Efficiency in Family Bargaining:
Living Arrangements and Caregiving Decisions of
Adult Children and Disabled Elderly Parents

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Comments Welcome

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Abstract

In this paper, we use a two-stage bargaining model to analyze the living arrangement of a disabled elderly parent and the assistance provided to the parent by each of her adult children. The first stage determines the living arrangement: the parent can live in a nursing home, live alone in the community, or live with any child who has invited coresidence. The second stage determines the assistance provided by each child in the family. Working by backward induction, we first calculate the level of assistance that each child would provide to the parent in each possible living arrangement. Using these calculations, we then analyze the living arrangement that would emerge from the first stage game. A key assumption of our model is that family members cannot or will not make binding agreements at the first stage regarding transfers at the second stage. Because coresidence is likely to reduce the bargaining power of the coresident child relative to her siblings, coresidence may fail to emerge as the equilibrium living arrangement even when it is Pareto efficient. That is, the equilibrium of the two-stage game need not be Pareto efficient.
1. Introduction

Family long-term care, the provision of in-kind services to elderly members due to disability, represents an important mode of assistance to elderly persons. The provision of such care is often the product of numerous individual and joint decisions by family members with different preferences facing different constraints. Family members not only make caregiving decisions on behalf of disabled family members but often provide hands-on care themselves and share the financial consequences of caregiving decisions. Moreover, the preferences of the disabled elderly may differ from those of their spouses and their adult children, and the preferences of one child may differ from those of another. Differences may arise about the type of care desired for the disabled parent and the setting in which they receive it. For example, children may want a parent to enter a nursing home, while the parent prefers to live independently; or, a child may want a parent to receive care from a family member, but want another child to be the primary caregiver. The possibility of conflict regarding caregiving and the roles of different family members in providing care suggests that family members may have incentives to behave strategically.

In this paper we develop a model of family caregiving that captures important features of the complex interactions between elderly parents and adult children and among the children. We restrict our attention to the unpartnered elderly, a group of particular policy interest because they are far more likely to be institutionalized (Freedman 1996) and are also more likely to receive care from their children than are their married counterparts (Dwyer and Coward 1991). We analyze the caregiving role of spouses in Pezzin, Pollak, and Schone (2005b).

We propose and analyze a two-stage game with three players: a disabled parent and
two adult children. The first stage determines the living arrangements and the second intrafamily transfers. First-stage decisions affect second-stage bargaining power, but family members cannot or will not make binding commitments regarding their future behavior. We show that even if the second stage is conditionally efficient (i.e., efficient given the living arrangements determined in the first stage), the equilibrium of the two-stage game may be inefficient.

Long-term care for the disabled elderly has sparked a growing literature. The economics literature, surveyed in Norton (2000), focuses on the supply and demand for nursing home care and on long-term care insurance, but pays little attention to the family. Early studies that discuss the role of the family concentrate primarily on support from children to parents in the form of shared housing, analyzing the determinants of living arrangements (Börsch-Supan, 1989; Börsch-Supan, et al., 1989; Ellwood and Kane, 1990; Kotlikoff and Morris, 1990; Börsch-Supan, et al., 1992; Börsch-Supan, McFadden and Schnabel, 1993).

The first generation of research on families’ care arrangements relied on Becker’s model of the family (Wolf and Soldo, 1994; Ettner, 1995 and 1996; Kemper and Pezzin, 1996). More recent work has used game-theoretic bargaining models to examine family care arrangements. Two examples will suffice. Pezzin and Schone (1999a, 2002a) specify and analyze a model of living arrangements, informal caregiving, labor force participation, and cash transfers. Their game involves two players, an elderly parent and an adult daughter, each attempting to maximize a utility function defined over a vector of private goods, leisure (for the daughter), and a public good, representing the parent's physical
health. Conditional on the parent’s disability, the production of the parent's physical health requires that the parent receive some form of care, be it formal (i.e., paid) or informal (i.e., unpaid). Parent and daughter make decisions that determine the levels of private consumption, leisure, cash transfers from the daughter to the parent, the combination of formal and informal care used to produce the parent's physical health or well-being and the living arrangement (either separate or coresidence). As in the separate spheres model of Lundberg and Pollak (1993), Pezzin and Schone assume that intrahousehold allocation is determined as the solution to a cooperative Nash bargaining game in which the threat point is the Cournot-Nash equilibrium of a noncooperative game.

