Borrower Runs *

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Abstract

Microfinance institutions and other lenders in developing countries rely on the promise of future loans to induce repayment. We show that such a promise is not always credible. If borrowers expect that others will default, and so loans will no longer be available in the future, then they will default as well. We refer to such contagion as a borrower run. The optimal lending contract must provide additional repayment incentives to counter this tendency to default.

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1 Introduction

Microfinance is an increasingly important form of financial intermediation. The success of the Grameen Bank in making group loans to poor (and predominantly female) borrowers in Bangladesh is especially well known. Microfinance institutions (henceforth MFIs) such as the Grameen Bank in Bangladesh, the Bank Rakyat Indonesia, the Bank for Agriculture and Agricultural Cooperatives in Thailand, and BancoSol in Bolivia, are among the largest banks in their respective countries. There are over 2500 MFIs worldwide, reaching at least 67 million people (Daley-Harris 2003).

The starting point for our paper is the familiar observation that since MFI borrowers possess limited collateral, an important source of repayment incentives is the prospect of receiving future credit.[1] A promise of future credit, along with a concomitant threat of credit denial, can induce repayment as follows. A borrower who repays today's loan effectively receives a claim to (valuable) future financial access. The borrower repays if the value of this claim exceeds the benefit of defaulting on the loan. Notice, however, that the expected value of a repaying borrower's claim depends on how likely other borrowers are to repay since that in turn affects the viability of the MFI.

We show that such repayment externalities can lead to a coordination failure in which borrowers choose to default because they expect that others will. We label this coordination failure as a *borrower run*. Unlike the depositor runs that have been widely analyzed in the literature (Diamond and Dybvig, 1983; Goldstein and

[1] This is clearest in the case of MFIs like Bank Rakyat Indonesia that grant individual loans (Churchill 1999). Armendariz and Morduch (2000) present a formal model based on Bolton and Scharfstein (1990). It is equally true of group lending schemes: while many academic papers have highlighted the role of groups in ameliorating information asymmetries (Ghatak and Guinnane, 1999), borrowers must still be induced to repay an uncollateralized loan. Reflecting this, most group lending schemes offer a group of borrowers repeated loans over time (Morduch 1999).

Pauzner, 2005) borrower runs arise on the asset side of the intermediary's balance sheet.[2]

We model the strategic interaction between borrowers in a global games framework (Carlsson and van Damme, 1993; Morris and Shin, 2003). Each borrower receives a private signal of economic fundamentals in the future. These fundamentals and the MFI's financial position affect the value of future financial access, and hence the incentive to repay. We compare two situations: with and without strategic interaction between borrowers. In both models, borrowers repay if the value of their future relationship exceeds a threshold. The threshold for repayment is higher if there is strategic interaction between borrowers. Strategic complementarity between borrowers in their repayment decisions makes borrowers default even when collectively they would prefer to repay. Borrower runs therefore weaken repayment incentives and lower welfare.

We examine the effect of borrower runs on the MFI's choice of lending contract. The MFI can increase incentives to repay in two ways. First, it can make loans that are more profitable, thereby increasing the value of future financial access to repaying borrowers. Secondly, the MFI can lower the repayment required on its loan. We show that the MFI will always use at least one of these two repayment incentives as an optimal response to borrower runs. We also show that the MFI's initial financial resources are valuable in preventing borrower runs.

Borrower runs may be a concern in any context where repayment is supported by

[2]The borrower runs we analyze are also distinct from the default equilibrium that Besley and Coate (1995) discuss as a drawback of group lending. In their model, an individual will default if others in his group choose to do so because he is liable for their repayment and will be punished even if he repays. In our model co-ordination failures do not arise because of the joint liability terms of the group loan contract. Instead, as we will show, there are repayment externalities across borrowers — even though contracts make no explicit use of joint liability.

the threat of credit denial. In this paper we focus on the implications of borrower runs for microfinance practice, and discuss other possible applications in Section 5. There is some anecdotal evidence that borrower runs have contributed to the collapse of lending programs, and are a concern for MFIs. For example, in the case of Childreach in Ecuador, "the number of residents defaulting on loans multiplied as the word spread that few people were paying, that what had been repaid was being pilfered by community leaders in at least a quarter of the communities, and that Childreach was taking little action" (see Goering and Marx, 1998). In terms of our model, since the viability of Childreach had been called to question, default became more attractive for each individual borrower. Related, Paxton *et al* (2000) empirically analyze repayment behavior *within* groups in a Burkina Faso microfinance program, but also write:

> In one urban sector that experienced widespread default, rumors of unethical behavior led the entire sector to collapse. In any sector, the first group may default for any number of reasons, but once this occurs the whole sector tends to collapse. In the words of PPPCR [the microfinance program analyzed] founder Konrad Ellsasser, the success of group lending can be likened to an airplane: if even one part fails, the plane cannot fly.

Not surprisingly, microfinance practitioners appear to be actively concerned about "contagion" default effects of this kind. For example, van Maanen (2004), a former managing director of one of the world's largest private capital providers of microfinance, writes:

> Once the [repayment] percentage sinks below 80% then it is very difficult to reverse that trend, because the virus travels faster than any medicine: [a borrower thinks to himself] 'why should I repay an MFI that is likely to go down? Let me wait and see what happens!'

