Energy Efficiency and Directed Technical Change: Implications for Climate Change Mitigation

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Abstract

I build a quantitative model of economic growth that can be used to evaluate the impact of environmental policy interventions on final-use energy consumption, an important driver of carbon emissions. In the model, energy demand is driven by endogenous and directed technical change (DTC). Energy supply is subject to increasing extraction costs. Unlike existing DTC models, I consider the case where multiple technological characteristics are embodied in each capital good, a formulation conducive to studying final-use energy. The model is consistent with aggregate evidence on energy use, efficiency, and prices in the United States. I examine the impact of new energy taxes and compare the results to the standard Cobb-Douglas approach used in the environmental macroeconomics literature, which is not consistent with data. When examining a realistic and identical path of energy taxes in both models, the DTC model predicts 22% greater cumulative energy use over the next century. I also use the model to study the macroeconomic consequences of R&D subsidies for new energy efficient technologies. I find large rebound effects that undo short-term reductions in energy use.

Keywords Energy, Climate Change, Directed Technical Change, Growth

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1 Introduction

Discussions of climate change mitigation often focus on substitution between clean and dirty sources of energy. Carbon accounting, however, shows that improvements in energy efficiency have been the primary driver of long-run reductions in the carbon intensity of output (e.g., Raupach et al., 2007; Nordhaus, 2013). In this paper, I further demonstrate that increases in energy efficiency are driven by improvements in final-use energy efficiency. In other words, reductions in the carbon-intensity of output tend to occur when capital goods and consumer durables require less energy to run, not when an economy uses more renewable energy sources or gets better at turning primary energy (e.g., coal) in to final-use energy (e.g., electricity).

Despite the importance of final-use energy efficiency in determining long-run trends in carbon emissions, this margin has received relatively little attention in the environmental macroeconomics literature. Studies with directed technical change and climate change focus on clean versus dirty sources of energy and do not consider energy efficiency as a separate source of technological improvement (e.g., Acemoglu et al., 2012, 2016). Studies with exogenous technology, other the other hand, frequently assume that final-use energy is combined with capital and labor in a Cobb-Douglas fashion (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014), even though this is at odds with well known data patterns (Atkeson and Kehoe, 1999; Hassler et al., 2012, 2016b).

This paper constructs the first quantitative macroeconomic model focusing on final-use energy. I show that a simple and tractable growth model is consistent with evidence on aggregate energy use, efficiency, and prices in the United States, as well as the standard Kaldor (1961) facts. I then calibrate the model to macroeconomic data from the United States and data on energy extraction costs. I use the model to investigate the impact of environmental policy interventions on final-use energy consumption.

The demand side of the model highlights the role of endogenous and directed technical change (Hassler et al., 2012, 2016b). Existing evidence suggests that there is a near-zero short-run elasticity of substitution between energy and non-energy inputs, but a unitary long-run elasticity. The model captures this fact by differentiating between *ex post* substitution – which captures substitution when technology is fixed – and *ex ante* substitution, which occurs through the choice of technology (e.g., Jones, 2005; Caselli et al., 2006; León-Ledesma and Satchi, 2018). For a given set of technologies, energy and non-energy inputs must be combined in fixed proportions. Capital good producers, however, respond to increases in the relative price of energy by lowering the energy input ratio through directed research and development activity. Thus, the long-run elasticity of substitution is higher than the short-run elasticity.

I develop a new underlying model of directed technical change. The standard Acemoglu (1998, 2002) approach focuses on the role of innovation in different sectors. To focus on final-use energy efficiency – rather than the efficiency of the energy sector – I consider the case where two types of technology are embodied in each capital good. One type captures the ability of the capital good to produce output. The other captures the energy efficiency of final good production.

Since the model focuses on the role of final-use energy, I consider a simple representation of primary energy supply. There is a single aggregate energy composite that is available in infinite supply, but is subject to increasing extraction costs. Most directed technical change models of energy use consider the case where the finite supply of energy resources drives long-run trends in prices (Hassler et al., 2012, 2016b; André and Smulders, 2014).[1] I show, however, that this Hotelling (1931) approach is at odds with aggregate data, which demonstrate that energy use is increasing on the balanced growth path. The increasing extraction cost model is consistent with data and suggests that energy consumption growth will continue at current rates in the absence of policy intervention or an environmental disaster.

The primary quantitative analysis in this paper studies the case of energy taxes and compares the new model to the standard Cobb-Douglas approach with exogenous technology (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014). Existing studies argue that the Cobb-Douglas model is an appropriate stand-in for a model with endogenous and directed technical change, because both approaches feature unit-elastic long-run substitution between energy and non-energy inputs (e.g., Golosov et al., 2014; Hassler et al., 2016c, 2017; Barrage, forthcoming). When explicitly comparing the two models, however, I find that they yield significantly different predictions about the impact of environmental policy.

The rationale for the conventional wisdom is straightforward: the study of climate change is inherently concerned with long-run outcomes. The Cobb-Douglas and directed technical change models have identical long-run predictions in the absence of policy. Thus, they are likely to yield similar long-run predictions in a world with policy. I show, however, that this reasoning does not hold up to quantitative scrutiny. Climate change is a function of the stock of carbon in the atmosphere, rather than the flow of emissions. Since the directed technical change model accurately captures the low short-run elasticity of substitution observed in the data, it predicts that energy efficiency will react slowly to new energy taxes. By contrast, the Cobb-Douglas approach predicts that reactions will occur immediately, because it is relatively easy to substitute between energy and non-energy inputs. In other words, the directed technical change model features slower transitions. As a result, the two models yield different predictions for medium-term and cumulative energy use.

The easiest way to compare the quantitative predictions of the two models is to examine their predicted reactions to the same path of future energy taxes. I simulate taxes in the Cobb-Douglas model that are needed to reduce energy use by 15% between 2005 and 2055.[2] With the same path of taxes, the directed technical change model misses the energy use target by over 20 percentage points. Thus, the slow transition path implies that policy designed with the Cobb-Douglas model is unlikely to achieve intended targets for medium-run flows of emissions. The directed technical change model also predicts 22% greater cumulative energy use over the next century. Given that

[1] This literature is focused on the economic consequences of exhaustible resources, rather than climate change and environmental policy.

[2] This is consistent with goals laid out in the Paris Agreement, which suggests that the United States adopt policies consistent with a 80% reduction in carbon emissions by 2050 (Heal, 2017; Williams et al., 2014). The goals are outlined in the Intended Nationally Determined Contribution (INDC) submitted by the United States to the United Nations Framework Convention on Climate Change (UNFCC), which is available at: https://www4.unfccc.int/sites/submissions/indc/Submission%20Pages/submissions.aspx.

the new model better explains past patterns of energy use, these results provide evidence that existing analyses are too optimistic about the impacts of climate change mitigation policy, at least when considering the important margin of final-use energy efficiency.

A second analysis investigates the macroeconomic consequences of research subsidies and mandates for energy efficient technologies. These policies are commonly used in attempts mitigate climate change and achieve energy security (Gillingham et al., 2009; Allcott and Greenstone, 2012). Despite their popularity, these policies may be ineffective due to rebound in energy use. Rebound occurs when economic behavior lessens the reduction in energy use following efficiency improvements. A long literature attempts to indirectly evaluate the effectiveness of such policies by estimating the size of rebound effects, usually in partial equilibrium or static settings (Gillingham et al., 2016). The directed technical change model, however, makes it possible to directly analyze the broader motivating question: can policies aimed at improving energy efficiency achieve long-term reductions in energy use, even if they do not increase energy prices? I start by considering the standard rebound exercise of a one-off improvement in energy efficiency. Consistent with existing evidence, such shocks lead to short-run reductions in energy use (e.g., Davis, 2017), but they also lower the incentive for future investment in energy efficient technology. As a result, the interventions lead to temporary increases in medium-term energy use relative to world without policy, an extreme form of rebound known as 'backfire.' Eventually, the short-term reductions and medium-term backfire offset, leaving cumulative energy use unchanged. Permanent policy interventions can overcome rebound effects to achieve long-run reductions in energy use relative to laissez faire, but cannot achieve absolute decreases in energy use.

Related Literature. This paper contributes to several literatures. The first is carbon accounting. A literature focusing on the Kaya identity demonstrates that energy efficiency, rather than the carbon intensity of energy, has driven reductions in the carbon intensity of output in the United States and around the world (Raupach et al., 2007; Nordhaus, 2013; Peters et al., 2017). I take this finding one step further and show that these reductions in energy use are driven by final-use energy efficiency.

Second, this paper contributes to the quantitative macroeconomic literature on climate change by constructing a growth model focusing on final-use energy. Studies on directed technical change and climate change focus on clean versus dirty sources of primary energy and do not consider energy efficiency as a separate source of technological improvement (e.g., Acemoglu et al., 2012, 2016; Fried, 2018). Meanwhile, the literature on endogenous, but not directed, energy efficiency improvements focuses on the efficiency of the energy sector (e.g., Popp, 2004; Bosetti et al., 2006). While both of these margins are important, the data strongly suggest that the overlooked margin of final-use energy efficiency is an important long-run driver of carbon emissions. Studies with exogenous technology frequently assume that final-use energy is combined with capital and labor in a Cobb-Douglas fashion (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014; Barrage, forthcoming), but this is at odds with existing data (Atkeson and Kehoe, 1999; Hassler et al., 2012, 2016b). As noted above, I show that this modeling difference has important consequences for understanding the impacts of climate change mitigation policy.

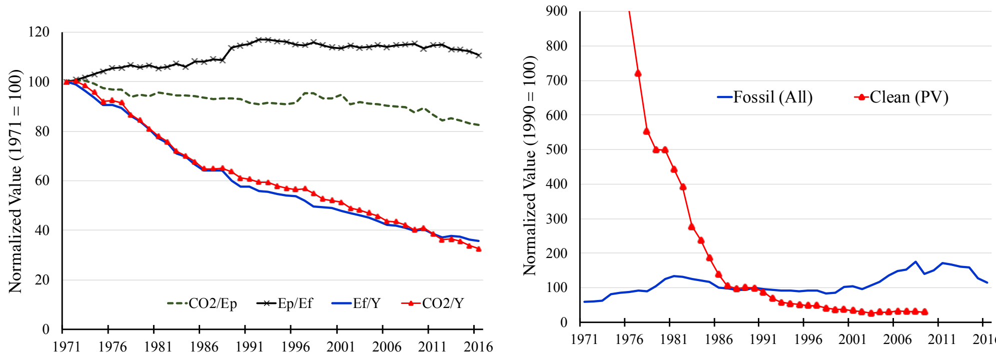
Third, this paper is related to the literature on directed technical change and energy use, which focuses on questions of long-run sustainability (e.g., Di Maria and Valente, 2008; André and Smulders, 2014). The most closely related paper is that of Hassler et al. (2012, 2016b), who show that a model of directed technical change is consistent with data on energy demand from the United States. They use this observation to examine how a social planner should manage a finite resource and generate predictions for future consumption growth. I build on their findings in several ways. First, I develop a decentralized model that can be used to investigate the impacts of environmental policy. While Hassler et al. (2012, 2016b) do not analyze the impacts of policy, their findings have been used to support the use of the Cobb-Douglas assumption in climate change economics (e.g., Golosov et al., 2014; Hassler et al., 2016c, 2017; Barrage, forthcoming). By explicitly analyzing the impact of policy in both models, I show the opposite result: constraining the model to match short-run data leads to different long-run reactions to policy. I also build on the work of Hassler et al. (2012, 2016b) by considering an alternative model of primary energy supply. In particular, I consider the case of increasing extraction costs, rather than finite energy supplies. I show that (i) unlike the Hotelling (1931) model, the increasing cost formulation is consistent with aggregate data and (ii) the increasing cost formulation leads to different predictions about growth in consumption and energy use in the absence of climate change mitigation policy.

Finally, this paper contributes to the broader literature on the modeling of directed technical change. Employing the directed technical change model of Acemoglu (1998, 2002) in the context of energy efficiency requires focusing on technological improvements in the energy sector (e.g., Smulders and De Nooij, 2003; André and Smulders, 2014). In order to focus on final-use energy efficiency, therefore, I construct a new model where both types of technology are embodied in the capital good. This allows for labor to move between production and research even as population grows, which is important for the current context because (i) growing population is an important driver of energy use and (ii) environmental policy may increase the incentive for investment in energy efficient technologies. Microeconomic evidence that changes in energy prices affects the direction of R&D is presented by Newell et al. (1999), Popp (2002), Newell and Stavins (2003), and Aghion et al. (2016), among others.

Roadmap. Section 2 discusses the empirical motivation underlying the theory. The model is presented in Section 3 and the calibration in Section 4. Section 5 reports the results of the quantitative analyses, and Section 6 concludes.

2 Empirical Motivation

In this section, I discuss the stylized facts that motivate the focus and modeling decisions in this paper. Section 2.1 presents evidence on the importance of final-use energy efficiency in determining



(a) Carbon Accounting

(b) Energy Prices

Figure 1: Panel (a) decomposes the decline in the carbon intensity of output using the identity $\frac{CO_2}{Y} = \frac{CO_2}{E_p} \cdot \frac{E_p}{E_f} \cdot \frac{E_f}{Y}$, where CO_2 is yearly carbon emissions, Y is GDP, E_p is primary energy, and E_f is final-use energy. The results demonstrate that the fall in the carbon intensity of output, $\frac{CO_2}{Y}$, has been driven by decreases in final-use energy intensity of output, $\frac{E_f}{Y}$, rather than the use of cleaner energy sources, $\frac{CO_2}{E_p}$, or a more efficient energy transformation sector, $\frac{E_p}{E_f}$. Panel (b) plots the real price of fossil fuel energy and the real cost of generating electricity from solar energy over this period.

long-run trends in carbon emissions in the United States. Section 2.2 presents evidence on the demand for energy use, which is well captured by a model of directed technical change. Finally, section 2.3 presents evidence on the supply of primary energy, which is consistent with a model of increasing extraction costs.