Heidemann and Stern (1999) and Engers and Stern (2002) develop a game theoretic model of family bargaining designed to motivate a structural empirical model of family long term care decisions. In particular, they focus on determining whether the parent enters a nursing home, lives independently with no care provided by her children, or, if the parent does receive care from her children, which child becomes the primary caregiver. In their formulation, each adult child decides independently whether to attend a meeting in which living and care arrangements for the disabled parent are determined. Both voluntary and compulsory participation versions are analyzed and estimated, with results favoring the voluntary model. The children who participate reach a binding agreement while the non-participating children are excluded from family decision making and bear no responsibility for caring for the parent. For each child, the decision of whether to attend the meeting depends on the value she places on participating in the decision, the side payments that she anticipates, and the effect that she anticipates her presence at the meeting would have on the
family's decision.

Much of the work examining family caregiving has focused on the parent-child dyad. Although most studies have included variables summarizing characteristics of the remaining family network (Kotlikoff and Morris 1990; Pezzin, Pollak and Schone 2005c; Pezzin and Schone 1999b and 2002b; Stern 1993 and 1995), little work has analyzed interactions among the adult children. Papers that analyze caregiving with interactions of two or more children include Engers and Stern (2002), Checkovich and Stern (2002), Pezzin and Schone (2001), and Pezzin, Pollak, and Schone (2005a).

To analyze interactions among adult children, we build on research that has modeled intrahousehold allocation within a game theoretic framework (Manser and Brown 1980; McElroy and Horney 1981; Lundberg and Pollak 1993, 1994, and 2003). Game theoretic models are especially suitable for analyzing intergenerational living and transfer arrangements because they recognize the divergent and often conflicting interests of family members and specify a process for translating these divergent interests into outcomes. In the next three sections we analyze the parent-two-children game, examining living arrangements, interhousehold transfers, and intrahousehold transfers. In section 2 we describe the two-stage caregiving game with two children. Like all dynamic games, our two-stage sequential game is solved by backward induction. In section 3 we analyze the second-stage game. We show that, for plausible specifications, the equilibrium of the second stage game need not be Pareto efficient given the living arrangement. In section 4 we analyze the first stage game which determines the living arrangement. We show that, even if the equilibrium of the second stage game is conditionally efficient, the equilibrium
of the full game need not be Pareto efficient. Section 5 is a brief conclusion.

2. A Caregiving Game with Two Children

We consider four possible living arrangements for the parent: living in a nursing home (A^n), living on her own in the community (A^o), living with child 1 (A^1), and living with child 2 (A^2). The parent's utility in each of these living arrangements depends on the living arrangement itself and on her consumption of a private consumption good (C). We use the superscripts \{n, o, 1, 2\} to indicate the living arrangement and the subscripts \{p, 1, 2\} to indicate family members. Thus, C^o_2 denotes private consumption by child 2 when the parent lives alone, and C^2_1 denotes consumption by child 1 when the parent lives with child 2. We assume that children care about their own private consumption and about the parent's private consumption. We also assume that the children care about the parent's living arrangement, which affects the parent's well being and the child's privacy.

Economists' usual assumption about preferences -- what Becker (1981) calls "altruistic" preferences -- implies that the children defer to the disabled parent's preferences over her own consumption and, in some cases, over her living arrangements as well. For example, Becker’s children would defer to the parent’s preference between living in a nursing home and living on her own in the community. We regard Becker's altruistic preferences as an implausible special case. We also dislike Becker's terminology, which forces us to say that a child who rejects a disabled parent's preference for living alone (e.g.,
because the child believes that the parent cannot safely live alone) is not altruistic.\footnote{In the introduction to the 1991 edition of his *Treatise on the Family*, Becker concedes that "The most unsatisfactory aspect of my discussion...[in the 1981 edition is]...the failure to combine the discussion of 'merit goods' and altruism" (p. 10). Just as merit goods can motivate paternalistic governments to provide tied transfers (e.g., food stamps), merit goods can motivate paternalistic family donors, when they have the ability, to provide tied rather than untied transfers. Pollak (1988) proposed a model in which family members have paternalistic preferences. In place of Becker's term, "altruistic" preferences, Pollak (2003) argues that "deferential" preferences is more descriptive.}

In this paper we treat consumption as one dimensional, avoiding the issue of merit goods except in regard to living arrangements. Pezzin and Schone (1999a; 2002a) analyze a model with two goods, one of which is a merit good.

