 | 0.3028 | 0.1008 | 0.2024 | 1.0022 | -0.5055 | 0.7848 | 1.0297 |
| | | | *(0.3998)* | *(2.0948)* | *(6.1769)* | *(5.8675)* | *(0.0013)* | *(3.5176)* | *(0.0004)* | *(0.0001)* |
| 4 | Low Wealth | **-0.3785** | 0.5985 | 1.5022 | 0.5993 | 0.5007 | 0.4999 | -0.5007 | 0.7998 | 0.2097 |
| | | | *(0.0005)* | *(0.0003)* | *(0.0086)* | *(0.0755)* | *(0.0077)* | *(5.4484)* | *(0.0005)* | *(0.0011)* |
| 5 | High Wealth | **-0.4054** | 0.1008 | 1.5114 | 0.4003 | 0.8061 | 0.9989 | -0.5039 | 1.5099 | 0.5027 |
| | | | *(3.2544)* | *(2.9450)* | *(4.4511)* | *(4.4511)* | *(4.4511)* | *(2.1891)* | *(2.9479)* | *(3.2544)* |
| 6 | access to credit | **-0.4363** | 1.2941 | 0.8158 | 0.6007 | 0.2003 | 1.0049 | -0.5006 | 0.2002 | 0.9990 |
| | | | (0.0223) | (0.0067) | (0.0001) | (0.1363) | (0.0000) | (4.5591) | (0.0001) | (0.0004) |
| 7 | no access to credit | **-0.3526** | 0.1003 | 1.4055 | 0.3944 | 0.8045 | 0.9974 | -0.6177 | 1.4991 | 0.4976 |
| | | | *(0.3425)* | *(0.0829)* | *(0.1165)* | *(0.0596)* | *(0.1165)* | *(1.7516)* | *(0.0777)* | *(0.2816)* |
| 8 | Education < 4 yrs | **-0.2988** | 0.6012 | 0.3013 | 0.1005 | 0.2004 | 0.9977 | -0.5027 | 0.8185 | 1.0053 |
| | | | *(0.3868)* | *(0.0004)* | *(0.0004)* | *(0.0004)* | *(0.0019)* | *(5.3579)* | *(0.4057)* | *(0.0023)* |
| 9 | Education = 4 yrs | **-0.3907** | 0.1007 | 1.5137 | 0.4021 | 0.8075 | 1.0014 | -0.5038 | 1.4838 | 0.5049 |
| | | | *(1.8332)* | *(1.2111)* | *(2.5953)* | *(1.8332)* | *(3.7738)* | *(1.7666)* | *(4.9455)* | *(0.0001)* |
| 10 | Education > 4 yrs | **-0.5054** | 1.3034 | 1.4814 | 0.6042 | 0.7861 | 1.0155 | -0.5161 | 1.5127 | 0.2014 |
| | | | *(0.0150)* | *(0.4678)* | *(0.1819)* | *(0.3523)* | *(0.57300* | *(1.7784)* | *(0.2329)* | *(0.3536)* |

Note: Standard errors in the parentheses.