1.1 Paper outline

The paper proceeds as follows. Section 2 describes the basic model. Section 3 discusses a benchmark with no strategic interaction. Section 4 explores the effect of borrower runs on welfare and on lending terms. Section 5 discusses other possible applications. Section 6 concludes.

2 Model

There is a continuum of identical borrowers who need outside finance to make investments. Loans are made by a microfinance institution (MFI) that aims to maximize the welfare of the borrowers. The MFI has funds A_0 per borrower. The MFI uses these funds to make loans, with (endogenously determined) loan size L and required repayment (face value) F. It earns a rate of return of $\rho > 1$ on any funds $A_0 - L$ that it does not lend out. In order to apply results from the global games literature (see below), it is necessary to rule out "loan" contracts with very low values of F, that is, grants. We assume that there is a strictly positive lower bound on the required repayment, i.e., $F \geq \underline{F} > 0$,[3] where $\underline{F}$ can be arbitrarily small.

The timing is as follows. The MFI determines the contract terms L and F, and makes loans. Borrowers invest any funds they receive. If a borrower invests L today, his return is $H(L)$, where $H(L)$ is concave and $H'(L) \to 1$ as $L \to \infty$. After output is realized, borrowers simultaneously decide whether to repay or to default. Let $\alpha \in [0,1]$ denote the fraction of borrowers who repay. The MFI's funds per borrower

[3] This is natural in a richer model: suppose there exist a large number of agents, some with projects and some without. Suppose further that each agent in the economy has a small amount of collateral $\underline{F}$. Then the MFI needs to set $F > \underline{F}$ in order to screen out the project-less borrowers. Rajan (1992) makes a similar assumption to rule out grants.

after repayment are thus

$$A(\alpha; L, F) \equiv \rho(A_0 - L) + \alpha F. \tag{1}$$

The only difficulty that the MFI faces is that of enforcing repayments. To enforce repayment F, the MFI promises future financial access to borrowers who repay and denies future financial access to borrowers who default. The value of future loans from the MFI depends on the MFI's financial resources A, on the fraction α of borrowers who repay, and on future economic fundamentals. We denote future economic fundamentals by x, where x is a random variable drawn uniformly from $[0, \bar{x}]$. Higher values of x indicate more profitable investment opportunities for all borrowers and hence increase the value of future financial access. Let $v(x, A, \alpha)$ denote the value of the future loans from the MFI where v is assumed to be continuous in fundamental x, funds A and fraction who repay α.[4] One simple parameterization is $v(x, A, \alpha) = x \Pr\left(A + \xi \geq \bar{A}\right)$, where ξ is a shock to MFI funds, $\bar{A}$ is the minimum amount of funds required for the MFI to continue operation, and x represents the borrower's value of a continued relationship with the MFI.

More generally, we conduct our analysis under the following assumptions on $v(x, A, \alpha)$:

A1. $v(x, A, \alpha)$ is strictly increasing and linear in x: The value of future loans is higher when economic conditions are favorable.

A2. $v(x, A, \alpha)$ strictly increasing in A: The value of future loans is higher if the MFI has more financial resources.

[4] Fully specified models of financial market exclusion can be found in, for example, Bolton and Scharfstein (1990), Bond and Krishnamurthy (2004), Kehoe and Levine (1993) and Kocherlakota (1996).

A3. Lower dominance, $v(0, A, \alpha) = 0$: Default is a dominant strategy for realizations of the fundamental x that are sufficiently low.

A4. Upper dominance, $v(\bar{x}, 0, 0) > H(A_0)$: The value of future loans exceeds the highest repayment that can possibly be required, $H(A_0)$, for x sufficiently high, independent of what other borrowers repay. As such, repayment is a dominant strategy for high enough fundamentals.[5]

A5. Strict strategic complementarity,

$$\frac{\partial}{\partial \alpha} v(x, A(\alpha), \alpha) = F v_A + v_\alpha > 0. \tag{2}$$

The incentive to repay is strictly increasing in the proportion of borrowers who repay. By $A2$, the term v_A is positive. In general, the term v_α may be either positive (if, for example, donors reward MFIs with high repayment rates); negative (if a fixed quantity of MFI resources are shared among more repaying borrowers); or zero. The content of the assumption is that even if v_α is negative the first term dominates.

A6. Diminishing importance of funds as repayment rises:

$$\frac{\partial^2}{\partial \alpha \partial A_0} \ln v(x, A(\alpha), \alpha) \leq 0.$$

That is, the percentage improvement in the value of future loans caused by an increase in A,

$$\frac{v_A(x, A(\alpha), \alpha)}{v(x, A(\alpha), \alpha)},$$

diminishes as repayment rates rise. In the special case when v has no direct dependence on α, i.e. if the continuation utility is $v(x, A)$, this assumption is

[5] The fact that $v(\bar{x}, A, 0) > H(A_0)$ even when $A = 0$ can be motivated by assuming that even an MFI with no funds ($A = 0$) has a small chance of receiving new outside financing. (This probability of new funds can be made arbitrarily small if the best fundamental $\bar{x}$ is simultaneously made large.)

just log concavity of v in A. This assumption is used only for Propositions 3 and 4, for which it is sufficient but not necessary.