We model family interactions as a two-stage game in which both stages may contain substages. The first stage is noncooperative and determines the living arrangement. The second stage determines consumption. We assume that family members cannot or will not make binding agreements at the first stage regarding assistance or allocations at the second stage. Hence, the assistance that a child provides at the second stage, although it may be predictable at the first stage, is determined at the second stage.\footnote{We cannot directly observe the impact of not having binding agreements on decisions made at the second stage. However, the lack of binding agreements may partially explain some empirical regularities associated with caregiving. For example, the concentration of caregiving by coresident caregivers (and the relatively small amount of assistance provided by the noncoresident caregiver) may reflect the fact that, once the parent begins to live with one child, the other children can reduce the care they provided without any real consequence.}

Suppose, for example, that the first stage begins with the children deciding, separately and simultaneously, whether or not to invite the parent to coreside and ends with the parent choosing among the feasible living arrangements: she can move into a nursing
home, live on her own, or accept the invitation of any child who has invited her to coreside. At the second stage, taking as given the living arrangement determined at the first stage, the children and the parent make decisions that determine resource allocation under that living arrangement. We can model the second stage as a noncooperative game or as a cooperative game; alternatively, we can finesse some but not all of the difficulties of modeling the second-stage game by postulating an "allocation rule." We define an allocation rule as a specification of each family member's second-stage behavior as a function of the living arrangement, which is determined in the first stage, of the economic and demographic characteristics of all players, and of any relevant policy parameters (e.g., tax subsidies for dependent care, caregiver allowances).

We have adopted and adapted the idea of an allocation rule from Chiappori (1988, 1992) who proposed a "sharing rule." In the context of allocation between spouses within marriage, Chiappori postulates a single-valued, Pareto-efficient sharing rule which determines each spouse's utility. Chiappori postulates the sharing rule directly, and does not attempt to derive it from an underlying model of bargaining. Unlike Chiappori's sharing rule, our allocation rule describes the allocation of goods rather than utility and, again unlike Chiappori’s sharing rule, ours need not be single-valued or Pareto efficient. By beginning with the allocation rule, we avoid not only the need to analyze the second-stage game but also the need to specify it, or even to specify whether it is cooperative or noncooperative.

A priori, it is unclear whether a game of family decision making should be modeled as cooperative or noncooperative. Arguing against noncooperative game theory, Shubik (1989, p. 103) asserts that it "is generally not so useful to describe complex, loosely
structured social interaction." Family bargaining -- whether between spouses within marriage, between an adult child and a disabled parent, or among adult children -- exemplifies such interactions. However, there are also concerns about the usefulness of cooperative game theory. First, although cooperative game theory allows us to proceed without specifying the "rules of the game" -- the strategies available to each player, or in extensive form, the sequence of moves and the information available to each player at each move – noncooperative game theory teaches that the rules of the game are often crucial determinants of the outcome. Thus, by modeling interactions as a cooperative game we are forced to disregard strategic factors. Second, the efficiency of social arrangements and practices is a central concern of economics and of public long-term care policy, yet cooperative bargaining models assume that outcomes are Pareto efficient. Hence, cooperative bargaining models are incapable of investigating the conditions that make families more or less likely to achieve and sustain efficient outcomes because they refuse to recognize the possibility of inefficiency. Because complex, loosely structured social interactions are very difficult to model, we regard the choice between modeling family interactions as a noncooperative or a cooperative game not a matter of principle but of research strategy. In the following section we consider alternative specifications of the second stage game.

3. The Second Stage Game: Child to Parent Transfers

For each of the four living arrangements, we now consider several formulations of the second stage game which determines transfers from children to the parent. In all of
these specifications we assume that the parent accepts whatever transfers the children offer her; hence, unlike the children, the parent is not a strategic player in the second stage game. Each child is concerned with her own consumption and with the parent's consumption. Each child would prefer that her sibling contribute more so that she could contribute less herself and increase her own consumption.

$A^\circ$. Suppose that the parent lives on her own in the community.

When the parent lives independently in the community, the game between the children can take many possible forms. For example, the children may play a one-shot noncooperative game, a repeated noncooperative game, or a cooperative game. We consider each of these in turn.

$A^\circ.i$. A noncooperative, one-shot, voluntary contribution game.

First suppose that the children play a one-shot Cournot-Nash game. Using their "reaction functions" which show each child's best response to the transfers made by the other, we can calculate the equilibrium. (Figure 1). When family public good provision is a simultaneous move one-shot game, the public good will be underprovided. More specifically, if both children make positive contributions to the public good, then the equilibrium is Pareto inefficient: there exist strictly greater contribution levels that both children would prefer to the Cournot-Nash equilibrium.$^3$

$^3$ This underprovision result does not hold if only one child makes positive contributions in the Cournot-Nash equilibrium. In the one-child case and, more generally, in the one
A noncooperative, repeated, voluntary contribution game.

Now suppose the children play a repeated voluntary contribution game. More specifically, suppose that at the beginning of every period each child has the opportunity to make transfers to the parent. To simplify the analysis, suppose that neither the parent nor the children can carry over resources from one period to another, so that the stage games played in successive periods are identical. Hence, the stage games are related only because the children can punish each other for misbehavior by reducing their own contributions to the public good.