# Table 7a - Model Comparisons

| Case # | # obs | Stratification | Normalized Likelihoods Sav. NL | B/L NL | Ins. NL | Sav. L | B/L L | Ins. L |
|---|---|---|---|---|---|---|---|---|
| 1 | 2313 | All Sample | -0.4121 | -0.3953 | -0.3950 | -0.3966 | -0.3921 | -0.3924 |
| 2 | 1091 | Central | -0.4884 | -0.4810 | -0.4808 | -0.4808 | -0.4734 | -0.4776 |
| 3 | 1222 | North-East | -0.3193 | -0.3067 | -0.3045 | -0.3083 | -0.3047 | -0.3037 |
| 4 | 1157 | Low Wealth | -0.3792 | -0.3792 | -0.3789 | -0.3793 | -0.3793 | -0.3785 |
| 5 | 1156 | High Wealth | -0.4112 | -0.4112 | -0.4142 | -0.4029 | -0.4028 | -0.4054 |
| 6 | 1072 | access to credit | -0.4476 | -0.4351 | -0.4350 | -0.4347 | -0.4321 | -0.4363 |
| 7 | 1241 | no access to credit | -0.3754 | -0.3545 | -0.3538 | -0.3571 | -0.3549 | -0.3526 |
| 8 | 1927 | no formal credit | -0.3911 | -0.3732 | -0.3745 | -0.3728 | -0.3744 | -0.3743 |
| 9 | 596 | agric. Credit | -0.4689 | -0.4524 | -0.4509 | -0.4506 | -0.4508 | -0.4510 |
| 10 | 381 | access to BAAC | -0.5038 | -0.4733 | -0.4740 | -0.4744 | -0.4726 | -0.4723 |
| 11 | 1932 | no access to BAAC | -0.3939 | -0.3735 | -0.3749 | -0.3784 | -0.3726 | -0.3730 |
| 12 | 455 | Education < 4 yrs | -0.3008 | -0.2987 | -0.2985 | -0.2988 | -0.2987 | -0.2988 |
| 13 | 1554 | Education = 4 yrs | -0.4144 | -0.3922 | -0.3915 | -0.3963 | -0.3939 | -0.3907 |
| 14 | 304 | Education > 4 yrs | -0.5066 | -0.5025 | -0.5092 | -0.5010 | -0.5041 | -0.5054 |
| 15 | 1388 | has any debt | -0.4553 | -0.4356 | -0.4350 | -0.4350 | -0.4350 | -0.4312 |
| 16 | 826 | has informal debt | -0.4107 | -0.3910 | -0.3901 | -0.3933 | -0.3891 | -0.3885 |
| 17 | 377 | has formal debt | -0.4977 | -0.4707 | -0.4819 | -0.4763 | -0.4764 | -0.4737 |
| 18 | 231 | NE, access to BAAC | -0.4251 | -0.4222 | -0.4202 | -0.4253 | -0.4247 | -0.4210 |
| 19 | 150 | C, access to BAAC | -0.5937 | -0.5351 | -0.5326 | -0.5259 | -0.5346 | -0.5314 |

## Table 7b - Model Comparisons

| Case # | # obs | %ents | Stratification | SNL v BNL | SNL v INL | BNL v INL | BL v SL | SL v IL | BL v IL |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Comparisons | | | | | |
| 1 | 2313 | 13.8% | All Sample | -3.1208** | -3.8197** | -0.0966 | 2.0094** | -1.9946** | 0.2515 |
| 2 | 1091 | 19.2% | Central | -1.5358 | -1.6860* | -0.6282 | 1.8330* | -0.7806 | 1.2289 |
| 3 | 1222 | 9.1% | North-East | -1.1218 | -1.2779 | -0.8367 | 0.6472 | -2.0217** | -0.2337 |
| 4 | 1157 | 12.6% | Low Wealth | 0.0270 | -0.1906 | -0.3879 | -0.0357 | -0.9423 | -1.0632 |
| 5 | 1156 | 15.1% | High Wealth | 0.2517 | 0.3785 | 0.3764 | 0.2858 | 0.5400 | 0.5503 |
| 6 | 1072 | 16.1% | access to credit | -2.2296** | -2.1397** | -0.0353 | 0.9737 | 0.8237 | 1.1333 |
| 7 | 1241 | 11.9% | no access to credit | -2.9821** | -2.9277** | -0.2215 | 1.4024 | -1.2893 | -0.9921 |
| 8 | 1927 | 12.8% | no formal credit | -3.9299** | -3.2705** | 0.4424 | -0.4150 | 0.3766 | -0.0145 |
| 9 | 596 | 18.0% | agric. Credit | -2.5389** | -2.1941** | -0.5781 | -0.0661 | 0.1461 | 0.2889 |
| 10 | 381 | 19.4% | access to BAAC | -2.6094** | -2.1376** | 0.1549 | 0.4409 | -0.2677 | -0.0379 |
| 11 | 1932 | 12.7% | no access to BAAC | -3.6756** | -4.0439** | 0.5975 | 2.1797** | -2.2109** | 0.4108 |
| 12 | 455 | 9.0% | Education < 4 yrs | -0.6780 | -0.5392 | -0.1611 | 0.1069 | -0.0424 | 0.0920 |
| 13 | 1554 | 13.8% | Education = 4 yrs | -3.3103** | -4.2849** | -0.2351 | 0.5026 | -1.7535* | -0.9667 |
| 14 | 304 | 21.1% | Education > 4 yrs | -0.3912 | 0.3121 | 0.8931 | -0.5030 | 0.4651 | 0.1087 |
| 15 | 1388 | 16.1% | has any debt | -2.7410** | -3.2053** | -0.1076 | -0.0033 | -1.2939 | -1.2085 |
| 16 | 826 | 13.3% | has informal debt | -2.4713** | -2.6708** | -0.2966 | 1.2555 | -1.2412 | -0.4262 |
| 17 | 377 | 19.9% | has formal debt | -1.5532 | -1.7196* | 0.8111 | -0.0823 | -0.3068 | -0.3403 |
| 18 | 231 | 15.1% | NE, access to BAAC | -1.1406 | -0.5817 | -0.2598 | 1.6763* | -1.2935 | -1.1210 |
| 19 | 150 | 26.0% | C, access to BAAC | -1.8704* | -2.4783** | -0.2349 | -0.8478 | 0.6003 | -0.2251 |