A7. $v_A(\bar{x}, A, 1) > 1$, i.e., at the highest realization of x an additional dollar is more valuable to the borrower in the hands of the MFI. This assumption is only used to establish Lemma 1.

Strategic complementarity (A5) is a natural feature of the repayment game we study — the more funds an MFI has, the more value a borrower places on a continued relation with the MFI. Economically, strategic complementarity potentially generates multiple equilibria in repayment behavior (Cooper and John, 1988). Moreover, together with the dominance region assumptions $A3$ and $A4$, and the state monotonicity assumption $A1$, strategic complementarity allows us to exploit well-known global games results on equilibrium uniqueness (Morris and Shin 2003).

3 Single-borrower benchmark

In this section we abstract from strategic interaction between borrowers. To do so, we discuss the MFI's contracting problem when there is just one borrower in the economy. We assume that the borrower directly observes the fundamental x.[6]

We take the MFI's objective to be the maximization of borrower welfare. The MFI chooses the loan terms L and F. In keeping with MFI practice, we restrict attention to standard debt contracts in which F is not contingent on the realization of the fundamental x.[7] Let $\alpha^1(x; L, F) \in \{0, 1\}$ denote the borrower's repayment

[6] Our results would be unchanged if the borrower instead received a noisy signal of x, and we took limits as the noise approached 0.

[7] In Appendix B we consider the opposite extreme in which the MFI can both discover x and write a contract in which the repayment F is contingent on x. Our main result – borrower runs reduce repayment incentives – is largely unaffected by allowing such contingencies.

decision for a given realization of the fundamental x and the loan contract (L, F), where $\alpha^1(x; L, F) = 1$ denotes repayment. (The borrower gains nothing from partial default.) Expected borrower welfare is thus

$$W^1(L, F) \equiv H(L) + E_x\left[\alpha^1(x; L, F)\left(v\left(x, A\left(\alpha^1(x; L, F); L, F\right), \alpha^1(x; L, F)\right) - F\right)\right],$$

where the first term is the project return from investing and the second term is the expected gain in future utility, less repayment, if the borrower repays. We assume that the MFI cannot lend out more than its initial funds (i.e., $L \leq A_0$) and that borrowers cannot repay more than their project return (i.e., $F \leq H(L)$).

Absent strategic interaction, the borrower repays if and only if the value of future loans from the MFI exceeds the direct cost of repayment. Define $X^1(L, F)$ as the realization of the future economic fundamental for a given loan contract (L, F) that makes the borrower indifferent between repaying and defaulting, i.e.

$$v\left(X^1(L, F), A(1; L, F), 1\right) = F. \tag{3}$$

Given $A1$, the borrower repays if and only if $x \geq X^1(L, F)$.

Lemma 1 *The threshold $X^1(L, F)$ exists and is unique.*

The proof is in Appendix A.

The MFI's problem can thus be rewritten as:

$$\max_{L \leq A_0, F \in [\underline{F}, H(L)]} H(L) + \frac{1}{\bar{x}} \int_{X^1(L,F)}^{\bar{x}} \left(v(x, A(1; L, F), 1) - F\right) dx. \tag{4}$$

For use below, note that providing the optimal loan size is interior (i.e., $L < A_0$) the borrower's marginal return is lower than the rate of return on unlent funds, ρ. The reason is that the MFI only benefits from holding onto funds if the borrower repays, and this occurs with a probability less than one.

Lemma 2 *The optimal loan size in the single-borrower benchmark is such that either $H'(L) < \rho$, or $L = A_0$.*

The proof is in Appendix A.

4 Many borrowers

We now turn to the heart of our analysis, and examine the effects of strategic interaction among borrowers on repayment.

4.1 Perfect information

First, suppose as before that borrowers observe the fundamental x. For a given loan contract (L, F) this perfect information coordination game can have multiple pure strategy equilibria. In one equilibrium all borrowers repay, while in another equilibrium all borrowers default. Such multiple equilibria are a standard consequence of strategic complementarities.

In the repayment equilibrium borrowers anticipate that others will repay, which strengthens the MFI's future financial position and makes individual repayment attractive. The repayment equilibrium occurs if the value of future loans (given repayment by other borrowers) exceeds the required repayment,

$$v\left(x, A\left(1; L, F\right), 1\right) \geq F.$$

In the default equilibrium, borrowers default because they anticipate others will default. There is a default equilibrium if the value of future loans (given default by other borrowers) is less than the required repayment F,

$$v\left(x, A\left(0; L, F\right), 0\right) \leq F.$$

As such, multiple equilibria exist whenever

$$v\left(x, A\left(1; L, F\right), 1\right) \geq F \geq v\left(x, A\left(0; L, F\right), 0\right).$$

Such multiplicity makes it difficult to specify the MFI's optimal lending contract and makes welfare comparisons with the benchmark problematic. Following the global games literature we next introduce slight uncertainty to borrower information about the fundamental x, which generates a unique equilibrium.