If the children are sufficiently patient, then the folk theorem asserts that any feasible, individually rational allocation is a subgame perfect equilibrium of the repeated game. Hence, the repeated game has a very large set of subgame perfect equilibria some of which are Pareto efficient, but many of which are not. A cogent objection to modeling children's transfers to a disabled elderly parent as a repeated game is the assumption that the game will continue indefinitely. Because the game evolves and will eventually stop, end game considerations may affect behavior from the outset.4

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4 The theoretical argument that finite games unravel is strong, but empirical evidence suggests that unraveling need not occur. The “centipede game” is the standard example of an experimental game in which unraveling does not occur. See Kreps (1990) and McKelvey and Palfrey (1992).
A Cooperative Game.

Now suppose, as Shubik would have us assume in analyzing "complex, loosely structured social interactions," that the second-stage game is cooperative. We can conclude immediately that, conditional on the living arrangement, the second stage equilibrium is Pareto efficient. Unlike noncooperative games, cooperative games make no mention of strategies or moves, but instead require a specification of the payoffs attainable by each coalition and a solution concept. We discuss two solution concepts: the core and the Nash bargaining solution.

The core is the set of feasible, undominated allocations -- allocations that cannot be improved upon by any coalition. Because a core allocation cannot be improved upon by the coalition of all players, every core allocation is Pareto efficient. A drawback of the core as a solution concept is that it not only fails to predict a unique equilibrium but that the set of equilibria is large. The Nash bargaining solution, the leading solution concept in bargaining models of marriage, selects a particular core outcome as the solution. (Figure 2). Which core allocation it selects depends on the specification of the threat point $(T^1, T^2)$.  

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5 If the players are sufficiently patient, the folk theorem implies that the set of equilibria in the repeated game is much larger than the core.
6 Bargaining models of marriage have emphasized Nash bargaining and neglected other cooperative bargaining models and solution concepts. For example, although Manser and Brown considered both the Nash and the Kalai-Smorodinsky (1975) bargaining solutions, subsequent work on bargaining in families has virtually ignored Kalai-Smorodinsky. Gugl (2004) provides an interesting exception, considering both the Nash and Kalai-Smorodinsky bargaining solutions. Gugl’s work suggests that the difficulty of doing comparative statics with Kalai-Smorodinsky may account for its eclipse by the Nash bargaining solution.
The core, despite its prominence in game theory, has received almost no attention as a solution concept in the economics of the family, perhaps because it does not yield a unique solution in two-person games, and perhaps because, in games with more than two players, the core may be empty.

\(A^a\). The Parent Lives in a Nursing Home.

The logical structure of the second-stage game when the parent lives in a nursing home is essentially identical to its structure when she lives on her own in the community. As when the parent lives on her own in the community, in the second stage game the children take the parent's living arrangement as given. Not surprisingly, some specifications of the second stage game imply efficient equilibria, other specifications imply inefficient equilibria, and still others imply multiple equilibria, some efficient and some inefficient. Because living in a nursing home presents no new issues, we discuss it no further.

\(A^1, A^2\). The Parent Resides with a Child.

Coresidence increases the strategic asymmetry between the children and weakens the bargaining power of the coresident child. Of course strategic asymmetry is always present. Even when the parent lives independently in the community or in a nursing home, the children may differ in gender, family responsibilities, labor force attachment, and attachment
or proximity to the parent.\textsuperscript{7} Like coresidence, many of these differences are endogenous. For definiteness, and without loss of generality, we suppose that the parent coresides with child 1.

We emphasize the strategic asymmetry between the coresident and the noncoresident child because it has implications for the choice of living arrangements in the first stage game. We begin by assuming that the parent is a passive spectator rather than a strategic player. Using the model proposed by Weiss and Willis (1985) in the context of child support following divorce, we examine the implications of coresidence for the noncoresident child's ability to monitor the way transfers are used by the coresident child. We then consider the way in which coresidence (and the frequency of contact it implies) is likely to affect the coresident child's awareness of the parent's needs or her attachment to the parent. Finally, we allow the parent to be a strategic player, assuming that allocation within the coresident household is the outcome of a cooperative game between the coresident child and the parent.

Weiss and Willis provide a framework for analyzing the effect of coresidence on bargaining power. Their concern is child support following divorce, but the strategic position of the noncoresident child contemplating contributing to the coresident household is similar to that of the noncustodial parent contemplating child support.