| Case # | # obs | %ents | Stratification | SNL v SL | BNL v BL | INL v IL |
|---|---|---|---|---|---|---|
| 1 | 2313 | 13.8% | All Sample | -3.4909** | -1.8614* | -1.5230 |
| 2 | 1091 | 19.2% | Central | -1.8316* | -2.0320** | -0.7869 |
| 3 | 1222 | 9.1% | North-East | -0.9802 | -0.3595 | -0.7900 |
| 4 | 1157 | 12.6% | Low Wealth | 0.0602 | 0.1422 | -0.3609 |
| 5 | 1156 | 15.1% | High Wealth | -1.2570 | -1.3105 | -2.4186** |
| 6 | 1072 | 16.1% | access to credit | -2.2578** | -1.2449 | 0.7603 |
| 7 | 1241 | 11.9% | no access to credit | -2.3314** | 0.1098 | -0.4432 |
| 8 | 1927 | 12.8% | no formal credit | -3.6252** | 0.3307 | -0.1956 |
| 9 | 596 | 18.0% | agric. Credit | -2.5279** | -0.5954 | 0.0187 |
| 10 | 381 | 19.4% | access to BAAC | -2.5255** | -0.3624 | -0.1919 |
| 11 | 1932 | 12.7% | no access to BAAC | -3.2398** | -0.7641 | -0.8766 |
| 12 | 455 | 9.0% | Education < 4 yrs | -0.7483 | -0.0001 | 0.1864 |
| 13 | 1554 | 13.8% | Education = 4 yrs | -3.3584** | 0.3564 | -0.5525 |
| 14 | 304 | 21.1% | Education > 4 yrs | -0.4867 | 0.1391 | -0.7295 |
| 15 | 1388 | 16.1% | has any debt | -3.3626** | -0.4036 | -1.1727 |
| 16 | 826 | 13.3% | has informal debt | -2.2364** | -0.7637 | -0.6941 |
| 17 | 377 | 19.9% | has formal debt | -1.5016 | 0.8683 | -0.7570 |
| 18 | 231 | 15.1% | NE, access to BAAC | 0.1186 | 2.2584** | 0.0893 |
| 19 | 150 | 26.0% | C, access to BAAC | -2.5230** | -0.024 | -0.2479 |

Note: ** means significant at 5% level and * means significant at 10% level.

## Table 8 - Descriptive Statistics, 1976 SES Data

| | |
|---|---|
| Observations | 10,613 |
| Mean Wealth (Baht) | 93,492 |
| Median Wealth | 57,462 |
| St. Dev. Wealth | 110,783 |
| Max Wealth | 1,341,589 |
| Min Wealth | 2,560 |
| Wealth skewness | 3.3689 |
| Fraction of entrepreneurs | 22% |
| Mean Wealth (entrepreneurs) | 133,813 |
| Mean Wealth (workers) | 83,022 |

## Table 9a - Static Parameter Estimates, 1976 SES Data

| Model | logL | $\gamma_1$ | $\gamma_2$ | $\lambda$ | $\alpha$ | q | m | $\kappa$ | w |
|---|---|---|---|---|---|---|---|---|---|
| SL | **-0.3972** | 0.6185 | 0.9481 | 0.1001 | 0.1795 | 1.0159 | -0.3605 | 0.8654 | 0.5720 |
| BL | **-0.4019** | 0.0906 | 0.2619 | 0.0955 | 0.4912 | 3.0488 | -0.5858 | 0.1914 | 1.1069 |
| IL | **-0.4049** | 0.1046 | 1.5149 | 0.1132 | 0.7983 | 0.9991 | 0.4243 | 1.4966 | 0.5002 |

## Table 9b - Dynamic Parameter Estimates, 1976 SES Data

| Model | logL | s | g | Metric |
|---|---|---|---|---|
| SL | **-0.3972** | 0.2758 | 0.0887 | 1.5675 |
| BL | **-0.4019** | 0.3379 | 0.1593 | 9.9826 |
| IL | **-0.4049** | 0.2767 | 0.0614 | 1.6989 |