4.2 Near-perfect information

Suppose that borrowers do not directly observe x, but instead each borrower i receives a signal $y_i = x + \sigma\varepsilon_i$, where ε_i are independently and identically distributed across borrowers. The parameter σ indexes the variance of the noise term in the signal. When the variance σ is sufficiently small standard results from the theory of global games imply that there is a unique equilibrium for each realization of the fundamental x. Specifically, as the noise becomes small each borrower follows a threshold strategy — default when $y_i < X^*$, and repay when $y_i \geq X^*$ — where X^* is defined by[8]

$$\int_0^1 \left(v\left(X^*, A\left(\alpha; L, F\right), \alpha\right) - F\right) d\alpha = 0. \tag{5}$$

Moreover, note that as noise becomes small ($\sigma \to 0$) borrower signals coincide with the fundamental x, and so the equilibrium converges to one in which all borrowers default for fundamentals $x < X^*$ and all borrowers repay for fundamentals $x \geq X^*$.

As in the perfect information case (section 4.1), repayment externalities lead borrowers to default. In particular, given a loan contract (L, F), there exist realizations of economic fundamentals that are higher than the repayment threshold $X^1(L, F)$ in the single-borrower benchmark but lower than the repayment threshold $X^*(L, F)$

[8]See Proposition 2.2 in Morris and Shin (2003).

in the many-borrower case. For such realizations, the MFI is repaid in the single-borrower benchmark but faces widespread default if borrowers interact strategically. We refer to the behavior of borrowers at fundamentals $x \in \left(X^1(L,F), X^*(L,F)\right)$ as a *borrower run*.

Our main results below all compare outcomes in the benchmark single-borrower problem (section 3) to the many-borrower problem with near-perfect information. Because by definition borrower runs only arise in the latter case, we will often describe any differences as stemming from borrower runs.

It remains to establish that borrower runs actually occur, i.e. that $X^*(L,F) > X^1(L,F)$. This is easily shown. By strategic complementarity ($A5$), $v(X^*, A(\alpha), \alpha)$ is increasing in α, and so (5) implies that[9]

$$v(X^*, A(1), 1) - F > 0 = v\left(X^1, A(1), 1\right) - F.$$

Since $v_x > 0$ (by $A1$), it follows that $X^*(L,F) > X^1(L,F)$. Hence we have shown:

Proposition 1 $X^*(L,F) > X^1(L,F)$*: With many borrowers the MFI is subject to a coordination failure, where borrowers fail to repay because they anticipate others failing to repay. That is, borrower runs occur.*

In contrast with the perfect information case (section 4.1), Proposition 1 establishes that the default equilibrium is played in equilibrium with positive probability.

By making a repayment each borrower is improving the MFI's financial position, and hence increasing the repayment incentive of other borrowers. This externality is not taken into account, however, by individuals in the repayment game. For that reason, there is too little repayment in the many-borrower case relative to the benchmark. Borrowers would collectively prefer to repay if $X^1(L,F) < x < X^*(L,F)$

[9] To see this, note that if instead $v(X^*, A(1), 1) - F \leq 0$, then the integral in (5) is strictly negative.

since the value of future loans dominates defaulting but repayment externalities lead to default instead. Borrower runs therefore lower welfare.

Put more formally, as $\sigma \to 0$ (the variance of the noise term approaches zero), the MFI's welfare converges to

$$W(L,F) = H(L) + \frac{1}{\bar{x}} \int_{X^*(L,F)}^{\bar{x}} (v(x, A(1), 1) - F)\, dx.$$

For a given loan contract (L, F), the difference in borrower welfare between the single-borrower and many-borrower cases is thus

$$W^1(L,F) - W(L,F) = \frac{1}{\bar{x}} \int_{X^1(L,F)}^{X^*(L,F)} (v(x, A(1), 1) - F)\, dx.$$

Recall that by definition $v(X^1(L,F), A(1), 1) - F = 0$. Hence by $A1$ welfare is higher in the single-borrower case, $W^1(L,F) > W(L,F)$. Since this is true for *any* loan contract, it follows that:

Proposition 2 *The maximal attainable welfare is lower when there are many borrowers. That is, borrower runs lower welfare.*

Holding the loan contract fixed, higher initial per-borrower resources A_0 act as a repayment incentive in both the single-borrower and the many-borrower cases. Since a higher A_0 implies higher future resources A, this increases the future utility of a repaying borrower, $v(x, A, \alpha)$, and in turn increases the likelihood that a borrower will repay. In addition, a higher A_0 reduces the likelihood of borrower runs – and so initial funds are more valuable in the many-borrower case than in the benchmark.

Proposition 3 $W_{A_0} > W^1_{A_0}$*: Holding the loan contract fixed, initial funds are more valuable in the many-borrower problem. That is, borrower runs increase the importance of initial funds.*

The proof is in Appendix A.

An increase in the loan size L makes repayment less attractive since it lowers the MFI's future resources A. Consequently the MFI will scale back L for any given level of F, an immediate consequence of Proposition 3:

Corollary 1 *$W_L < W_L^1$, and so for any repayment level F the MFI chooses a smaller loan size when there are many borrowers. That is, borrower runs lead to smaller loans (holding F fixed).*

An important implication of Corollary 1 is that for any given face value of debt F borrower runs lead the MFI to increase the profitability/reduce the subsidy of its loans, in the sense of increasing $F - \rho L$.