2.1 The Importance of Final-Use Energy

In this section, I demonstrate that final-use energy efficiency has played a crucial role in reducing the carbon-intensity of output in the United States. To analyze the determinants of the carbon intensity of output, I consider the following decomposition:

$$\frac{CO_2}{Y} = \frac{CO_2}{E_p} \cdot \frac{E_p}{E_f} \cdot \frac{E_f}{Y}, \tag{1}$$

where CO_2 is yearly carbon emissions, Y is gross domestic product, E_p is primary energy use (e.g., coal, oil), and E_f is final-use energy consumption (e.g., electricity, gasoline). The carbon intensity of primary energy, $\frac{CO_2}{E_p}$, captures substitution between clean and dirty sources of energy (e.g., coal versus solar). The efficiency of the energy sector, which transforms primary energy into final-use energy, is captured by $\frac{E_p}{E_f}$. For example, the ratio decreases when power plants become more efficient at transforming coal into electricity. The final-use energy intensity of output, $\frac{E_f}{Y}$, measures the quantity of final-use energy used per unit of output. For example, the ratio decreases when manufacturing firms use less electricity to produce the same quantity of goods.

Panel (a) of figure 1 plots each component from equation (1) from 1971-2014. Data are normalized to 1971 values.[3] The carbon intensity of output fell over 60% during this time period, and this decline is matched almost exactly by the decline in the final-use energy intensity of output. The carbon intensity of primary energy, $\frac{CO_2}{E_p}$, declined approximately 15% over this period. While this is a significant improvement for environmental outcomes, it is small compared to the overall improvements in the carbon intensity of output. Finally, the efficiency of the energy transformation sector, as measured by the inverse of $\frac{E_p}{E_f}$, actually declined roughly 15% over this period.[4]

To the best of my knowledge, this is the first paper to perform a carbon accounting exercise using equation (1). Existing studies often focus on the Kaya Identity, which only considers the role of primary energy. These studies show that aggregate energy efficiency is the main driver of long-run trends in carbon emissions (e.g., Raupach et al., 2007; Peters et al., 2017). This is the first study to demonstrate the relative importance of final-use energy efficiency, rather than the efficiency of the energy sector.

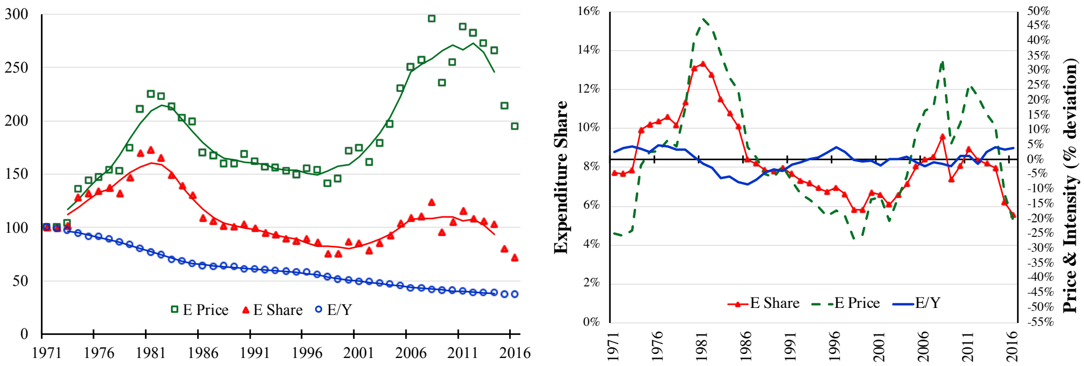
The carbon accounting evidence strongly suggest that final-use energy efficiency cannot be ignored when thinking about the determinants of carbon emissions. Without price data, however, it is difficult to know exactly how well existing trends capture the impact of environmental policy interventions that raise the price of carbon. A tax on carbon has two main effects. First, it will raise the price of fossil fuel-based primary energy sources relative to renewable sources of primary energy. Second, it will raise the price of final-use energy relative to non-energy inputs in production. Panel (b) of figure 1 demonstrates that the average real price of final-energy derived from fossil fuels increased over this period. Unfortunately, a broad measure of the price of renewables is not currently available over this period or level of aggregation. Existing evidence, however, strongly suggests that the real price of renewable energy has been declining (e.g., Nemet, 2006; Covert et al., 2016; Gillingham and Stock, 2018). To highlight this point, panel (b) also plots an estimate of the real cost of generating electricity from solar energy over this period.[5] Improvements in final-use energy efficiency were the dominant source of reductions in the carbon-intensity of output, even during a time period when the price of renewable energy decreased relative to fossil fuels. This result suggests that the margin of final-use energy efficiency will be important in understanding the impact of carbon taxes.

Motivated by these findings, this paper focuses on final-use energy efficiency and its role in climate change mitigation. As explained above, this channel has not received much attention in the existing macroeconomic literature focusing on climate change (e.g., Acemoglu et al., 2012, 2016; Fried, 2018) or energy use (e.g., Smulders and De Nooij, 2003; André and Smulders, 2014). The quantitative analyses in this paper focus on energy use reductions that are necessary to meet environmental policy goals even in the presence of large-scale substitution toward clean sources of

[3] Appendix Section A describes the data and provides links to the original sources.

[4] This result is driven by differences in the efficiency of transformation across different sources of primary energy, rather than technological regress.

[5] The data are originally from Nemet (2006) and were accessed via the Performance Curve Database from the Sante Fe Institute Nagy et al. (2013). These data are for illustrative purposes and will not be used the quantitative analysis.



(a) Normalized Values

(b) Detrended

Figure 2: This figure shows the energy expenditure share (E_{share}), the final-use energy intensity of output (E/Y), and the average real energy price (p_E) in the United States from 1971-2014. These objects are related through the following identity: $E_{share} = p_E \cdot \frac{E}{Y}$. Panel (a) presents data normalized to 1971 values (markers) and 5-year moving averages (lines). Panel (b) presents detrended values for E/Y and p_E, along with the expenditure share. Trends are estimated via OLS, assuming a constant growth rate over the period. Data are taken from the Energy Information Administration (EIA) and the Bureau of Economic Analysis (BEA).

primary energy.[6]

2.2 Energy Demand

The model presented in this paper can recreate key stylized facts regarding energy use, prices, efficiency and expenditure observed in U.S. data, suggesting that it is a useful framework to think about the impacts of policy on final-use energy. Figure 2 summarizes evidence on the demand for energy. It shows the expenditure share of energy (E_{share}), the energy intensity of output (E/Y), and the average real energy price (p_E) in the United States from 1971-2014. These objects are related through the following identity:

$$E_{share} = p_E \cdot \frac{E}{Y}. \tag{2}$$

Panel (a) plots annual values, along with five-year moving averages, to highlight medium- and long-run trends. Panel (b) plots detrended annual values for E/Y and p_E against E_{share} to highlight short-run fluctuations.

The data indicate that the expenditure share, but not the energy intensity of output, reacts to short-term price fluctuations, suggesting that it is difficult to substitute between energy and non-energy inputs in the short run. Hassler et al. (2012, 2016b) provide a formal maximum likelihood

[6]Ideally, it would be possible to estimate the elasticity of substitution between different sources of primary energy, as well as the elasticity between energy and non-energy inputs in production. This would allow for a simple decomposition of these channels in a unified structural model. In this paper, I study substitution between energy and non-energy inputs, which is dynamic. Unfortunately, there is no consensus on the elasticity of substitution between clean and dirty sources of primary energy, largely because aggregate price data do not exist. See section 5.1 for further discussion.

estimate of the short-run elasticity of substitution between energy and non-energy inputs and find a value very close to zero. As seen in both figures, the expenditure share can deviate from its long-run average (8.4%) by a substantial amount and for a significant period of time. Despite increasing prices, however, there is no long-run trend in the energy expenditure share of output.[7]

Hassler et al. (2012, 2016b) show that a directed technical change (DTC) model can recreate these facts. With fixed technology, the elasticity of substitution between energy and non-energy inputs is essentially zero. Over longer time horizons, agents in the economy respond to higher energy prices by investing in energy efficiency, driving down energy use. As a result, the expenditure share is constant on the balanced growth path, despite increasing prices and a low short-run elasticity of substitution.[8,9]

I build on their work by constructing a decentralized model that can be used to examine the impacts of policy. Motivated by the evidence presented in figure 1, the model focuses on the demand for final-use energy coming from final good production, rather than the demand for primary energy coming from the energy sector. To capture this important margin, the new model departs from the DTC approach of Acemoglu (1998, 2002) and considers the case where multiple types of productivity – including energy efficiency – are embodied in each capital good. When combined with Leontief production in the short-run, this yields a tractable model that can be used to study the impact of environmental policy on final-use energy consumption.[10]

In macroeconomic studies of climate change, it is common to assume that energy and non-energy inputs are combined in a Cobb-Douglas (CD) fashion, even though this is at odds with the short- and medium-run data provided in figure 2 (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014; Barrage, forthcoming). While Hassler et al. (2012, 2016b) do not investigate the impacts of policy, their work has been used to motivate the CD assumption, because the two models have the same long-run elasticity of substitution (e.g., Golosov et al., 2014; Hassler et al., 2016c, 2017; Barrage, forthcoming).

Using the decentralized model developed in this paper, I compare the two approaches and show that they lead to significantly different quantitative predictions about the impacts of policy. The difference can be seen through equation (2). New taxes effectively increase the price of energy. The CD model assumes that energy intensity must immediately fall by enough to leave the expenditure

[7] Focusing on primary energy, Hassler et al. (2012, 2016b) show that this is true over a longer time horizon.

[8] See also, Hart (2013) and André and Smulders (2014). For related results focusing on the elasticity of substitution between capital and labor, see Jones (2005), Caselli et al. (2006), and León-Ledesma and Satchi (2018), among others.

[9] Not all improvements in aggregate energy efficiency need to be driven by technical change. In particular, sectoral reallocation could potentially explain changes in agggregate energy use. Decomposition exercises suggest that improvements in intra-sectoral efficiency, rather than reallocation, have been the key driver of falling energy intensity over this period (Sue Wing, 2008; Metcalf, 2008). They also suggest that, prior to 1970, sectoral reallocation was the primary driver of falling energy intensity. The calibration will focus on the post-1970 period. Existing work suggests that there was a significant regime shift in both energy prices and energy efficiency improvements after this period (e.g., Hassler et al., 2012, 2016b; Baumeister and Kilian, 2016; Fried, 2018). See Hart (2018) for a model focusing on earlier periods where energy efficiency was driven by sectoral reallocation.

[10] The Acemoglu (2002) approach focuses on innovation in different sectors, rather than the efficiency with which different inputs are combined. This has been a fruitful way to examine, for example, directed technical change in clean versus dirty energy sectors (Acemoglu et al., 2012, 2016; Fried, 2018), but it cannot speak to final-use energy, which is inherently concerned with the ratio with which energy in combined with other inputs.

share unchanged. Both the data and the new DTC model, however, suggest that energy intensity will be unchanged in the very short-run and the expenditure share will spike. Then, energy efficiency will improve over time and the expenditure share will converge back to its long-run level. The difference in transition paths implies that the two models will have different predictions for both cumulative and medium-term energy use, even in cases where they have identical predictions for energy use at some point in the future.

2.3 Energy Supply

Since this paper is focused on the role of final-use energy, I consider a simple representation of the supply of primary energy. As discussed above, a trendless energy expenditure share, increasing energy prices, and falling energy intensity of output are all consistent with the balanced growth path of a DTC model. To differentiate between possible causes of the rising prices, I now turn to trends in energy use.[11]

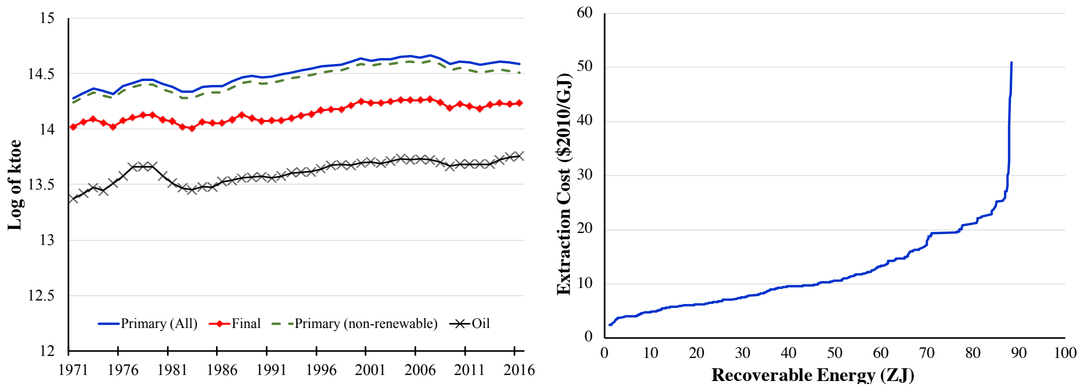
Studies with aggregate energy use almost always use one of two underlying models of energy supply to explain long-run trends in prices, optimal depletion of finite resources (e.g., Hotelling, 1931; Dasgupta and Heal, 1974) or increasing extraction costs (e.g., Pindyck, 1978; Slade, 1982). Existing work on directed technical change and the environment focuses on the former (Di Maria and Valente, 2008; Hassler et al., 2012, 2016b; André and Smulders, 2014). Of the two approaches, however, only the increasing extraction cost model is consistent with aggregate evidence from the United States. If rising prices are driven by forward looking behavior and finite supplies, then energy use must decrease on the balanced growth path, which is when the energy expenditure share is constant. Figure 3, however, shows that energy use has been increasing over the period of study. Thus, the aggregate data are inconsistent with a model where increasing prices are driven by scarcity rents.