In Weiss and Willis the child's well-being is a parental public good valued by both

\footnote{Konrad, \textit{et al.} (2002) argue that older children exploit their first mover advantage by moving away from their parents, leaving younger children to bear a disproportionate share of long-term care responsibilities.}
parents. Each parent, however, is also concerned with his or her private consumption and unconcerned with the private consumption of the ex-spouse. The noncustodial parent, for definiteness, the divorced father, because he does not coreside with the child, is poorly positioned to monitor his ex-wife's allocation of child support between herself and the child. Weiss and Willis view the inability of the father to monitor the mother's allocation of resources between herself and the child as the crucial feature of the strategic situation. The inability to monitor precludes binding, enforceable agreements between the parents: the father is rationally concerned that if he increases his contribution, his ex-wife will respond by reducing her own.

Weiss and Willis model child support as a Stackelberg game: the first mover, the father, contributes resources to the mother; the mother then allocates resources between herself and the child. As Weiss and Willis show, the equilibrium allocation is Pareto inefficient: both parents would prefer an allocation in which they both reduced their private consumption and increased their transfers to the child. The Stackelberg game captures the intuition that, because the inability to monitor precludes binding agreements, the child will receive less than the Pareto-efficient level of resources. The Weiss and Willis conclusion is appealing, although asymmetric information and inability to monitor play no role in their formal model.

Neither child support nor long-term care is a Stackelberg game, but the Weiss and Willis insight about the strategic importance of the inability to monitor applies to both. The analogy between child support and long-term care is closest when the parent has a cognitive disability such as severe Alzheimer's that precludes her active participation in the allocation
process. Under these circumstances, the coresident child allocates resources between herself and the parent, just as in Weiss and Willis the mother allocates resources between herself and the child.

Once coresidence is established, the coresident child may have incentives to maintain it because termination would impose high psychic costs or adversely affect instrumental or affective relationships with other family members. That is, once coresidence becomes the status quo, the coresident child may find termination difficult and costly. We can interpret the coresident child's incentives to continue coresidence in terms of rewards offered for continuing or, equivalently, in terms of punishments threatened for terminating. The noncoresident child, knowing that her sister cannot easily terminate coresidence, realizes that if she reduces her contribution, the coresident child will respond by increasing hers. England and Folbre (2003), describing the predicament of paid care workers, write: "these emotional bonds [to those receiving care] put care workers in a vulnerable position...We might call the workers 'prisoners of love'; a kind of emotional 'hostage effect' comes into play" (p. 73). The logic of their argument applies with at least equal force to care provided by family members. In a dynamic model, the contributions by the noncoresident child might decrease over time while contributions by the coresident child might increase; but our static models cannot accommodate this behavior.

We now drop our assumption that the parent is a passive spectator and assume instead that she is an active player. We begin with the one-child case and consider the implications of coresidence for bargaining between the coresident child and the parent. We then return to the two-child case, and consider the implications of coresidence for
bargaining between the noncoresident child, the coresident child, and the parent.

Following Pezzin and Schone (1999a, 2002a) we assume that when the parent and the child coreside their interactions are cooperative but when they do not coreside their interactions are noncooperative. We assume that, within the coresident household, control over resources affects allocation. That is, government and family transfers to the coresident child have a different effect than transfers to the parent. More formally, resources controlled by the coresident child and resources controlled by the parent are separate arguments of the coresident household's allocation rule. Empirical work by Hayashi (1995) and by Pezzin and Schone (1997) on allocation within two-generation households finds that resources controlled by the elderly parents have a different effect on household expenditure patterns than resources controlled by their adult children.8

Government programs that provide direct payments, in-kind services, or tax incentives to households with a disabled elderly member are uniform across households rather than tailored to the allocation rules of particular households. Government transfers, like private transfers, affect allocations in the coresident household through its allocation rule. Hence, government policies that provide resources to disabled elderly parents will have predictably different effects than policies that provide resources to coresident children.

We now turn to the two child case. To model the asymmetry between the positions of the noncoresident child and the coresident child, we assume that the second stage game

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8 In a similar vein, empirical work by Lundberg, Pollak and Wales (1997) on allocation within married couple households finds that resources controlled by husbands have a different effect on household expenditure patterns than resources controlled by wives.
contains two substages: in the first substage, the noncoresident child makes transfers to the coresident child and to the parent; in the second substage, the coresident child and the parent play a cooperative game. For definiteness, it is often convenient to think of the coresident household's behavior as the outcome of a cooperative Nash bargaining game in which the threat point depends on the allocation of transfers between the coresident child and the parent. Thus, the noncoresident child must decide not only how much to transfer to the coresident household, but also on the allocation of transfers between the coresident child and the parent. We represent the allocation that emerges from the full second stage game by an allocation rule.