Raising the repayment F required on loans has both a direct effect (higher repayments are costly to the borrower) and an indirect effect (the MFI's financial position is stronger, giving the borrower more incentive to repay). The net effect is hard to sign. However, if the MFI raises F by one dollar and raises L by $\frac{1}{\rho}$ or more, the loan is less profitable (even if repaid) and the MFI has a weaker financial position. In this case both the direct and indirect effects act in the same direction, and discourage repayment.

Because borrower runs reduce repayment incentives, the MFI needs to change the loan terms in some way to increase repayment. From the above, it follows that it either increases the profitability of the loan, or decreases the required repayment F, or does both. In contrast, the MFI definitely *does not* both increase F and reduce the profitability of the loan.

Formally, let (L^1, F^1) and (L^*, F^*) be optimal loan contracts in the single and many-borrower problem, respectively. We prove:

Proposition 4 *At least one of the following is true: (A) The many-borrower contract is more profitable, in the sense that $F^* - \rho L^* \geq F^1 - \rho L^1$; or (B) the required* *repayment is lower in the many-borrower contract, $F^* \leq F^1$. Both relations are strict if $L^1 < A_0$.*

The proof is in Appendix *A*.

Proposition 4 says that borrower runs cause the MFI to either increase loan profitability, so as to increase the relationship value it can offer to repaying borrowers, or decrease the repayment request. Moreover, in a couple of special cases one can say even more.

Suppose first that the feasibility constraint binds in the single-borrower problem. Then if the MFI were to ask for a higher repayment F in the many-borrower problem it would need to offer a higher loan L and that would reduce its profits (by Lemma 2). This is clearly a contradiction of Proposition 4. Therefore, the MFI must reduce the repayment required:

Corollary 2 *If the feasibility constraint binds in the single-borrower problem, then the optimal repayment request is lower in the many-borrower problem, $F^* \leq F^1$ (strict if $L^1 < A_0$). That is, borrower runs lead to a reduction in F.*

Suppose next that the feasibility constraint binds in *both* the single-borrower and many-borrower problems. From Corollary 2, the optimal loan repayment F is lower in the many-borrower problem. Since the borrower repayment constraint binds in both problems, it follows that the loan size is also smaller. Finally, since output H is subject to decreasing returns, profitability is higher in the many-borrower problem. So borrower runs have the following effect:

Corollary 3 *If the feasibility constraint binds in both the single-borrower and many-borrower problem, then the optimal loan size and repayment request are lower in the many-borrower problem, i.e. $L^* \leq L^1$ and $F^* \leq F^1$, and loan profitability is higher, $F^* - \rho L^* \geq F^1 - \rho L^1$ (all strict if $L^1 < A_0$).*

5 Other applications

Thus far we have focused on the impact of borrower runs on microfinance. However, in principle borrower runs can occur in any context where repayment is supported by the threat of credit denial. Informal lending relationships and credit cooperatives resemble microfinance in this respect, and are obvious examples.

Like microfinance loans, international debt transactions are widely believed to be supported by the promise of future credit. Consequently commercial banks that specialize in international lending or the World Bank and IMF may themselves be susceptible to borrower runs. Empirically, the possibility of a borrower run occurring could generate a form of financial contagion: if investors fear that country B will default because country A has done so, then yields will rise on country B's bonds.[10]

In our model, default by one borrower reduces the repayment incentives of other borrowers because it reduces a borrower's expected value of future finance from the MFI. As discussed, in microfinance the promise of future finance is one of the main (and sometimes the only) motives for a borrower to repay. In contrast, most traditional bank loans are heavily collateralized. However, even in this context the large literature on relationship banking (see, e.g., Petersen and Rajan, 1994) suggests that default by one borrower imposes a negative externality on other borrowers. Evidence for this negative externality is provided by Hubbard, Kuttner and Palia (2002), who show that small borrowers pay higher interest rates when their lending bank suffers losses. It follows that to the extent to which bank loans are *less* than 100% collateralized borrower runs may impact even traditional banks. As with sovereign debt, one

[10] Financial contagion may arise for a variety of reasons including trade or financial links between countries and/or herding behavior of lenders (see Kaminsky et al (2003) for a review). Our model differs from other theories in that contagion stems from an increase in default probabilities caused by a decrease in the viability of a shared lender.

implication is a contagion effect whereby default by one borrower increases default by other borrowers. Moreover, since borrower runs reduce the profitability of lending, and are more likely for a lending institution with low assets (A_0 in our model), our model provides a possible explanation for "credit crunches."[11]

6 Conclusion

In this paper we analyze coordination failures in the repayment of loans to microfinance institutions. We label these coordination failures borrower runs. If borrowers expect that the defaults of others will lower their own future gains from microfinance, then they too will have an incentive to default. We show that such contagion defaults occur with positive probability in the unique equilibrium of the imperfect information repayment game.

Microfinance institutions may have a hard time establishing credibility because of borrower runs. Proposition 3 establishes that initial funds are more crucial to an MFI when it is faced with borrower runs. Without sufficient donor funds or enough start-up capital, MFIs may not be able to make it off the ground as strategic interaction between borrowers who are unsure of the MFI's viability may lead to its failure.

There is considerable emphasis on profit making (or financial self sustainability) in current microfinance practice (Drake and Rhyne, 2002). This is one possible response of MFIs to borrower runs (Proposition 4). Under some circumstances

[11] We thank an anonymous referee for this suggestion. Credit crunches are episodes in which bank losses lead to a reduction in lending activity, and are the object of study of a large literature (see, e.g., Bernanke et al, 1991). The main problem in definitively identifying credit crunches is that they are hard to empirically distinguish from economic shocks that reduce the *demand* for loans. Much of the literature is concerned with this issue.