Panel (b) of figure 3 provides direct evidence for the existence of increasing extraction costs. It shows estimates fossil fuel extraction costs and availability from McGlade and Ekins (2015). The estimates were developed for the TIMES Integrated Assessment Model in University College London ("TIAM-UCL"), which has a detailed representation of energy supply. The figure aggregates across the three sources of fossil fuels – oil, natural gas, and coal – and converts primary energy availability into final-use energy availability using average transformation rates.[12]

Based on the evidence presented in figure 3, I consider the case of increasing extraction costs, which allows for increasing energy use on the balanced growth path. As discussed in Sections 3 and 4, the convexity in the supply curve for energy is not sufficient to explain rising energy prices relative to other goods. I attribute the remaining increase in relative prices to slower technological

[11] As demonstrated in figure 2, the price of energy in the United States had an upward trend from 1971-2016. Once again, this is a good match for post-1970 data, but not for U.S. data in the preceding two decades, where energy prices actually declined. Consistent with the predictions of the model, decomposition exercises suggest that intra-sectoral energy efficiency declined during this period of falling prices (Sue Wing, 2008). In this paper, I focus on the case where prices increase in the long run.

[12] See appendix sections A and B.6 for more detail.



(a) Energy Use (b) Extraction Costs

Figure 3: Panel (a) demonstrates that aggregate energy use has been increasing in the United States over the period 1971-2014, even as the energy expenditure share was constant (see figure 2). *ktoe* is kilotons of oil equivalent, a measure of energy content. Panel (b) presents estimates of the availability and extraction cost of fossil fuel resources. Original estimates are from McGlade and Ekins (2015). Quantities are measured as zettajoules (ZJ) of final-use energy that can be extracted from available primary energy resources. Transformations from primary to final-use energy availability were performed using information from Rogner et al. (2012), the EIA, and the IEA.

progress in the energy extraction sector.

The underlying model of energy costs will help determine the equilibrium impacts of environmental policy. When policy decreases energy use, extraction prices decrease as well, partially offsetting the environmental benefits of the original intervention. The strength of this feedback depends on the nature of supply. Also, the fact that energy use will increase in the absence of policy implies that larger interventions are needed to hit specific environmental policy goals, when compared to a world where energy use would decrease even without targeted policy.

3 Model

3.1 Structure

3.1.1 Final Good Production

The model extends the standard endogenous growth production function to account for energy use. Final good production is perfectly competitive. To match the low short-run elasticity of substitution between energy and non-energy inputs, I will consider a Leontief structure

$$Q_t = \int_0^1 \min\left[\left(A_{N,t}(i) X_t(i)\right)^\alpha L_t^{1-\alpha}, A_{E,t}(i) E_t(i) \right] \, di, \tag{3}$$

$$s.t. \quad A_{E,t}(i) E_t(i) \leq A_{N,t}(i) X_t(i)^\alpha L_t^{1-\alpha} \; \forall i, \tag{4}$$

where Q_t is gross output at time t, $A_{N,t}(i)$ is the the quality of capital good i, $X_t(i)$ is the quantity of capital good i, L_t is the aggregate (and inelastic) labor supply, $A_{E,t}(i)$ is the energy efficiency of capital good i, and $E_t(i)$ is the amount of energy devoted to operating capital good i. Several components of the production function warrant further discussion. As in the standard endogenous growth production function, output is generated by a Cobb-Douglas combination of aggregate labor, L_t, and a series of production processes, each of which uses a different capital good, indexed by i. Unlike the endogenous growth literature, each production process also requires energy to run. Thus, the usual capital-labor composite measures the potential output that can be created using each production process, and the actual level of output depends on the amount of energy devoted to each process, $E_t(i)$. The notion of potential output is captured by constraint (4). Each capital good i has two distinct technological characteristics. The quality of the capital good, $A_{N,t}(i)$, improves its ability to produce output. The energy efficiency of the capital good, $A_{E,t}(i)$, lowers the amount of energy needed to operate the production process at full potential.[13]

3.1.2 Energy Sector

Energy supply is subject to increasing extraction costs (see, e.g., Heal, 1976; Pindyck, 1978; Lin and Wagner, 2007). Extraction costs are paid in final goods, and energy is provided by a perfectly competitive sector with open access.[14] The increasing extraction cost incorporates two main forces that govern long-run energy availability. First, it captures the increase in cost needed to extract conventional energy resources from harder-to-access areas.[15] Second, it captures the increase in cost that may occur when a particular energy source is exhausted, necessitating a switch to a type of energy that is more difficult to extract.

The fact that production is open-access implies that Hotelling (1931) rents play no role in determining prices. In other words, agents do not behave as if energy resources are finite. As discussed in Section 2.3, this is necessary to match data on long-run energy use. When examining the implications of the model, I also assume that the underlying energy supply limits are never reached. This is consistent with existing models and geological evidence. In particular, the infinite supply of energy and increasing extraction costs capture the existence of 'unconventional' energy sources, which have high extraction costs but are available in vast quantities (Rogner, 1997; Rogner et al., 2012).[16] As in Golosov et al. (2014), the treatment of energy sources as infinite in potential

[13]Consistent with the econometric literature on energy use, energy requirements depend both on the amount of capital and the amount of labor being used in the production process (Van der Werf, 2008; Hassler et al., 2012, 2016b). Consistent with both the econometric and DTC literatures, improvements in non-energy technology, $A_N(i)$, raise energy requirements (e.g., Smulders and De Nooij, 2003; Van der Werf, 2008; Hassler et al., 2012, 2016b; Fried, 2018). This framework is isomorphic to one in which $A_N(i)$ is the relative price of investment goods.

[14]Since energy extraction is not forward looking, the competitive equilibrium will not be pareto optimal.

[15]For example, recent research suggests that most new oil production comes from the exploitation of new geographic areas, rather than improved technology applied to existing sources of energy (Hamilton, 2012).

[16]For example, Rogner et al. (2012) estimate a resource base of 4,900 – 13,700 exajoules (EJ) for conventional oil, compared with annual production of 416 EJ across all energy sources. Thus, constraints on availability of conventional oil sources may be binding. The ability to exhaust fossil fuel energy sources, however, appears much less likely when considering other options. The resource base for unconventional sources of oil is estimated to be an additional 3,750 – 20,400 EJ. Meanwhile, the resource base for coal and natural gas (conventional and unconventional) are 17,300– 435,000 EJ and 25,100 – 130,800 EJ, respectively. These estimates rely on projections regarding which resources will be profitable to extract from the environment. When considering the full range of energy sources that could become profitable to extract as resource prices tend towards infinity, the numbers grow even larger. In particular,

supply also incorporates the abundance of coal, which is predicted to be the major driver of climate change (van der Ploeg and Withagen, 2012; Hassler et al., 2016a).[17] Together, the vast quantities of coal and 'unconventional' energy sources imply that using too much fossil energy, rather than exhausting supply, is the relevant environmental concern (Covert et al., 2016).

The marginal cost of extraction, which will also be equal to the price, is given by

$$p_{E,t} \quad = \quad A_{V,t}\bar{E}_{t-1}^{\psi}, \tag{5}$$

where $\bar{E}_{t-1}$ is *cumulative* energy ever extracted at the start of period t, and $A_{V,t}$ captures differential state of technology between the energy extraction and final good sectors.

As discussed in Section 2, the price of energy relative to output has been increasing since the early 1970s. This can occur for two reasons. First, $A_{V,t}$ could increase over time, implying the technological progress in the extraction sector is slower than technological progress in the final goods sector. Second, $\psi > 0$ implies that the cost of energy extraction increases over time, even when there is no differential technological progress. There is ample evidence that extraction costs increase with fixed technology (e.g., McGlade and Ekins, 2015; Rogner et al., 2012). For simplicity, I assume that differential technological progress is given by the exogenous process

$$A_{V,t} = (1 - g_V)A_{V,t-1}, \tag{6}$$

and focus on endogenous and directed technical change in energy demand. If $g_V > 0$, then technological progress is faster in the extraction sector (i.e., the rate at which capital and labor can be used to extract energy improves faster than the rate at which capital and labor can be used to produce final goods).

The law of motion for the stock of extracted energy is given by

$$\bar{E}_t = E_t + \bar{E}_{t-1}, \tag{7}$$

where $\bar{E}_t$ is *cumulative* energy used at the end of period t and E_t is *flow* energy use during period t. The fact that extraction costs are constant within each period is a useful simplification. As motivation, it is intuitive to consider the case where energy producers exploit new sources of energy in each period and the difficulty of extraction is constant within each source.[18,19]

such 'additional occurrences' are estimated to be larger than 1 million EJ for natural gas and 2.6 million EJ for uranium.

[17]Technically, Golosov et al. (2014) specify a finite amount of coal, but assume it is not fully depleted. Thus, it has no scarcity rent, although it does have an extraction cost. Oil, by contrast, is assumed to have no extraction cost, but does have a positive scarcity rent. Hart and Spiro (2011) survey the empirical literature and find little evidence that scarcity rents are a significant component of energy costs. They suggest that policy exercises focusing on scarcity rents will give misleading results.

[18]This is consistent, for example, with recent evidence from the oil industry, where drilling, but not within-well production, responds to changes in prices (Anderson et al., 2014).

[19]A primary goal of this paper is to compare the results of the new DTC model to the standard CD approach used in integrated assessment models (IAMs) (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014). Since IAMs examine worldwide outcomes, it is crucial to consider the equilibrium effect of policy on energy prices. Hence, the

3.1.3 Final Output

Final output is given by gross production less total energy extraction costs, which are equal to energy expenditures by the final good producer. As long as equation (4) holds with equality,[20] final output is given by

$$Y_t \;\;=\;\; L_t^{1-\alpha}\int_0^1 \left[1-\frac{p_{E,t}}{A_{E,t}(i)}\right](A_{N,t}(i)X_t(i))^{\alpha}\;di. \tag{8}$$

This formulation further illuminates the continuity between the production function used here and the standard approach in endogenous growth models. Output has the classic Cobb-Douglas form with aggregate labor interacting with a continuum of capital goods. The model developed in this paper extends the standard endogenous growth set-up model by considering a broader notion of aggregate productivity, $\left[1-\frac{p_{E,t}}{A_{E,t}(i)}\right]\cdot(A_{N,t}(i))^{\alpha}$. Productivity is determined two different types of embodied technology, as well as energy extraction costs. The functional form is driven by the fact that underlying production function is Leontief. I show that this updated formulation leads to a tractable growth model. Moreover, in the long-run, the updated technology index grows at a constant rate, and the model can explain all of the usual balanced growth facts.

Final output can either be consumed or saved for next period. In the empirical application, each period will be ten years. Following existing literature, I assume complete depreciation during production (Golosov et al., 2014). Market clearing for the final good implies

$$Y_t = C_t + K_{t+1} = L_t w_t + r_t K_t + \Pi_t + p_t^R + T_t, \tag{9}$$

where K_t is aggregate capital, Π_t is total profits, T_t is net tax revenue, and p_t^R is total payments to R&D inputs (discussed in the next section). I assume that the government balances the budget using lump-sum taxes or transfers.

3.1.4 Capital Goods and Research

Each type of capital good is produced by a single profit-maximizing monopolist in each period. This monopolist also undertakes in-house R&D activities to improve the embodied technological

comparison between models is most accurate when considering endogenous prices. At the same time, I also use the model to investigate the effect of policies pursued in the United States. In this case, endogenous energy prices can be motivated in two ways. First, it is possible to think of the United States as a closed economy, which is a good match for some, but not all, sources of primary energy. Alternatively, one can imagine the policies being applied worldwide with the United States making up a constant fraction of total energy. To ensure that the results of the paper are not driven by this assumption, I also consider the opposite case of exogenous energy prices, which implicitly treats the United States as a small open economy taking unilateral policy actions. In this case, energy prices will increase at a constant exogenous rate.

[20] To ensure that equation (4) holds with equality, it is sufficient, but not necessary, to assume that capital fully depreciates after each period. If capital fully depreciates, then forward looking consumers will never over invest in capital and drive its return to zero.

characteristics, $A_{N,t}(i)$ and $A_{E,t}(i)$. The R&D production function is given by

$$A_{J,t}(i) = \left[1 + \eta_J R_{J,t}(i)^{1-\lambda}\right] A_{J,t-1}, \quad J = N, E, \tag{10}$$

where $R_{J,t}(i)$ is R&D inputs assigned to characteristic J by firm i in period t, and $A_{J,t-1} \equiv \max\{A_{J,t-1}(i)\}$. I also define $R_{J,t} \equiv \int_0^1 R_{J,t}(i)di$. R&D builds on aggregate knowledge, $A_{J,t-1}$, and within-period research allocations, $R_{J,t}(i)$. There are decreasing returns to R&D within a period. When the period ends, patents expire and all technology becomes available to all firms. Monopolists make decisions to maximize single period profits.[21]

There are a unit mass of R&D inputs, yielding

$$R_{N,t} + R_{E,t} = 1 \ \forall t. \tag{11}$$

Thus, $R_{J,t}$ can be interpreted as the total share of research inputs used to improve technology of type J. The fixed set of research inputs is a stand in for two offsetting forces, an increase in aggregate research inputs and an increase in the cost of generating a given aggregate growth rate (Jones, 1995, 2002; Bloom et al., 2016). This approach is consistent with both existing literature on DTC and the environment (Acemoglu et al., 2012; Fried, 2018; Hassler et al., 2019) and the social planner model of Hassler et al. (2012, 2016b).[22,23]

I assume that the investment price is fixed at unity. Thus, market clearing implies that

$$\int_0^1 X_t(i)di = K_t, \tag{12}$$

where K_t is aggregate capital.