4. The First Stage Game: Living Arrangements

In this section we analyze the first stage game. In section 4a we show that the equilibrium of our two-stage game can be Pareto inefficient even when the second stage game is Pareto efficient conditional on the living arrangement. In section 4b we show that the equilibrium may depend on the precise specification of the first-stage game (e.g., when

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9 We do not assume that the parent and the coresident child play as a "team." The team assumption would imply that the parent and the coresident child act as a single player. That is, the coresident household would act as if it had a single preference ordering and a single budget constraint (i.e., as if the parent and the coresident child pooled their resources). If the coresident child and the parent are a team, then the behavior of the coresident household would be independent of how the noncoresident child or the government allocated transfers between the coresident child and the parent.

10 The government may allocate a portion of its transfers to the coresident child rather than the disabled elderly parent, and such an allocation may induce a child to offer coresidence. A child cannot use the prospect of such transfers to induce a sibling to offer coresidence unless the child can make binding commitments.
the game is sequential, the equilibrium may depend on which child moves first), and we consider a game in which the parent can commit herself to rejecting certain invitations. Finally, in section 4c we show that the game may have multiple equilibria; more specifically, we consider a game in which all family members move simultaneously and show that 5 of its 12 strategy profiles are equilibria.

We begin by describing the first stage game. Because the parent cares about living arrangements as well as private consumption, she might prefer to live independently with fewer consumption goods than live with child i with more. We assume that, given values for \{C^n, C^o, C^1, C^2\}, the parent can rank the four possible living arrangements (e.g., \(A^o\) preferred to \(A^1\), \(A^1\) preferred to \(A^n\), etc.). We also assume that, given these values, each child also can rank the four possible living arrangements. Each child's preferences reflect his or her concern for the parent's private consumption as well as the implications of each living arrangement for the assistance the child will provide and, hence, for the resources remaining for the child's private consumption.

As an initial example, we model the first stage as consisting of a simultaneous moves by the siblings, followed by a decision by the parent, who chooses among the living arrangements available to her. Each child has two moves: inviting coresidence (v) or not inviting coresidence (v'). For each profile of moves (e.g., both invite coresidence; child 1 invites coresidence and child 2 does not), we assume that the resulting levels of utility are known to each child (e.g., if both invite coresidence, they know that the parent will choose to live with child 1), or, more precisely, they can assign probabilities to each living arrangement.
In the first stage example described above, the parent will choose her preferred living arrangement from the available options determined by the children's invitations. This choice is based on the parent’s calculation of the utility levels attainable in each living arrangement. The parent faces at most four alternatives, depending on whether both children invite coresidence, neither child invite coresidence, or one child invites coresidence and the other does not. Two loose ends remain: nonuniqueness in the solution to the second stage game, and nonuniqueness in the parent's choice.

If the allocation rule associates a unique allocation with each living arrangement, then the parent will prefer one to the other or be indifferent between them. We assume that the parent's ranking of living arrangements is an ordering. If the ordering is strict (i.e., no ties), then we can proceed directly to the analysis of the first stage game. If the ordering is not strict, then the parent will sometimes face situations in which the "best" living arrangement is not unique. These ties pose no problem for the parent, but they do pose a problem for the children who must decide at the first stage whether to invite coresidence. Henceforth, we ignore such ties.

The multiplicity of equilibria in the second stage game poses a more troubling problem.\footnote{Multiple equilibria can arise in cooperative second stage games (e.g., when the core is the solution concept) and in noncooperative second stage games (e.g., in repeated games as a consequence of the folk theorem.} The problem arises because the parent cannot choose among living arrangements unless she can assign probabilities to each allocation in the set. If the allocation rule associates probabilities with each allocation in the set, then choosing among
living arrangements is like choosing among lottery tickets. In the absence of such probabilities, however, we encounter difficulties modeling parental choice.

Before turning to our examples, we dispose of an expositional complication. We want to treat family members as if they have direct preferences over living arrangements. Provided the parent and the children can predict the transfers that would take place in each possible living arrangement, we can legitimately focus on induced preferences over living arrangements, relying on the fact that each living arrangement is associated with unique levels of private consumption, of care for the parent, and of privacy for the children and for the parent. Hence, instead of carrying forward notation that explicitly recognizes the role of private consumption for the children and for the parent, we work with the induced preferences over living arrangements.