(for example, Corollaries 1 and 3), the MFI will always respond to borrower runs by making its loans more profitable. While there are certainly other reasons that microlenders stress profit making and their desire to reduce reliance on subsidies, our paper suggests that providing repayment incentives in the face of borrower runs could be a possible motivation.

Finally, we have analyzed how the MFI can change the terms of its current loan contract to reduce the welfare impact of borrower runs. The model in our paper is a static model and the value of future loans is represented by $v(x, A, \alpha)$, which we have taken as exogenous to the MFI and borrowers. Economically, one can think of this restriction as reflecting limited commitment on the part of the MFI,[12] so that $v(x, A, \alpha)$ is determined by optimizing decisions made after repayment. If instead one relaxes this assumption, the MFI could also potentially mitigate or even eliminate borrower runs by changing its future loan terms. In particular, since runs arise from strategic complementarity in repayments, the MFI could offer especially generous loans to borrowers who repay when others do not. Such future loan terms could eliminate strategic complementarity and hence prevent borrower runs. We leave a formal analysis for future research.

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[12] That is, the MFI can commit not to deal with defaulting borrowers, but to nothing more.

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A Proofs

Proof of Lemma 1: By $A1$, v is strictly increasing in x, with $v(x=0,A,\alpha)\equiv 0$ by $A3$. At the other extreme,

$$v(\bar{x},A(1;L,F),1)=v(\bar{x},A(1;L,0),1)+\int_0^F v_A\left(\bar{x},A(1;L,0)+\tilde{F},1\right)d\tilde{F}.$$

By $A7$, $v_A(\bar{x},A,1)>1$. So $v(\bar{x},A(1;L,F),1)>F$. So a solution to equation (3) exists and is unique. **QED**

Proof of Lemma 2: Differentiating W^1 with respect to L and F gives

$$\begin{aligned}
W_L^1 &= H'(L)-\frac{\rho}{\bar{x}}\int_{X^1(L,F)}^{\bar{x}} v_A(x,A(1;L,F),1)\,dx. \\
W_F^1 &= \frac{1}{\bar{x}}\int_{X^1(L,F)}^{\bar{x}} (v_A(x,A(1;L,F),1)-1)\,dx.
\end{aligned}$$

We have used $v(X^1(L,F),A(1;L,F),1)=F$ in calculating these terms. If $L<A_0$ it is always possible to increase L by ε and F by $\varepsilon H'(L)$, without violating the borrower feasibility constraint. So the solution satisfies

$$W_L^1+H'W_F^1=0,$$

unless it is at the corner $L=A_0$. Expanding, the lefthand side equals

$$\begin{aligned}
& H'(L)-\frac{\rho}{\bar{x}}\int_{X^1(L,F)}^{\bar{x}} v_A(x,A(1;L,F),1)\,dx \\
& +H'(L)\frac{1}{\bar{x}}\int_{X^1(L,F)}^{\bar{x}} v_A(x,A(1;L,F),1)\,dx-H'(L)\Pr\left(x\geq X^1(L,F)\right) \\
= \; & H'(L)\left(1-\Pr\left(x\geq X^1(L,F)\right)\right) \\
& +\left(H'(L)-\rho\right)\frac{1}{\bar{x}}\int_{X^1(L,F)}^{\bar{x}} v_A(x,A(1;L,F),1)\,dx.
\end{aligned}$$

If $H'(L)\geq\rho$ this expression is clearly strictly positive. It follows that $H'(L)<\rho$ at the optimal loan size. **QED**

Proof of Proposition 3: The following derivatives are used in this proof (and in the subsequent proof of Proposition 4):

$$\begin{aligned}
\frac{\partial X^*}{\partial L} &= \frac{\rho\int_0^1 v_A\left(X^*, A\left(\alpha\right), \alpha\right) d\alpha}{\int_0^1 v_x\left(X^*, A\left(\alpha\right), \alpha\right) d\alpha} \\
\frac{\partial X^*}{\partial F} &= -\frac{\int_0^1 \left(\alpha v_A\left(X^*, A\left(\alpha\right), \alpha\right) - 1\right) d\alpha}{\int_0^1 v_x\left(X^*, A\left(\alpha\right), \alpha\right) d\alpha}.
\end{aligned}$$

and

$$\begin{aligned}
W_L &= H'\left(L\right) - \frac{\rho}{\bar{x}}\int_{X^*(L,F)}^{\bar{x}} v_A\left(x, A\left(1\right), 1\right) dx \\
&\quad - \frac{X_L^*\left(L, F\right)}{\bar{x}}\left(v\left(X^*\left(L, F\right), A\left(1\right), 1\right) - F\right). \\
W_F &= \frac{1}{\bar{x}}\int_{X^*(L,F)}^{\bar{x}} \left(v_A\left(x, A\left(1\right), 1\right) - 1\right) dx \\
&\quad - \frac{X_F^*\left(L, F\right)}{\bar{x}}\left(v\left(X^*\left(L, F\right), A\left(1\right), 1\right) - F\right).
\end{aligned}$$

Next, note that the partial derivative with respect to A_0 is related to W_L by

$$W_{A_0} = -\left(W_L - H'\left(L\right)\right).$$

The analogous relation holds for the single-borrower problem. We prove that $W_L^1 - W_L > 0$, which is equivalent to $W_{A_0}^1 - W_{A_0} < 0$.