[21] This can be motivated in several ways. Most directly, the identity of the firm producing capital good i could change after each period. Alternatively, it could be the case that firms are infinitely lived but myopic, which seems reasonable considering the ten year period length. The set-up presented here is isomorphic to one where firms are infinitely lived and the aggregate technology, $A_{J,t-1}$, is given by the average of the previous period's technology as in Fried (2018). This would open up the possibility of technological regress at the firm level, though it would not occur in equilibrium.

[22] Often, models of directed technical change refer to the fixed set of research inputs as scientists (e.g., Acemoglu, 2003; Acemoglu et al., 2012; Fried, 2018). This would be applicable here, though generating the standard Euler equation would require the representative household to ignore scientist welfare (in the environmental literature, directed technical change and capital accumulation are generally not included simultaneously). This would be a close approximation to a more inclusive utility function as long as scientists made up a small portion of the overall population. For simplicity, I refer to research inputs, which could be scientists, research labs, etc.

[23] An extension of the model presented in Appendix Section B.8 incorporates aspects of second-wave endogenous growth theory (e.g., Peretto, 1998; Young, 1998; Howitt, 1999) to eliminate the scale effects present in most directed technical change models (e.g., Acemoglu, 2002, 2003; Hassler et al., 2012, 2016b). As a result, the extended model has a BGP with a growing set of R&D inputs, as well as free mobility of workers between production and R&D and free entry of new capital good producers. Following the existing quantitative literature on the macroeconomics of climate change (Acemoglu et al., 2016; Fried, 2018), the analyses conducted in this paper have a fixed set of R&D inputs. The extended model demonstrates that the core intuition of the baseline model holds in a richer setting.

3.1.5 Consumer Problem

The consumer side of the problem is standard. In particular, the representative household chooses a path of consumption such that

$$\{C_t\}_{t=0}^{\infty} \quad = \quad \operatorname{argmax} \sum_{t=0}^{\infty} \beta^t L_t \frac{\tilde{c}_t^{1-\sigma}}{1-\sigma}, \tag{13}$$

where $\tilde{c}_t = C_t/L_t$. Population growth is given exogenously by

$$L_{t+1} = (1+n)L_t. \tag{14}$$

I am interested in the decentralized equilibrium. Thus, I consider the case where the representative household takes prices and technology as given.

3.2 Analysis

As demonstrated in Appendix Section B.1, the first order conditions for the final good producer yield the following inverse demand functions:

$$p_{X,t}(i) \quad = \quad \alpha A_{N,t}(i)^{\alpha} \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right] L_t^{1-\alpha} X_t(i)^{\alpha-1}, \tag{15}$$

$$w_t \quad = \quad (1-\alpha) \int_0^1 \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right] L_t^{-\alpha} \left(A_{N,t}(i) X_t(i)\right)^{\alpha} di, \tag{16}$$

where $\tau_t \geq 1$ is a value-added tax on energy. The intuition for the result is straightforward. The final good producer demands capital goods until marginal revenue is equal to marginal cost. Unlike the usual endogenous growth model, marginal revenue is equal to marginal product minus the cost of energy needed to operate capital goods. Consider the case where the final good producer is already operating at a point where $\left(A_{N,t}(i)X_t(i)\right)^{\alpha} L_t^{1-\alpha} = A_{E,t}(i)E_t(i)$. If the final good producer purchases more capital, it receives no increase in output unless there is a corresponding increase in energy purchased. The final good producer realizes this when making optimal decisions and adjusts demand for capital accordingly. This iso-elastic form for inverse demand maintains the tractability of the model.

Monopolist providers of capital goods must decide on optimal production levels and optimal research allocations. See Appendix Section B.2 for a formal derivation of monopolist behavior. Given the iso-elastic inverse demand function, monopolists set price equal to a constant markup over unit costs. Since capital goods must be rented from consumers, the unit cost is given by $\tau_t^K r_t$, where r_t is the rental rate and τ_t^K is a subsidy for capital purchases. For all subsequent analyses, I assume that $\tau_t^K = \alpha\ \forall t$, which undoes the monopoly distortion generated by embodied technological progress. This facilitates comparison between the new DTC model and existing work, which generally assume perfect competition. Thus, monopolist optimization yields

$$p_{X,t}(i) = r_t, \tag{17}$$

$$X_t(i) = \alpha^{\frac{1}{1-\alpha}} r_t^{\frac{-1}{1-\alpha}} A_{N,t}(i)^{\frac{\alpha}{1-\alpha}} L_t \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}}, \tag{18}$$

$$\bar{\pi}_{X,t}(i) = (\frac{1}{\alpha} - 1)\alpha^{\frac{2}{1-\alpha}} r_t^{\frac{-\alpha}{1-\alpha}} A_{N,t}(i)^{\frac{\alpha}{1-\alpha}} L_t \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}}, \tag{19}$$

where $\bar{\pi}_{X,t}(i)$ is production profits (i.e., profits excluding research costs) of the monopolist.

To understand research dynamics, it is helpful to look at the relative prices for research inputs,

$$\frac{(1-\eta_t^S)p_{E,t}^R(i)}{p_{N,t}^R(i)} = \frac{\tau_t p_{E,t} A_{N,t}(i)}{\alpha A_{E,t}(i)^2 \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]} \frac{\eta_E R_{E,t}(i)^{-\lambda} A_{E,t-1}}{\eta_N R_{N,t}(i)^{-\lambda} A_{N,t-1}}, \tag{20}$$

where $p_{J,t}^R(i)$ is the rent paid to research inputs used by firm i to improve technological characteristic J at time t and $\eta_t^S \in [0, 1)$ is a subsidy for energy efficient research. There are several forces affecting the returns to R&D investment. First, increases in the tax-inclusive price of energy increase the relative return to investing in energy efficiency. Second, the return to investing in a particular type of R&D is increasing in its efficiency. Research efficiency, in turn, depends on inherent productivity, η_J, accumulated knowledge, $A_{J,t-1}$, and the degree of decreasing returns, $R_{J,t}(i)^{-\lambda}$. Third, since energy and non-energy inputs are complements in production, increases in $A_{N,t}(i)$ raise the return to investing in $A_{E,t}(i)$ and vice versa. These effects, however, are asymmetric. The asymmetry occurs because energy efficiency, $A_{E,t}(i)$, has a negative and convex effect on the cost of energy per unit of final output, $\frac{\tau_t p_{E,t}}{A_{E,t}(i)}$. Finally, the return to investing in the quality of capital goods is increasing in elasticity of output with respect to technology, α.

In the usual DTC model, this analysis would demonstrate the role of market size effects and price effects in research incentives (Acemoglu, 1998, 2002). As demonstrated in equation (20), however, aggregate inputs do not affect R&D decisions in this model. In other words, market size effects play no role in this model. This is due to the short-run complementarity between energy and non-energy inputs. Moreover, the price effects in this model differ from those in the usual DTC model. Since the price of the final good is the numeraire, $\frac{\tau_t p_{E,t}}{A_{E,t}(i)}$ is the cost of energy per unit of final good production, and $1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}$ is the cost of non-energy inputs in final good production. Thus, the relative input prices do affect research allocations, but the relative price is completely determined by the cost of energy extraction. Moreover, as explained above, the relative price of energy – along with lagged technology levels – enter asymmetrically, unlike in the seminal model. These theoretical differences highlight the importance of considering the case where improvements in energy efficiency are driven by final-use energy, rather than using the more common approach where innovation occurs in different sectors.

To understand the intuition of the model, it is helpful to consider the laissez-faire case ($\tau_t = 1,\ \eta_t^S = 0$). Then, noting that the price of research inputs must be the same for each technology, equation (20) can be re-written as:

$$\frac{1+g_{E,t}(i)}{1+g_{N,t}(i)} = \frac{\theta_{E,t}(i)}{\alpha}\frac{\eta_E R_{E,t}(i)^{-\lambda}}{\eta_N R_{N,t}(i)^{-\lambda}}, \tag{21}$$

where $\theta_{E,t}(i) = \frac{\frac{p_{E,t}}{A_{E,t}(i)}}{1-\frac{p_{E,t}}{A_{E,t}(i)}}$. This equation has a natural interpretation. Monopolists must trade off the relative benefits and costs of investing in the two types of technology. The ratio $\frac{\theta_{E,t}(i)}{\alpha}$ is a summary measure of the relative return to investment in energy efficiency. As discussed below, $\theta_{E,t}(i)$ measures the energy expenditure share, which captures the benefit of energy efficiency improvements. Meanwhile, α gives the fraction of increased final output that will be paid to capital good producers. The remaining terms on the right-hand side capture the inverse of relative costs – i.e. research efficiencies – of investing in the two types of technology, which are determined by inherent productivity and the degree of decreasing returns.

Given that all firms use common technology at the start of the period, they make identical R&D decisions and end the period with identical technology. Moreover, there is a unit mass of monopolists. Thus, $R_{J,t}(i) = R_{J,t}\ \forall i, J, t$. The optimal research allocations are given by the implicit solution to (22) and (23),

$$R_{E,t} = \frac{\sqrt{\frac{\tau_t p_{E,t}}{A_{E,t-1}}}\sqrt{\frac{1}{\alpha(1-\eta_t^S)}\left[\frac{\eta_E R_{E,t}^{-\lambda}}{\eta_N(1-R_{E,t})^{-\lambda}} + \eta_E R_{E,t}^{-\lambda} - \eta_E R_{E,t}^{1-\lambda}\right] + (1+\eta_E R_E^{1-\lambda})} - 1}{\eta_E R_{E,t}^{-\lambda}}, \tag{22}$$

$$R_{N,t} = 1 - R_{E,t}. \tag{23}$$

To analyze the determinants of research activity, it is instructive to consider multiplying both sides of (22) by $\eta_E R_{E,t}^{-\lambda}$ so that the growth rate of energy efficiency technology is given as a function of the other parameters. Since $\eta_t^S \in [0,1)$, the left-hand side is strictly increasing in $R_{E,t}$ in this formulation and the right-hand side is strictly decreasing in $R_{E,t}$. Thus, $R_{E,t} = \Gamma(\frac{\tau_t p_{E,t}}{A_{E,t-1}})$, for some function $\Gamma(\cdot)$.

Two implications can be immediately read from equations (22) and (23). First, on a balanced growth path, $\frac{\tau_t p_{E,t}}{A_{E,t-1}}$ must be constant. As discussed in Section 3.1.3, this implies that the relevant technology index in the economy will grow at a constant rate and that the model will have a balanced growth path that resembles the standard neoclassical growth model. Second, the model is relatively easy to investigate computationally, because conditional on the price of energy, it is possible to solve for the full sequence of technology parameters independently of capital, the other state variable.

Utility maximization yields

$$\left(\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\right)^{-\sigma} = \beta r_{t+1}. \tag{24}$$

Noting that all monopolists make the same decisions and that there is a unit mass of monopolists, the real interest rate is given by

$$r_t \;=\; \alpha A_{N,t}^{\alpha}\left[1-\frac{\tau_t p_{E,t}}{A_{E,t}}\right]L_t^{1-\alpha}K_t^{\alpha-1}, \tag{25}$$

where the market clearing condition from equation (12) has been applied.

3.3 Equilibrium

Definition 1. *A* competitive equilibrium *is a sequence of prices, $\{w_t, p_{X,t}, r_t, p_t^R, p_{E,t}\}_{t=0}^{\infty}$, allocations, $\{C_t, K_t, L_t, E_t, R_{N,t}, R_{E,t}\}_{t=0}^{\infty}$, technology levels, $\{A_{N,t}, A_{E,t}\}_{t=0}^{\infty}$, and environmental policies, $\{\tau_t, \eta_t^S\}_{t=0}^{\infty}$, such that each of the following conditions holds $\forall t$:*

- *The economy obeys market clearing conditions for final goods, (9), and capital goods, (12).*
- *Optimal research allocations solve (22) and (23).*
- *The dynamics for technology follow (6) and (20), noting that all monopolists make identical decisions.*
- *Consumer behavior follows the Euler equation, (24).*
- *Factor prices are given by (5), (16), (17), and (25), noting that all monopolists make identical decisions and that the market for capital goods clears.*
- *The economy obeys laws of motion for total extracted energy, (7), and population, (14).*
- *Initial Conditions $A_{J,-1}$ for $J = E, N, V$, K_0, L_0, and $\bar{E}_{-1}$ are given.*

3.4 Balanced Growth under Laissez Faire

In this section, I examine long-run outcomes in the absence of environmental policy.

Definition 2. *A* laissez-faire equilibrium *is a* competitive equilibrium *without environmental policy, i.e., $\tau_t = 1$ and $\eta_t^S = 0$ $\forall t$.*

Definition 3. *A* balanced growth path (BGP) *is a path along which equilibrium quantities and technology levels $\{Y_t, K_t, C_t, A_{E,t}, A_{N,t}\}$ grow at constant rates $\forall t \geq s \geq 0$.*

I use asterisks (*) to denote BGP values. On a BGP, research allocations must remain fixed. Consider the laissez-faire case. From equations (22) and (23), it is immediate that $\frac{p_{E,t}}{A_{E,t-1}}$ is constant. Intuitively, this occurs because of the non-linear relationship between energy efficiency, $A_{E,t}$, and the cost of energy per unit of output, $\frac{p_{E,t}}{A_{E,t}}$. When energy prices increase, monopolists have greater incentive to invest in energy efficient technology, but this incentive dissipates as energy technology improves and the cost of energy per unit of falls. As a result, energy prices and energy efficient technology grow at the same constant rate, $g_P^* = g_E^*$, on the BGP.