4a. Pareto Inefficient Equilibria

To construct an example of a game with an inefficient equilibrium, we begin by specifying the preferences of each family member. Suppose preferences for the parent’s living arrangement (conditional on the transfers that would be made) are represented by:

<table>
<thead>
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<th>Parent’s Ranking</th>
<th>Child 1’s Ranking</th>
<th>Child 2’s Ranking</th>
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<td>Parent lives:</td>
<td></td>
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<td>First Choice</td>
<td>with Child 1</td>
<td>with Child 2</td>
<td>with Child 1</td>
</tr>
<tr>
<td>Second Choice</td>
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<td>independently</td>
<td>independently</td>
</tr>
<tr>
<td>Third Choice</td>
<td>independently</td>
<td>with Child 1</td>
<td>with Child 2</td>
</tr>
<tr>
<td>Fourth Choice</td>
<td>in nursing home</td>
<td>in nursing home</td>
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</tbody>
</table>
That is, the parent prefers to live with child 1, but would rather live with child 2 than live independently. Each child prefers that the parent coreside with the other child, and each child would prefer that the parent live independently rather than coreside with the parent. The unique equilibrium, indeed, the dominant strategy equilibrium, of any game in which the children have these preferences has the parent living independently. With these preferences, the nursing home living arrangement is an option that the parent would never choose, and one that her children know that she would never choose.

Suppose, however, that each child would invite coresidence if she knew her sister would contribute "enough," and the sister would rather contribute enough than have the parent live independently. An omniscient and omnipotent social planner could impose a solution on the family that would make everyone -- the parent and both children -- better off by requiring that the parent live with child 1 and requiring child 2 to contribute "enough." But the family cannot achieve this or any other Pareto-efficient solution and is misguided, as if by an invisible hand, to a Pareto inefficient equilibrium. The demonstration of inefficiency depends on comparing a living arrangement and transfer pattern that a social planner could impose with the living arrangement and transfer pattern that would emerge as the equilibrium of a two-stage game. Our argument does not establish nor do we claim that for all configurations of preferences the equilibrium of the two-stage game is inefficient. It does establish that for some configurations of preferences the equilibrium is inefficient.

Lundberg and Pollak (2003) describe and analyze a related two-stage game in the context of the "two-earner couple location problem." In Lundberg and Pollak, spouses play a two-stage game in which the first stage determines the location (e.g., whether the couple
moves to the husband's preferred location or the wife's preferred location), and the second stage determines allocation within marriage. When the spouses prefer different locations, inefficient outcomes (e.g., inefficient divorces) are possible even when the second stage game is efficient conditional on the location determined in the first stage. A similar result holds in our long-term care game -- the equilibrium of the two-stage long-term care game may be an inefficient living arrangement, even when second stage transfers are efficient conditional on the living arrangement. The crucial features of both the two-earner couple location game and our long-term care game are that first-stage decisions affect future bargaining power and that family members cannot or will not make binding, enforceable agreements.

4b. Outcomes Depend on the Structure of the First-Stage Game

To construct an example in which the equilibrium living arrangement depends on the precise specification of the first stage game, we again begin by specifying each family member's preferences.

<table>
<thead>
<tr>
<th></th>
<th>Parent’s Ranking</th>
<th>Child 1’s Ranking</th>
<th>Child 2’s Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parent lives:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First Choice</td>
<td>with Child 1</td>
<td>with Child 2</td>
<td>with Child 1</td>
</tr>
<tr>
<td>Second Choice</td>
<td>with Child 2</td>
<td>with Child 1</td>
<td>with Child 2</td>
</tr>
<tr>
<td>Third Choice</td>
<td>independently</td>
<td>independently</td>
<td>independently</td>
</tr>
<tr>
<td>Fourth Choice</td>
<td>in nursing home</td>
<td>in nursing home</td>
<td>in nursing home</td>
</tr>
</tbody>
</table>

That is, the parent prefers to live with child 1, but would rather live with child 2 than live
independently. Each child prefers that the parent coreside with the other child, but each child prefers coresidence with the parent to having the parent live independently. Both children and the parent prefer having the parent live independently rather than in a nursing home.

Using these preferences, we consider alternative specifications of the first stage game. We first consider three specifications of the noncooperative first-stage game in which the children move before the parent, then three specifications in which the parent moves before the children. Finally, we consider a specification in which the parent and the children move simultaneously.

i. Children Move before the Parent.

We consider two sequential games that differ in which child moves first and a simultaneous move game. When Child 1 moves first, she does not invite (v') the parent to coreside; the best response of child 2 is to invite (v) the parent to coreside, and the parent accepts the invitation. When Child 2 moves first, she does not invite the parent to coreside; the best response of child 1 is to invite the parent to coreside, and the parent accepts the invitation. The simultaneous move game has two pure strategy equilibria: (1) Child 1 invites coresidence, and child 2 does not; the parent accepts the invitation of child 1. (2) Child 2 invites coresidence, and child 1 does not; the parent accepts the invitation of child 2.\(^{12}\)

These examples demonstrate that the equilibrium of our two-stage game can depend

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\(^{12}\) The simultaneous move game also has a mixed strategy equilibrium which we shall not discuss.
on the precise specification of the game (e.g., which child moves first in the sequential
game) and that some specifications of the game (e.g., the game in which the children move
simultaneously) can have multiple equilibria. In section 4c we offer an example with a
richer set of equilibria.

ii. Parent Moves before the Children.