Observe that

$$\bar{x}\left(W_L^1 - W_L\right) = X_L^*\left(v\left(X^*, A\left(1\right), 1\right) - F\right) - \rho\int_{X^1}^{X^*} v_A\left(x, A\left(1\right), 1\right) dx.$$

Substituting in for X_L^* and $F = v\left(X, A\left(1\right), 1\right)$, and recalling that v is linear in x by $A1$,

$$\begin{aligned}
\bar{x}\left(W_L^1 - W_L\right) &= \rho\left(v\left(X^*, A\left(1\right), 1\right) - v\left(X^1, A\left(1\right), 1\right)\right)\frac{\int_0^1 v_A\left(X^*, A\left(\alpha\right), \alpha\right) d\alpha}{\int_0^1 v_x\left(X^*, A\left(\alpha\right), \alpha\right) d\alpha} \\
&\quad -\rho\left[\frac{x^2}{2} v_{Ax}\left(x, A\left(1\right), 1\right)\right]_{X^1}^{X^*},
\end{aligned}$$

and so

$$\frac{\bar{x}\left(W_L^1 - W_L\right)}{\rho\left(X^* - X^1\right)} = \frac{v_x\left(X^*, A\left(1\right), 1\right)\int_0^1 v_A\left(X^*, A\left(\alpha\right), \alpha\right) d\alpha}{\int_0^1 v_x\left(X^*, A\left(\alpha\right), \alpha\right) d\alpha} - \frac{X^* + X^1}{2} v_{Ax}\left(X^*, A\left(1\right), 1\right).$$

To complete the proof, since by Proposition 1 (which follows from strategic complementarity) $X^* > X^1$, it suffices to show that

$$\frac{\int_0^1 X^* v_{Ax}\left(X^*, A\left(\alpha\right), \alpha\right) d\alpha}{\int_0^1 v_x\left(X^*, A\left(\alpha\right), \alpha\right) d\alpha} \geq \frac{X^* v_{Ax}\left(X^*, A\left(1\right), 1\right)}{v_x\left(X^*, A\left(1\right), 1\right)}.$$

This is true provided that for any $\alpha \in [0, 1]$,

$$\frac{v_{Ax}\left(X^*, A\left(\alpha\right), \alpha\right)}{v_x\left(X^*, A\left(\alpha\right), \alpha\right)} \geq \frac{v_{Ax}\left(X^*, A\left(1\right), 1\right)}{v_x\left(X^*, A\left(1\right), 1\right)},$$

which is indeed the case since by linearity $\frac{v_{Ax}(X^*, A(\alpha), \alpha)}{v_x(X^*, A(\alpha), \alpha)} = \frac{v_A(X^*, A(\alpha), \alpha)}{v(X^*, A(\alpha), \alpha)}$, and by $A6$ this is decreasing in α. **QED**

Proof of Proposition 4: If $L^1 = A_0$ the result is immediate, since the only way the many-borrower contract can be less profitable is if $F^* \leq F^1$. The remainder of the proof deals with the case in which $L^1 < A_0$. We must show that either $F^* - \rho L^* > F^1 - \rho L^1$ or $F^* < F^1$. Suppose to the contrary that $F^* \geq F^1$ and $F^* - \rho L^* \leq F^1 - \rho L^1$.

The key to this result is to show

$$W_L + \lambda W_F < W_L^1 + \lambda W_F^1 \text{ for any } \lambda \in [0, \rho]. \tag{6}$$

The result is implied by (6), as follows. If $L^* = L^1$ then $F^* = F^1$ also. Note that $H'\left(L^1\right) < \rho$ by Lemma 2. In this case, we have a contradiction since $W_L^1\left(L^1, F^1\right) + H'\left(L^1\right) W_F^1\left(L^1, F^1\right) = 0$, and so inequality (6) implies that $W_L^1\left(L^*, F^*\right) + H'\left(L^*\right) W_F^*\left(L^*, F^*\right) < 0$. This contradicts the optimality of L^*, F^* since it implies the MFI would be better of decreasing L^* by ε and F^* by $\rho\varepsilon$ (this perturbation is feasible since $H'\left(L^*\right) < \rho$).