Definition 4. *The energy share of expenditure, denoted by $\theta_{E,t}$, is the sum of resources paid to energy producers and energy taxes as a fraction of final output, i.e., $\theta_{E,t} \equiv \frac{\tau_t p_{E,t} E_t}{Y_t}$.*

Given that energy prices and energy efficiency grow at the same rate on the BGP, it is straightforward to show that the energy share of expenditure is constant on the BGP,

$$\theta_{E,t} \quad = \quad \frac{p_{E,t}/A_{E,t}}{1 - p_{E,t}/A_{E,t}}, \tag{26}$$

which must be constant given that $\frac{p_{E,t}}{A_{E,t-1}}$ is fixed and the growth rate of energy efficient technology is constant.[24] Thus, despite the Leontief nature of production, the model still delivers a constant long-run energy expenditure share. As demonstrated in Section 2, this is consistent with aggregate data from the United States. Importantly, the expenditure share is only constant on the BGP. It will not be constant on the transition path following the implementation of environmental policy.

The fact that energy technology and the price of energy grow at the same rate yields the first of two key BGP relationships. In particular,

$$(1 - g_V)(1 + g^*_{\overline{M}})^{\psi} = (1 + g^*_E), \tag{BGP-RD}$$

where $g^*_{\overline{M}}$ is the growth rate cumulative energy use. This equation summarizes the conditions for a BGP on the research side of the economy.

I now move to analyzing the remainder of the economy.

Definition 5. Total factor productivity *is defined as in the standard neoclassical growth model, i.e., $TFP_t \equiv \frac{Y_t}{K_t^{\alpha} L_t^{1-\alpha}}$.*

It is immediate that

$$TFP_t = A^{\alpha}_{N,t} \left[1 - \frac{p_{E,t}}{A_{E,t}}\right]. \tag{27}$$

Since $\frac{p_{E,t}}{A_{E,t}}$ is constant on the BGP in the absence of policy, TFP grows at rate $(1+g^*_N)^{\alpha} - 1$, which is also constant. Since the consumer problem and capital accumulation equation are standard, the model now reduces to the standard neoclassical growth model, implying that the DTC model will have the usual BGP properties. In particular, both final output and the capital stock will grow at rate $g^*_Y = (1 + g^*_N)^{\frac{\alpha}{1-\alpha}}(1 + n) - 1$. From equation (3), therefore,

$$1 + g^*_M = \frac{(1 + g^*_N)^{\frac{\alpha}{1-\alpha}}}{1 + g^*_E}(1 + n), \tag{BGP-QE}$$

where g^*_M is the growth rate of per period energy use. This is the key equation describing the output side of the economy. For the remainder of the paper, I assume that

$$\eta_E > n. \tag{A1}$$

[24]See Hart (2013) for a general discussion of the relationship between factor shares and directed technical change.

This rules of the case where energy use grows even when all R&D effort is devoted to energy efficiency.

To characterize the BGP, I compare equations (BGP-RD) and (BGP-QE). The growth rate of the cumulative stock of energy depends on the growth rate of flow energy use. If $g^*_M \leq 0$, $g^*_{\overline{M}} = 0$. Otherwise, $g^*_{\overline{M}} = g^*_M > 0$. To be consistent with data from the United States, I assume that

$$(1 - g_V) \geq \left[1 + \eta_N \left(1 + \left[\frac{g_V}{\eta_E}\right]^{\frac{1}{1-\lambda}}\right)^{1-\lambda}\right]^{\frac{\alpha}{1-\alpha}} (1+n). \tag{A2}$$

which implies that $g^*_M > 0$ in the absence of environmental policy. Combined with assumption (A1), this also implies that $g^*_N > 0$, which is again consistent with data. Energy prices have been increasing over the last four and half decades. So, I assume that

$$\left[(1+\eta_N)^{\frac{\alpha}{1-\alpha}}(1+n)\right]^{\psi} > (1-g_V)^{-1}, \tag{A3}$$

which implies that it is possible for energy prices to increase in the long run. More specifically, if there is no investment in energy efficiency, energy prices will increase over time. Given that $p_E/A_{E,t}$ is constant, this also implies that $g^*_E > 0$.

If $g^*_M = g^*_{\overline{M}} > 0$, as guaranteed by assumption (A2), then equations (BGP-RD) and (BGP-QE) determine the relative growth rates of technology on the unique BGP. Adding in market clearing for R&D inputs, equation (11), yields the optimal research allocations, and applying the law of motion for technology, equation (10), gives the technology and energy use growth rates. The technology growth rates are then sufficient to characterize the output-side of the BGP.

Proposition 1 summarizes and extends the results from this section. In particular, it uses the relationship between equations (BGP-RD) and (BGP-QE) to explicitly characterize the balanced growth path.

Proposition 1. *Let assumptions (A1) – (A3) hold. In a* laissez-faire equilibrium*, there exists an* unique BGP *on which each of the following holds true:*

1. *The research allocations are implicitly given by*
$$R^*_E = \left\{\frac{\left[(1+\eta_N(1-R^*_E)^{1-\lambda})^{\frac{\alpha}{1-\alpha}}(1+n)(1-g_V)^{\frac{1}{\psi}}\right]^{\frac{\psi}{1+\psi}} - 1}{\eta_E}\right\}^{\frac{1}{1-\lambda}} \text{ and } R^*_N = 1 - R^*_E.$$

2. *Technological growth rates are given by $g^*_E = \eta_E(R^*_E)^{1-\lambda}$ and $g^*_N = \eta_N(1-R^*_E)^{1-\lambda}$. The relationship between growth rates can be expressed as:*
$(1+g^*_E)^{\frac{\psi+1}{\psi}} = (1+g^*_N)^{\frac{\alpha}{1-\alpha}}(1+n)(1-g_V)^{\frac{1}{\psi}}$.

3. *Output per worker and consumption per worker grow at a constant rate, $g^*_R = (1+g^*_N)^{\frac{\alpha}{1-\alpha}} - 1$.*

4. *Total output and the capital stock grow at a constant rate, $g^*_Y = (1+g^*_R)(1+n) - 1$, which implies that the capital-output ratio is fixed.*

5. *The real interest rate, r_t, is constant.*

6. *Per period energy use grows at rate $g_M^* = \frac{1+g_R^*}{1+g_E^*}(1+n) - 1 > 0$.*

7. *The expenditure shares of energy, capital, labor, and R&D inputs are constant.*

Proof. The intuition is provided in the text, and a formal proof is provided in Appendix Section B.4. □

3.5 Balanced Growth with Environmental Policy

In this section, I consider long-run outcomes in the presence of environmental policy.

Definition 6. *An* equilibrium with environmental policy *is a* competitive equilibrium *where $\tau_t = \tau_0(1+g_\tau)^t$ and $\eta_t^S = \eta^S$, where $g_\tau > 0$ and $\eta^s, \tau_0 \geq 0\ \forall t$.*[25]

In a world with increasing energy taxes, equations (22) and (23) now imply that the growth rate of energy efficiency is equal to the product of growth in the energy price and the growth of the taxes. Thus, balanced growth on the research side of the economy requires

$$(1+g_\tau)(1-g_V)(1+g_M^*)^\psi = (1+g_E^*), \tag{BGP-RD$'$}$$

which is equivalent to the laissez-faire condition if $g_\tau = 0$. This also implies that, on a BGP, $\lim_{t\to\infty} \frac{p_{E,t}}{A_{E,t}} = 0$. Thus, $\lim_{t\to\infty}[Q_t - Y_t] = 0$ and $\lim_{t\to\infty} \theta_{E,t} = \frac{\tau_t p_{E,t}}{A_{E,t}}$, which is constant. The following condition ensures that energy efficiency can grow fast enough to keep up with changes in tax-inclusive energy prices:

$$\eta_E > (1-g_V)(1+g_\tau) - 1 > 0. \tag{A4}$$

In the limit, the model again reduces to that of the standard neoclassical growth model. As a result, the BGP condition for the output side of the economy is unchanged,

$$1 + g_M^* = \frac{(1+g_N^*)^{\frac{\alpha}{1-\alpha}}}{1+g_E^*}(1+n). \tag{BGP-QE$'$}$$

If $g_M^* = g_{\overline{M}}^* > 0$, then it is possible to characterize the BGP using the same steps as in Section 3.4. Noting the similarity between (BGP-RD$'$) and (BGP-QE$'$) on one hand and (BGP-RD) and (BGP-QE) on the other, it is immediate that the growth rate of technological progress is unaffected by the level of taxes or the research subsidy.

As demonstrated in equation (BGP-RD$'$), the existence of increasing energy taxes weakens the link between the cost of energy extraction, $p_{E,t}$, and energy efficient research. If energy taxes grow fast enough, energy use may not increase on the BGP. In this case, the research allocations can

[25] I restrict the formal analysis to the case of exponentially increasing taxes and a fixed research subsidy for analytic convenience. This restriction allows for the simple characterization of a balanced growth path, but does not drive any of the underlying intuition.

be found by setting $g_M^* = 0$ in (BGP-RD$'$). The following condition must hold for energy use to increase on the BGP:

$$(1 - g_V)(1 + g_\tau) \leq \left[1 + \eta_N \left(1 - \left[\frac{(1 - g_V)(1 + g_\tau) - 1}{\eta_E}\right]^{\frac{1}{1-\lambda}}\right)^{1-\lambda}\right]^{\frac{\alpha}{1-\alpha}} (1 + n). \qquad \text{(A5)}$$

Intuitively, this condition places restrictions on the exogenous growth in tax-inclusive energy prices. If energy prices grow quickly even when energy use does not, energy use need not grow on the BGP.

Remark 1. *In an* equilibrium with environmental policy*, $\frac{dg_M^*}{dg_\tau} < 0$. Moreover, $g_M^* > 0$ if and only if assumption (A5) holds.*

Even if assumption (A5) does not hold, the growth rate of technological progress is unaffected by the level of taxes or the research subsidy.

Remark 2. *In an* equilibrium with environmental policy*, changes in energy research subsidies and the level of energy taxes have no effect on the BGP growth rate of energy. Formally, $\frac{dg_M^*}{d\tau_0} = \frac{dg_M^*}{d\eta^S} = 0$.*

Proof. The intuition follows from the preceding discussion. Formally, the remark follows from Proposition 2. □

All of the results presented thus far are summarized and extended in Proposition 2. In particular, it uses the relationship between equations (BGP-RD$'$) and (BGP-QE$'$) to explicitly characterize the BGP in the presence of environmental policy.

Proposition 2. *Let assumptions (A1)–(A4) hold. In an* equilibrium with environmental policy*, there exists an* unique BGP *on which each of the following holds true:*

1. *If assumption (A5) holds, research allocations are implicitly given by*

$$R_E^* = \left\{\frac{\left[(1 + \eta_N(1 - R_E^*)^{1-\lambda})^{\frac{\alpha}{1-\alpha}}(1+n)[(1+g_\tau)(1-g_V)]^{1/\psi}\right]^{\frac{\psi}{1+\psi}} - 1}{\eta_E}\right\}^{\frac{1}{1-\lambda}} \quad \text{and } R_N^* = 1 - R_E^*.$$

   Otherwise, research allocations are given by

$$R_E^* = \left[\frac{(1 - g_V)(1 + g_\tau) - 1}{\eta_E}\right]^{\frac{1}{1-\lambda}} \quad \text{and } R_N^* = 1 - R_E^*.$$

2. *Technological growth rates are given by $g_E^* = \eta_E(R_E^*)^{1-\lambda}$ and $g_N^* = \eta_N(1 - R_E^*)^{1-\lambda}$. If assumption (A5) holds, the relationship between growth rates can be expressed as $(1 + g_E^*)^{\frac{\psi+1}{\psi}} = (1 + g_N^*)^{\frac{\alpha}{1-\alpha}}(1 + n)[(1 - g_V)(1 + g_\tau)]^{1/\psi}$.*

3. *Output per worker and consumption per worker grow at a constant rate, $g_R^* = (1+g_N^*)^{\frac{\alpha}{1-\alpha}} - 1$.*

4. *Total output and the capital stock grow at a constant rate, $g_Y^* = (1+g_R^*)(1+n)-1$, which implies that the capital-output ratio is fixed.*

5. *The real interest rate, r_t, is constant.*

6. *Per period energy use grows at rate $g_M^* = \frac{1+g_R^*}{1+g_E^*}(1+n)-1$.*

7. *The expenditure shares of energy, capital, labor, R&D inputs, and profits are all constant.*

Proof. The intuition is provided in the text, and a formal proof is provided in Appendix Section B.4. □

3.6 Comparison to Cobb-Douglas

As mentioned in the introduction, the standard approach in climate change economics is to treat energy as a CD component of the aggregate production function (Nordhaus and Boyer, 2000; Golosov et al., 2014). The standard CD production function is given by

$$Q_t^{CD} = A_t^{CD} K_t^{\gamma} E_t^{\nu} L_t^{1-\gamma-\nu},$$

where A_t^{CD} grows at an exogenous rate, g_{CD}. Since energy extraction costs $p_{E,t}$ units of the final good, final output is given by

$$Y_t^{CD} = (1-\frac{\nu}{\tau}) A_t^{CD} K_t^{\gamma} E_t^{\nu} L_t^{1-\gamma-\nu}.$$

As a result, the energy expenditure share under Cobb-Douglas is given by

$$\theta_{E,t}^{CD} = \frac{\nu}{1-\frac{\nu}{\tau_t}}.$$

In the absence of growing taxes, the energy expenditure share is constant, matching the long-run elasticity of substitution between energy and non-energy inputs, but not the near-zero short-run elasticity of substitution. This has important implications for climate policy. In the CD model, a tax on energy use – no matter how large – immediately generates declines in energy use that are sufficient to leave the expenditure share essentially unchanged.[26] This is at odds with data showing that the energy expenditure share increases when the price of energy increases.