When the parent moves first, she can be a strategic player. Suppose that the parent
can commit herself to reject particular invitations, if she should receive them.\footnote{Shelly Lundberg suggests that the parent might achieve the same result by insulting a child’s spouse.} \footnote{We assume that the parent cannot commit herself to reject living independently or in a nursing home. If she could commit herself to rejecting one or both of these living arrangements (e.g., by starving herself to death), she might be able to force coresidence.} More specifically, the game begins with the parent choosing among three moves.

(i) preemptively reject an invitation from Child 1 (r1)

(ii) preemptively reject an invitation from Child 2 (r2)

(iii) make no preemptive rejection of any invitation (r’)

The children then move, either sequentially or simultaneously, as described above. Finally,
the parent chooses a living arrangement: she can live independently or in a nursing home or
accept any invitation she has received except those she has preemptively rejected.

The analysis of these games is straightforward. The parent begins by committing
herself to rejecting an invitation from child 2. The equilibrium of all three games -- the two
sequential games and the simultaneous game -- is an invitation from child 1, which the
parent accepts. This example shows that the ability of a family member (in this case, the
parent) to commit can alter the equilibrium outcome and, in this case, commitment enables the parent to achieve the outcome she prefers.

4c. *Multiple Equilibria*

iii. *The Parent and the Children Move Simultaneously.*

The simultaneous game has 12 strategy profiles and 5 of these are equilibria. Recall that the disabled parent has 3 possible strategies (r1, r2, r'), and each child has two possible strategies (v, v'). The reader can verify that the 5 equilibria are:

- (r1, v', v)  parent lives with child 2
- (r2, v, v') parent lives with child 1
- (r2, v, v)  parent lives with child 1
- (r', v', v)  parent lives with child 2
- (r', v, v') parent lives with child 1.

As this example shows, some specifications of the game have a large number of Nash equilibria.\(^\text{15}\)

5. *Conclusion*

We have used a two-stage bargaining model to analyze the living arrangement of a disabled elderly parent and transfers to the parent from her adult children. The first stage determines the living arrangement, the second child-to-parent transfers. Working by backward induction, we first calculate an allocation rule that specifies the level of transfers

\(^\text{15}\) If the parent moves first and the children observe her move and then play a simultaneous move game, then all of these Nash equilibria are subgame perfect.
that each child would provide to the parent in each living arrangement. We then analyze the living arrangement(s) that emerge as equilibria of the first stage game. Because the living arrangement affects bargaining power in the second stage game, and because family members at the first stage are unwilling or unable to make binding agreements regarding transfers at the second stage, the equilibria of the two-stage game may be Pareto inefficient even if the equilibrium of the second stage subgame is conditionally efficient.

A better understanding of the process by which families come to assume the responsibility and share the burden of caring for the disabled elderly is essential for designing and evaluating long-term care policies. As governments increasingly explore policies to address the needs of their growing disabled elderly populations, the observation that families’ long-term care decisions may result in inefficient outcomes suggests an additional role for public policy. Initiatives may be tailored to family living arrangements, for example, by imposing a “tax” on non-coresiding adult children or compensating co-residing adult children for both their informal care services and relative loss of bargaining power, in order to promote more efficient outcomes.\textsuperscript{16} While government transfers that are independent of family living arrangements are analytically simpler than family transfers that are not, the effectiveness of long-term care public policy will be enhanced by

\textsuperscript{16} In the United States, examples of such an initiative are state programs under the Home and Community-Based Waivers that either condition eligibility for services on the presence of an “active” family caregiver or provide more, rather than less, hours of paid assistance to elderly persons who coreside with a family caregiver (Centers for Medicare and Medicaid Services 2004).
recognizing that the caregiving behavior of family members will be affected by the incentives created by public programs.
References


McKelvey, R. and T. Palfrey (1992) "An Experimental Study of the Centipede Game,"  
*Econometrica* 60(4): 803-836.


Figure 1: Children’s Reaction Functions for Transfers

Transfers from Child 2

$T_2^*$

Child 1's reaction function

Child 2's reaction function

$T_1^*$

Transfers from Child 1
Figure 2: The Nash Bargaining Solution between the Children