If instead $L^* > L^1$, we can write $F^* = F^1 + \lambda(L^* - L^1)$ for some $\lambda \in [0, \rho]$. The quantities $W(L^*, F^*)$ and $W^1(L^*, F^*)$ can then be written as

$$\begin{aligned} W(L^*, F^*) &= W\left(L^1, F^1\right) + \int_{L^1}^{L^*} \left(W_L\left(\tilde{L}, F^1 + \lambda\left(\tilde{L} - L^1\right)\right) + \lambda W_F\left(\tilde{L}, F^1 + \lambda\left(\tilde{L} - L^1\right)\right)\right) d\tilde{L}. \\ W^1(L^*, F^*) &= W^1\left(L^1, F^1\right) + \int_{L^1}^{L^*} \left(W_L^1\left(\tilde{L}, F^1 + \lambda\left(\tilde{L} - L^1\right)\right) + \lambda W_F^1\left(\tilde{L}, F^1 + \lambda\left(\tilde{L} - L^1\right)\right)\right) d\tilde{L}. \end{aligned}$$

From (6), $W^1(L^*, F^*) - W^1(L^1, F^1) > W(L^*, F^*) - W(L^1, F^1)$. Since L^* and F^* are optimal choices in the many-borrower problem, $W(L^*, F^*) - W(L^1, F^1) \geq 0$. But then $W^1(L^*, F^*) - W^1(L^1, F^1) > 0$, contradicting the optimality of L^1 and F^1 in the single-borrower problem.

To establish (6), note that

$$\bar{x}\left(W_L^1 + \rho W_F^1 - W_L - \rho W_F\right) = -\rho \int_{X^1}^{X^*} dx + \left(X_L^* + \rho X_F^*\right)\left(v\left(X^*, A(1), 1\right) - F\right).$$

Substituting for $X_L^* + X_F^*$ and $F = v(X^1, A(1), 1)$, and recalling that v is linear in x by $A1$,

$$\begin{aligned} &\frac{\bar{x}\left(W_L^1 + W_F^1 - W_L - W_F\right)}{\rho\left(X^* - X^1\right)} \\ &= -1 + \frac{1 + \int_0^1 (1-\alpha)\, v_A\left(X^*, A(\alpha), \alpha\right) d\alpha}{\int_0^1 v_x\left(X^*, A(\alpha), \alpha\right) d\alpha} v_x\left(X^*, A(1), 1\right). \end{aligned}$$

This is positive since by strategic complementarity $(A5)$, $v_x(X^*, A(1), 1) > v_x(X^*, A(\alpha), \alpha)$ for any α. Thus

$$W_L^1 - W_L + \rho W_F^1 - \rho W_F > 0.$$

Since $W_L^1 - W_L > 0$ (from Corollary 1), it follows that for any $\lambda \in [0, \rho]$,

$$W_L^1 - W_L + \lambda\left(W_F^1 - W_F\right) > 0,$$

i.e., inequality (6). **QED**

B Contingent Loan Contracts

In the main text we restricted attention to loan contracts in which the required repayment F is not allowed to depend on the realization of the fundamental x. Most MFIs appear to use simple non-contingent debt contracts of this form.

In this appendix we briefly consider the opposite extreme in which the required repayment F can be made contingent on the fundamental x in an arbitrary way. For expositional ease, we assume that the MFI directly observes the fundamental x. (We would obtain similar results if the MFI observes only a noisy signal of x, where the variance of the noise term is small.)

Specifically, suppose now that the MFI chooses loan terms L and $F : [0, \bar{x}] \to \Re$ to maximize borrower welfare

$$H(L) + E_x \left[\alpha(x; L, F(x)) \left(v\left(x, A\left(\alpha(x; L, F(x)); L, F(x)\right), \alpha(x; L, F(x))\right) - F(x) \right) \right],$$

where as before $\alpha(x; L, F(x))$ denotes the fraction of borrowers who repay for a given realization of the fundamental x and the loan contract $(L, F(\cdot))$. As in the main text we continue to assume that the MFI cannot lend out more than its initial funds (i.e., $L \leq A_0$) and that borrowers cannot repay more than their project return (i.e., $F(x) \leq H(L)$ for all x).

First, consider the repayment condition for the single-borrower problem (as in section 3). For any realization of the fundamental x, the borrower repays $F(x)$ if and only if $x \geq X^1(L, F(x))$, where $X^1(\cdot, \cdot)$ is as defined in the main text in equation (3).

Second, consider the repayment condition for the many-borrower problem with near perfect information (as in section 4.2). An issue that arises here is that if F is fully contingent on x (i.e., if $x_1 \neq x_2$ then $F(x_1) \neq F(x_2)$) the contract terms reveal the fundamental x to borrowers. That is, $F(x)$ acts as a public signal of the fundamental x. In this case, the repayment game is one of perfect information, and multiple equilibria may exist. To circumvent this problem we assume that the MFI introduces a small amount of noise into its repayment request, and that the variance of this noise approaches zero more slowly than does the standard deviation of borrowers' signals about the fundamental. Hellwig (2002) and Morris and Shin (2003) show[13] that under these conditions the repayment equilibrium in the near-perfect information case without public signals remains the unique equilibrium even when the public signal is introduced. Thus for any realization of the fundamental x the borrowers repay $F(x)$ if and only if $x \geq X^*(L, F(x))$, where $X^*(\cdot, \cdot)$ is as defined in equation (5). (Alternately, one could justify this equilibrium by simply assuming that borrowers do not update their estimate of x from the contract terms $F(x)$.)

From Proposition 1, we know that $X^*(L, F) > X^1(L, F)$ for any value of x. Consequently:

Corollary 4 *Suppose the loan contract has contingencies of the form $(L, F(x))$. Then the MFI is repaid after more realizations of the fundamental x in the single-borrower problem than in the many problem. That is, borrower runs reduce repayment.*

[13] See Theorem 1(*ii*) of Hellwig (2002) and section 3.3 of Morris and Shin (2003)