Since addressing climate change inherently involves long-run outcomes, the existing literature argues that the CD approach may provide accurate predictions about the reaction of energy use to policy interventions over the relevant time frame, even though it cannot match short-run responses (e.g., Golosov et al., 2014; Hassler et al., 2016c, 2017; Barrage, forthcoming). The analytic results from Section 3.5, however, cast doubt on this assertion. The DTC model matches both the short- and long-run elasticities, suggesting that it will more accurately predict the effect of environmental

[26] In response to new energy taxes, there is actually a slight *decrease* in the energy expenditure share, which is due to the tax rebate. This effect is quantitatively unimportant.

taxes on energy use. This new model predicts that, in response to policy, the energy expenditure share will not be constant on the transition path and the balanced growth level of the energy expenditure share may even increase permanently in response to policy. Thus, there is good reason to expect that the CD approach overestimates the decline in energy use following an environmental policy intervention. Section 5.1 quantifies the difference in predictions between the models.[27]

4 Calibration

4.1 External Parameters

I solve the model in 10 year periods. As discussed above, the consumer and non-energy production portions of the model are standard. I follow Golosov et al. (2014) and set $\alpha = .35$, $\delta = 1$, $\sigma = 1$, and $\beta = .860$. I assume that the economy starts without environmental policy. As a result, all taxes and subsidies can be thought of as relative to 'business as usual' case, which serves as the baseline.

In addition to standard neoclassical elements, the DTC model includes R&D and energy extraction. Thus, the parameters from these segments of the model cannot be taken from the existing literature. Data sources and details can be found in Appendix A. Due to limitations on energy expenditure data, I restrict attention to the period 1971-2016.

Over this period, the average growth rate of output in the United States was $g_Y = 0.33$ (2.9%/year). Population growth was $n = 0.10$ (1.0%/year). On the BGP, the growth rate of income per capita is given by $g_R^* = (1 + g_N^*)^{\frac{\alpha}{1-\alpha}} - 1$. In the data, $g_R^* = 0.20$ (1.9%/year), which yields $g_N^* = 0.41$. Final-use energy consumption in the United States grew at rate $g_M^* = 0.06$ (0.6%/year). So, $g_E^* = 0.25$ (2.2%/year), which is also the growth rate of energy prices, g_P^*, on the BGP. The average energy expenditure share in the data is 8.4%, and the expenditure share of R&D is 2.6%.

4.2 R&D Calibration

The R&D production function has three unknown parameters, η_N, η_E, and λ. The η terms capture the inherent efficiency of R&D in improving the two types of technology, while λ governs the the degree of diminishing returns.

From equation (21), research arbitrage implies that

$$\frac{1 + g_{E,t}}{1 + g_{N,t}} = \frac{\theta_{E,t}}{\alpha} \frac{\eta_E}{\eta_N} \left(\frac{R_{E,t}}{1 - R_{E,t}} \right)^{-\lambda} \quad \forall t, \tag{28}$$

which takes advantage of the fact that all capital good producers make identical decisions.[28] Next,

[27] In Appendix Section B.5, I explain the calibration procedure for the CD model and describe the balanced growth path. I calibrate both models so that they have identical predictions for output and energy use in the absence of policy.

[28] Hassler et al. (2016b) identify a similar relationship between equilibrium growth rates and the expenditure share

consider the following two equations,

$$g_{E,t} = \eta_E (R_{E,t})^{1-\lambda}, \tag{29}$$

$$g_{N,t} = \eta_N (1 - R_{E,t})^{1-\lambda}, \tag{30}$$

which ensure that rates of technological progress match the data. Taking the ratio of (29) and (30) yields

$$\frac{g_{E,t}}{g_{N,t}} = \frac{\eta_E}{\eta_N} \left(\frac{R_{E,t}}{1 - R_{E,t}} \right)^{1-\lambda}. \tag{31}$$

Substituting $\frac{\eta_E}{\eta_N}$ from equation (31) into (28) and rearranging yields

$$\frac{R_{E,t}}{1 - R_{E,t}} = \frac{g_{E,t}}{1 + g_{E,t}} \frac{1 + g_{N,t}}{g_{N,t}} \frac{\theta_{E,t}}{\alpha}, \tag{32}$$

which captures the equilibrium relationship between research allocation and growth rates. Conditional on the equilibrium energy expenditure share, the equilibrium research allocations can be found without knowing any of the parameters of the R&D production function. As expected, there is a positive relationship between $R_{E,t}$ and both $g_{E,t}$ and $\theta_{E,t}$. In equilibrium, research expenditure on energy efficiency is higher when the growth rate of energy efficiency is high and when the energy expenditure share is large. Bringing this equation to data yields, $R_E^* = 0.14$ and $R_N^* = 0.86$, which implies that 14% of all R&D effort is devoted to improving energy efficiency.

As shown in the Appendix Section B.4.2, the equilibrium R&D share of GDP is equal to

$$\theta_R^* \equiv \frac{p_t^R}{Y_t} = (1 - \lambda)\alpha^2 \frac{\eta_N \left(R_{N,t}^* \right)^{-\lambda}}{1 + g_N^*}, \tag{33}$$

on a BGP without environmental policy. Combined with equation (30), this gives

$$\theta_R^* = (1 - \lambda) \cdot \alpha^2 \cdot \frac{g_N^* / R_N^*}{1 + g_N^*}, \tag{34}$$

which yields $\lambda = 0.38$. Again, this result is independent of the other parameters in the R&D production function. With an estimate of λ, I use equations (29) and (30) to solve for $\eta_E = 0.85$ and $\eta_N = 0.45$. The results suggest that improving energy efficiency technology is inherently easier than improving non-energy technologies. Intuitively, this results follows from two facts observed in the data. First, the energy expenditure share is small compared to the capital share of income,

of energy when considering a social planner solution with a general CES production function and a finite set of energy resources that can be extracted from the environmental without cost (see also, Hart, 2013). In their framework, the long-run equilibrium must also conform to the social planner's optimal depletion condition for the energy resource. This pins down the long-run expenditure share and technology growth rates. Since energy use is currently rising, the data suggests that the BGP conditions are not met in the Hassler et al. (2016b) world, leading to the prediction that the energy expenditure share will increase and consumption growth will decrease in the long run.

suggesting that there is little incentive for firms to invest heavily in energy efficiency. Second, relative to the expenditure shares, the growth rate of energy efficiency is high. If the relative benefit of investing in energy efficiency in low, but investment is high, it must be the case that the cost of improving energy efficiency is low. In other words, it is inherently easier to improve energy efficiency.

Fried (2018) examines directed technical change in clean versus dirty sources of energy, as well as non-energy technology, and finds diminishing returns of $\lambda = 0.21$. I use this value in robustness analyses. Existing empirical work in endogenous growth models finds cost functions that are approximately quadratic in research effort (Acemoglu et al., 2018; Akcigit and Kerr, 2018). This would give $\lambda = 0.5$. As shown in Section 5.1, this would increase the difference between the DTC and CD models.

4.3 Energy Sector Calibration

To calibrate the energy sector parameters, I start by noting that, on the BGP, both cumulative and per period energy use grow at the same constant rate (i.e., $g^*_M = g^*_{\bar{M}}$). The per period growth rate of energy, g^*_M, is observed in the data. Thus, I calculate the initial level of extracted energy as

$$\frac{E_0}{\bar{E}_{-1}} = g^*_M, \tag{35}$$

where E_0 is flow energy use in the first period and $\bar{E}_{-1}$ is the cumulative energy used prior to the first period. The calibration yields $\bar{E}_{-1}/E_0 = 15.7$. Conditional on ψ, the ratio between the initial stock and the per period flow of energy use determines the degree to which energy prices fluctuate in response to policy-induced changes in energy use. If the stock of consumed energy is large, then per period energy use fluctuations will only have a small effect on extraction costs.

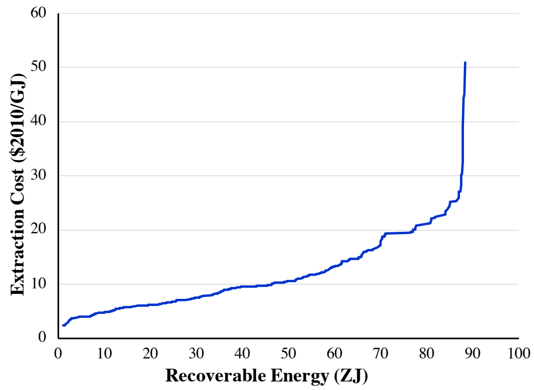
The parameter ψ captures the shape of the energy extraction cost curve. To calibrate this parameter, I use estimates of energy availability and extraction costs from McGlade and Ekins (2015). They estimate global extraction cost curves for coal, oil, and natural gas. I combine these curves to calculate total final-use energy availability at each extraction cost.

Figure 4 reproduces the resulting curve, which was also shown in figure 3. The outcome variable, $cost_i$, is the cumulative extraction cost and is measured in 2010 dollars. The explanatory variable, R_i, is the total amount of final-use energy available at $cost_i$ or less in 2010. It is measured in zettajoules (ZJ). I divide the range of R_i into 200 equally-spaced grid points and find the extraction cost at each point. These grid points are the unit of observation, i, in the regression described below.[29]

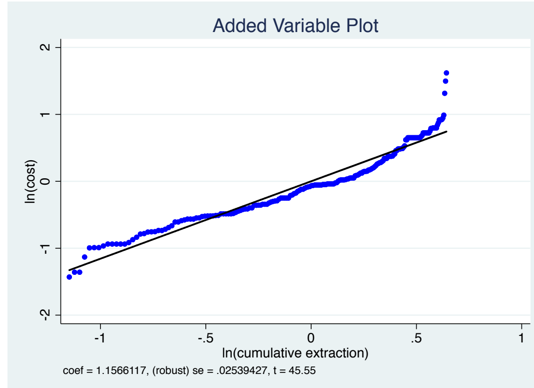
Taking logs, equation (5) is given by

$$\ln p_{E,t} = \ln A_{V,t} + \psi \ln \left(\bar{E}_{-1} + \sum_{s=0}^{t} E_s \right). \tag{36}$$

[29] See appendix section B.6 for further details about the data and construction of the figure.



(a) Extraction Cost Curve



(b) Estimating ψ

Figure 4: Extraction Costs. Panel (a) presents extraction cost curves in terms of final-use energy availability. Panel (b) shows the fit of equation (37) when estimated by OLS. Original cost and availability estimates are from McGlade and Ekins (2015), and adjustments for the average efficiency at which primary energy is converted to final-use energy are calculated using data from Rogner et al. (2012), the IEA and the EIA.

Here, $p_{E,t}$ is the extraction cost at time t, which corresponds to $cost_i$ in the data. Costs are measured at a single point in time. Also, $\sum_{s=0}^{t} E_s$ is the amount of energy used between periods 0 and t. This corresponds to R_i in the data, which is the amount of energy that could be extracted at a given cost. In the model, $\bar{E}_{-1}$ is cumulative energy used prior to start of the model. The estimates from McGlade and Ekins (2015) do not include past extraction. Rogner et al. (2012) estimate that cumulative extraction of coal, natural gas, oil prior to 2005 has been 16.5 ZJ,[30] which I take as given for the regression. Finally $A_{V,t}$ is the state of extraction technology at the time that costs are measured, which is held constant in the data. Thus, I estimate

$$\ln cost_i = b_0 + b_1 \ln(16.5 + R_i), \tag{37}$$

by ordinary least squares. The coefficient of interest is b_1, which provides the estimate of ψ. The results of the regression are show in panel (b) of figure 4. I find $\hat{\psi} = 1.16$.[31]

With an estimate of ψ, it is now possible to calibrate g_V, the growth rate of extraction technology relative to TFP. From equation (5), the growth rate of the real energy price is given by

$$(1 + g_P^*) = (1 - g_V)(1 + g_M^*)^{\psi}. \tag{38}$$

[30]See table 7.1 in their work. This estimate is in terms of primary energy. Converting primary to final-use energy using the conversion factors described in Appendix Section B.6 yields $\psi = 0.99$ instead of $\psi = 1.16$. Given that these estimates are relatively similar and it is not clear how accurate transformation data are for historical use, I use the more conservative estimate of ψ.

[31]The regression has heteroskedasticity-robust standard errors of 0.03. Given that the underlying data are themselves estimates and that they have been aggregated together to build a cumulative cost curve, I do not rely on these standard errors below. Instead, I show robustness with the extreme values of $\psi = 0$ (exogenous energy prices) and $\psi = 3.64$ ($g_V = 0$).

Table 1: Parameters

Parameter	Value	Description	Source
α	.35	Capital share of income	Golosov et al. (2014)
δ	1	Depreciation	Golosov et al. (2014)
β	.860	Discount factor	Golosov et al. (2014)
σ	1	Inter-temporal substitution	Golosov et al. (2014)
n	0.10	Population growth	EIA
λ	0.38	Research dim. returns	Calibrated
η_E	0.85	Research efficiency	Calibrated
η_N	0.45	Research efficiency	Calibrated
ψ	1.16	Extraction cost convexity	Calibrated
$\bar{E}_{-1}/E_0$	15.7	Energy stock/flow	Calibrated
g_V	-0.17	Extraction technology growth	Calibrated

Taking logs and rearranging, the BGP relationship becomes

$$\ln(1-g_V) = \ln(1+g_P^*) - \psi\ln(1+g_M^*). \tag{39}$$

Differential technological progress is capturing the growth in energy prices that cannot be explained by the shape of the extraction cost curve. This yields $g_V = -0.17$ over ten years (-1.8%/year). Technological progress in the energy extraction sector is significantly slower than technological progress in final good production, though extraction technology is still improving over time. Finally, the starting value $A_{V,0}$ is a scale parameter calibrated to the starting price,

$$A_{V,0} \quad = \quad \frac{p_{E,0}}{\bar{E}_{-1}^{\psi}}. \tag{40}$$

Conditional on the other parameters, $A_{V,0}$ simply reflects the choice of units.

For robustness, I also re-calibrate the energy extraction parameters under the assumption that $g_V = 0$. Then, ψ is directly identified from an updated version of equation (39): $\psi = \ln(1+g_P^*)/\ln(1+g_M^*)$. In this case, the increase in energy prices must be driven entirely by the convexity of the extraction cost curve, which gives $\psi = 3.64$. As shown in section 5.1.4, this is a conservative assumption in that it decreases the difference between the DTC and CD models. I also perform robustness analyses assuming that energy prices grow at a constant exogenous rate. The results are qualitatively and quantitatively similar to the baseline case, implying that none of the core results in this paper rely on the functional form for extraction costs or the energy sector calibration details.

4.3.1 Calibration Results

Table 1 presents the results of the baseline calibration.

4.4 Solving the Model

Conditional on the price of energy, the model can separated into three pieces: the R&D allocations, the standard neoclassical growth model, and the energy extraction sector. The fact that innovation occurs in different characteristics of capital goods, rather than in different sectors, facilitates the solution of the model. In particular, equations (22) and (23) demonstrate that, conditional on the price of energy, the R&D allocations and technology growth rates can be found independently of the consumer problem. To find the competitive equilibrium, I employ the following steps:[32]

1. Guess a vector of energy prices.

2. Solve for productivity paths and R&D allocations using equations (10), (22), and (23), noting that all monopolists make identical research decisions.

3. Solve the neoclassical growth model conditional on the path of productivities using equations (B.31) – (B.37) in Appendix Section B.4.1.

4. Back out implied energy use and energy prices using equations (3), (5), (6), and (7). This takes advantage of the fact that (4) holds with equality in all periods.

5. Check if the initial guess and resulting prices are the same. If they are, then consumers have made optimal decisions taking all future prices as given and the economy is in equilibrium.

6. If the economy is not in equilibrium, start from step 1 with a convex combination of initial guess and resulting prices.

5 Results

5.1 Energy Taxes

In this section, I examine the effect of energy taxes in the new directed technical change (DTC) model and compare the results to those in the standard Cobb-Douglas (CD) model. The period length in the model is ten years. Policies are announced in the initial period, which I take as 2005 to match the stated objectives of international climate agreements. Policies take effect in 2015. The gap between the announcement and implementation of the policy allows one round of endogenous and directed technical change to occur before comparing the outcomes across the two models. If the policy was not anticipated, the final good producer in the CD model could react, whereas there would be no adjustment in the DTC model due to the Leontief structure.

[32] In all quantitative applications, this procedure is sufficient to find a competitive equilibrium. I have not shown that such a procedure must converge to an equilibrium. In all cases, I use the BGP in the absence of energy taxes to generate the initial guess of energy prices.

5.1.1 Set-up

To understand the quantitative importace of the new model, it is helpful to consider a realistic path of future energy taxes. Under the Paris Agreement, the United States aims to adopt policies consistent with an 80% reduction in carbon emissions by the year 2050, when compared to 2005 levels (Heal, 2017). In a report for the United Nations, Williams et al. (2014) examine the technical feasibility of reducing carbon emissions in 2050 by 80% compared to 1990 levels. Across a wide range of scenarios, they find that total energy use needs to decrease by approximately 15% compared to 2005 levels.[33] This is true even though their analyses suggest that almost all electricity is generated from renewable sources by 2050 and that the share of final-use energy coming from electricity more than doubles. These results imply that a 15% reduction in energy use between 2005 and 2055 is necessary to meet climate policy goals, even in the face of large-scale substitution toward clean sources of primary energy. Based on this evidence, I adopt this target of a 15% reduction in energy use.[34] The remaining reduction in emissions necessary to meet the Paris Agreement goals is assumed to come from substitution between clean and dirty sources of energy, which is outside of the model.[35,36] As in Section 3.4, I consider energy taxes that grow at a constant rate,

$$\tau_t = 1 \cdot (1 + g_\tau)^{\frac{t-2005}{10}}. \tag{41}$$

Then, I search for the growth rate of energy taxes, g_τ, that is necessary to achieve the policy goal.[37]

5.1.2 Policy Designed with Cobb-Douglas Model

In this section, I find the growth rate of value-added taxes, g_τ, that achieves the policy target with the CD model and then examine the impacts of these same taxes in the DTC model. The thought experiment is straightforward. Suppose policy is designed with the models that employ the usual

[33] See figure 8 in Williams et al. (2014). According to the IEA, final energy use in 2005 was 65.4 EJ. See appendix section A for links to IEA data.

[34] Since the model is solved in ten year periods, I choose taxes such that the reduction occurs by 2055.

[35] To endogenously determine the portion of emission reductions due to lower energy use, it would be necessary to explicitly model substitution between energy and non-energy inputs and substitution between clean and dirty sources of energy. There is no consensus in the literature about the elasticity of substitution between different sources of primary energy. Adding this margin, therefore, would complicate the model without providing new theoretical or quantitative insights. Instead, I focus on the 15% reduction in energy use that is necessary to meet environmental policy goals even in the face of large-scale substitution towards clean energy sources.

[36] The existing literature uses a wide varying of estimates for the elasticity of substitution between clean and dirty sources of primary energy. Golosov et al. (2014) take the average of oil-coal, oil-electricity, and coal-electricity elasticities from a meta-study by Stern (2012), finding an average elasticity that is less than one. Given that electricity is produced with both clean and dirty inputs, however, it is not clear that these estimates measure elasticities between different primary sources of energy. They also impose the assumption that the relative price between oil and renewables is equal to one at all times. Stern (2012) stresses that the underlying studies do not necessarily capture directed technical change, implicitly attributing changes in technology to *ex post* substitution. In an econometric study, Papageorgiou et al. (2017) find elasticities greater than one in the energy sector and overall economy, but assume that all technical change is factor-neutral. In their economy-wide estimates, they also assume that the aggregate production function is CD. Acemoglu et al. (2016) assume that clean and dirty production technologies are perfect substitutes at the firm level and do not define an aggregate elasticity.

[37] To find the minimum tax necessary to meet the policy goal, I use a step size of 0.001.

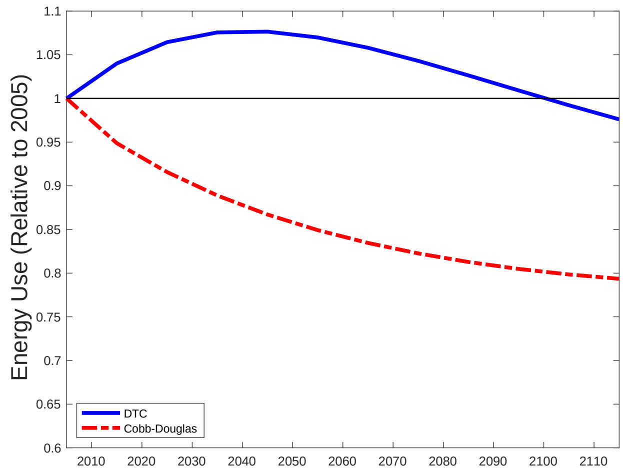


Figure 5: Energy Use. This figure presents energy use in the DTC and CD models, relative to 2005 levels. The policy goal is a 15% reduction in energy use between 2005 and 2055, which is achieved in the CD model. Energy taxes grow at a constant rate of $g_\tau = 0.100$ (1.0%/year). The policy is announced in 2005 and takes effect in 2015.

CD assumption about energy use, but reality actually follows the DTC model. How close will the economy come to meeting the policy goals?

To meet the policy goal in the CD model, $g_\tau = 0.100$ (1.0%/year). Taking into account the endogenous reduction in extraction costs, this implies that tax-inclusive prices are 52% higher than baseline values in 2055. Figure 5 presents energy use in both model, relative to 2005 levels. The DTC model misses the 2055 energy use target by 22 percentage points, implying that the path of energy taxes is not sufficient to be bring energy use in 2055 below its 2005 level.

Figure 6 presents more detailed results from this analysis. All outcomes are shown relative to a 'business as usual' case where the economy remains on the original BGP. Panel (a) presents the results for energy use. While policy targets are often expressed in terms of medium-run energy use, long-run cumulative energy use is most relevant for climate outcomes. In other words, climate outcomes depend on the total area between the two curves. With this path of energy taxes, the DTC model predicts 22% greater energy use over the next century.

Panel (b) demonstrates the mechanism driving these results. In the CD model, energy taxes cause immediate reductions in the energy intensity of output (E/Y) such that the expenditure share of energy is essentially unchanged.[38] In the DTC model, however, reductions in E/Y are not large enough to prevent the energy expenditure share from rising. As demonstrated in figure 2, this is consistent with data from the United States. As a result, the DTC model predicts higher medium-term and cumulative energy use, when compared to the CD model.

Panel (c) shows that energy extraction costs fall as the result of the policy intervention. Panel (d) examines consumption. In 2115, consumption in roughly 2% below baseline in both models, even though the DTC model does not hit the policy target. The differences in consumption will be

[38]The small reduction in the energy share is due to the tax rebate. The energy expenditure share is a constant fraction of gross, as opposed to final, output.

larger when considering a path a taxes that meets the policy goal in both models.

Panels (e) and (f) look at reductions in GDP and TFP. To make a relevant comparison, TFP is defined as $Y_t / \left(K_t^{\alpha} L_t^{1-\alpha}\right)$ in both models. In the DTC case, the loss of TFP comes from the reallocation of R&D inputs away from $A_{N,t}$ and towards $A_{E,t}$. In the CD model, the loss to productivity comes from direct substitution between energy, capital, and labor. In both cases, productivity is increased by the fall in energy extraction costs, holding all else constant. In 2115, TFP and GDP reductions are similar in the two models, but the transition is slower in the DTC model.

Overall, this section presents a set of pessimistic conclusions with regards to environmental policy. Suppose that the real world is captured by the DTC model, which is consistent with aggregate data from the United States. If policy is designed with the CD model, the economy will miss the medium-run energy use targets by a significant amount and have higher than expected cumulative energy use in the long-run.

5.1.3 Policy Designed with Directed Technical Change Model

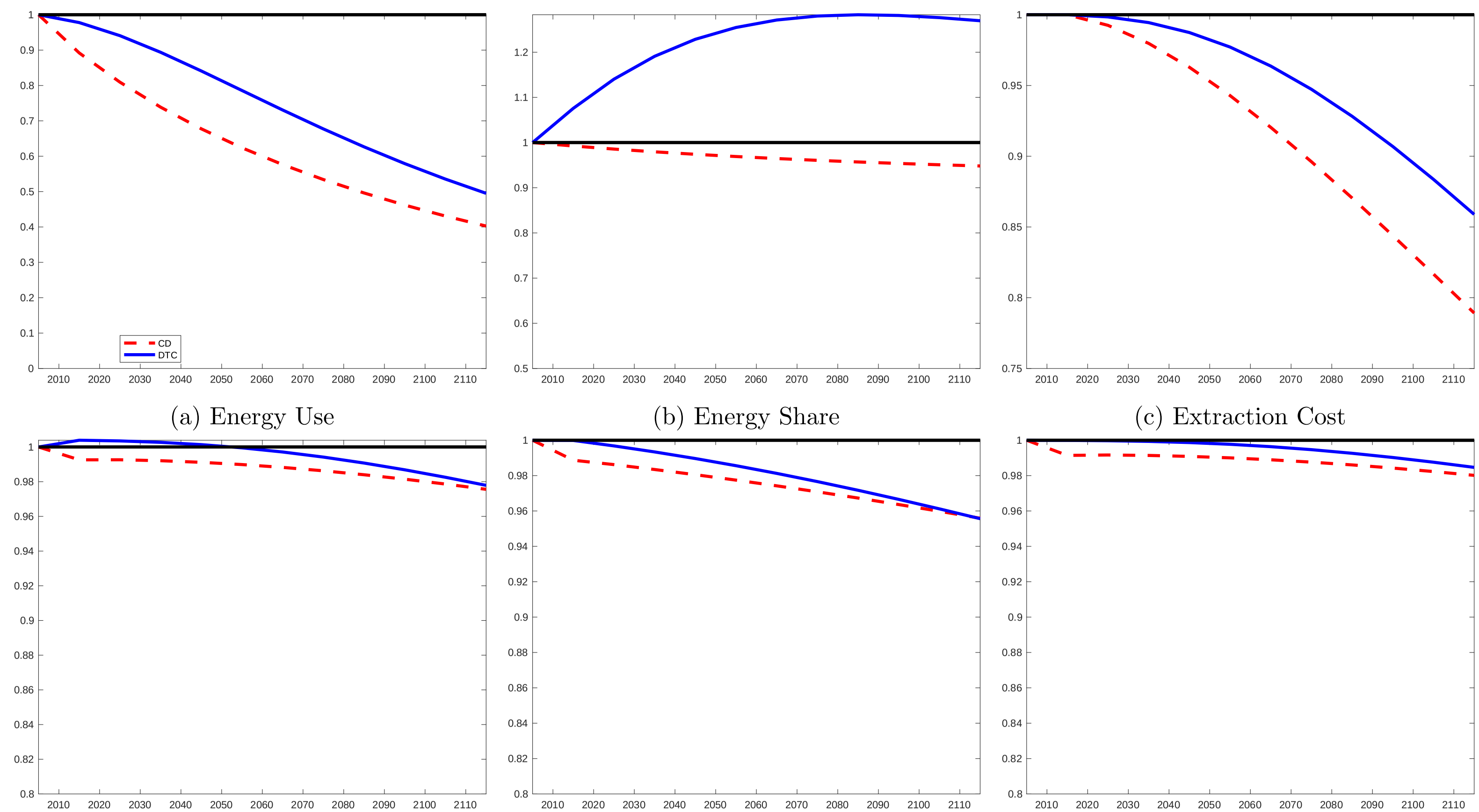
In this section, I find the path of taxes that are needed to achieve the policy goal in the DTC model. I compare the results for the DTC with the new path of taxes to those found for the CD model in the previous section. To achieve the environmental goals given above, the DTC model requires $g_{\tau} = 0.199$ (1.8% year), compared to 0.100 (1.0%/year) in the CD model. When taking into account the general equilibrium effect of energy use on extraction costs, this yields a tax-inclusive energy price that is 137% higher than the baseline level in 2055, compared to 52% in the CD model.

Figure 7 presents the outcomes when both models meet the policy target. Panel (a) shows that the two models now have identical energy use in 2055, which is the target year for environmental policy. In order to reach this reduction in energy use, the DTC model must have a large increase in the energy expenditure share, as shown in panel (b). The two models have nearly identical paths of extraction costs, as shown in panel (c), implying that cumulative energy extraction is also similar. Panels (d)–(f) present the results for consumption, output, and TFP, which are all lower in the DTC model. Consumption in the DTC model is 10% lower than baseline in 2115, compared to 2% in the CD model.[39] So, the DTC model requires more aggressive policy and more forgone consumption to meet environmental policy goals.

5.1.4 Robustness

In this section, I discuss the results of the robustness analyses. All analyses focus on the path of taxes that meet the policy goal in the CD model. Figure 8 provides a summary of the results that

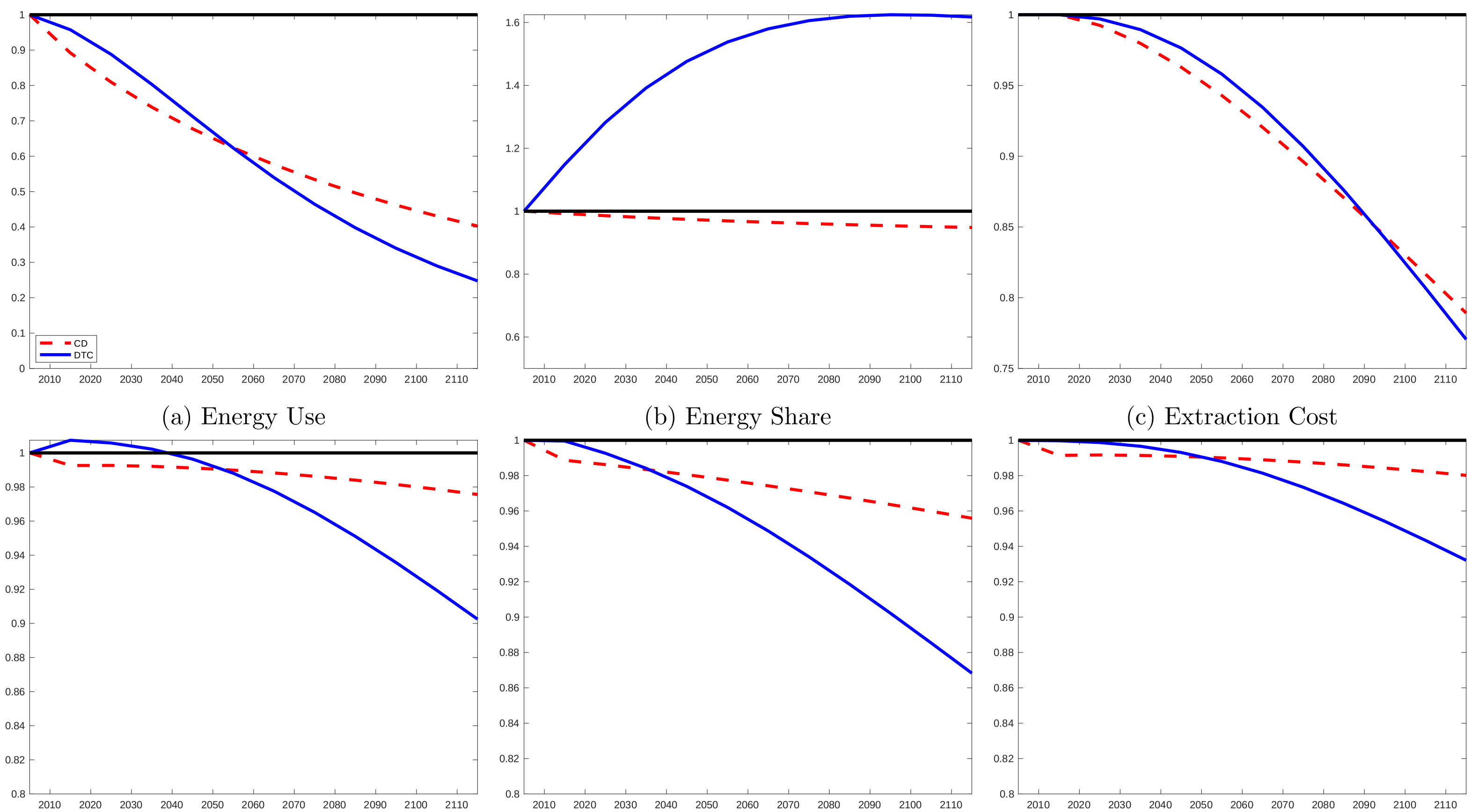
[39] Given that there are many unknowns about the impact of climate change on economic outcomes (e.g., the existence and consequences of non-linearities in natural systems), this paper does not try to calculate optimal carbon or energy taxes. Using a standard marginal abatement cost framework, however, it is clear that the DTC model will yield lower optimal taxes when compared to the CD model, as long as the damage function between the two models is held constant. This is true because the DTC model suggests that greater reductions in consumption are necessary to achieve a given reduction in energy use.



(a) Energy Use (b) Energy Share (c) Extraction Cost

(d) Consumption (e) GDP (f) TFP

Figure 6: Comparison of the DTC and CD models when using an identical path of taxes. In particular, $g_\tau = 0.100$, which meets the policy goal in the CD model. The policy goal is that energy use in 2055 is 15% lower than energy use in 2005. The policy is announced in 2005 and takes effect in 2015. All results are shown relative to a 'business as usual' case where the economy remains on the original BGP. Panel (a) compares flow energy use. Panel (b) gives the energy expenditure share of final output. Panel (c) shows the pre-tax price of energy, which is equal to the energy extraction cost. Panel (d) gives consumption by the representative household. Panel (e) gives final output. Panel (f) gives TFP. For ease of comparison between the models, TFP is calculated as $Y_t/(K_t^\alpha L_t^{1-\alpha})$ in both cases.



(a) Energy Use (b) Energy Share (c) Extraction Cost

(d) Consumption (e) GDP (f) TFP

Figure 7: Comparison of the DTC and CD models when both models meet the policy goal. In particular, $g_\tau = .199$ (1.9%/year) in the DTC model and $g_\tau = .100$ (1.0%) in the CD model. The policy goal is that energy use in 2055 is 15% lower than energy use in 2005. The policy is announced in 2005 and takes effect in 2015. All results are shown relative to a 'business as usual' case where the economy remains on the original BGP. Panel (a) compares flow energy use. Panel (b) gives the energy expenditure share of final output. Panel (c) shows the pre-tax price of energy, which is equal to the energy extraction cost. Panel (d) gives consumption by the representative household. Panel (e) gives final output. Panel (f) gives TFP. For ease of comparison between the models, TFP is calculated as $Y_t/(K_t^\alpha L_t^{1-\alpha})$ in both cases.

is directly comparable to figure 5. Panel (a) shows the baseline results for comparison. Appendix Section B.7 presents more detailed results that are directly comparable to figure 6 in the baseline case.

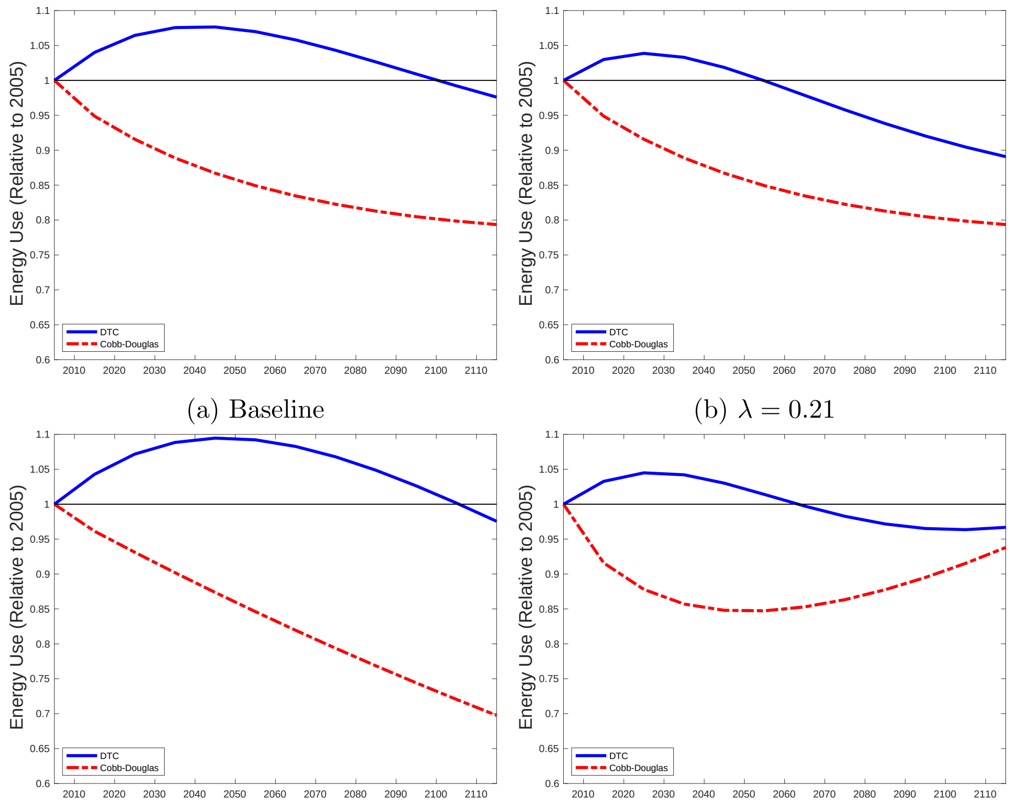
Alternate λ. — Panel (b) of figure 8 shows results with $\lambda = 0.21$, the degree of diminishing returns estimated by Fried (2018) in the context of substitution between clean and dirty sources of primary energy. The detailed results are presented in B.2. The smaller degree of diminishing returns makes it easier to reallocate R&D inputs in the DTC case. Still, the DTC misses the policy target by 15 percentage points, and cumulative energy use between 2015 and 2115 is 15% higher in the DTC model. Thus, the results are qualitatively unchanged by assuming this smaller degree of diminishing returns. As discussed above, existing empirical work in endogenous growth finds cost functions that are quadratic in research effort. Setting $\lambda = 0.5$ would increase the difference between the DTC and CD models when compared to the baseline analysis.

Exogenous energy prices. — As discussed above, the baseline model assumes that extraction costs depend only on cumulative extraction in the economy under study, capturing the case of a closed economy or the case where the all countries undertake identical actions and the U.S. is responsible for a constant fraction of total energy use. Panel (c) presents results with exogenous energy prices, capturing the case of a small open economy that takes unilateral policy action (see figure B.3 for complete results). In this case, energy prices are assumed to increase exogenously at the BGP rate in the baseline model. Importantly, this analysis still captures the main energy supply facts discussed in Section 2, rising energy prices and flow energy use on the BGP. The exogenous price scenario is equivalent to setting $\psi = 0$ and attributing all changes in prices to g_V.

With exogenous energy prices, the CD model requires $g_\tau = 0.088$ (0.9%/year) to hit the policy target. The DTC model model misses the target by 24 percentage points. From 2015 to 2115, cumulative energy use is 28% higher in the DTC model. Eliminating the general equilibrium feedback of endogenous energy prices increases the difference in predictions between the two models. Intuitively, this occurs because reductions in energy extraction costs partially offset initial reductions in energy use caused by environmental policy. The difference in outcomes, however, is qualitatively similar, implying that none of the baseline results are driven by the functional form assumptions for extraction costs.

Higher ψ. — In the baseline analysis, I estimated the convexity of the extraction costs curve using information from McGlade and Ekins (2015) and attributed the remaining increase in real energy prices to slower technological progress in the extraction sector ($g_V > 0$). Panel (c) shows that the results are robust to attributing all changes in prices to g_V. Panel (d) investigates the opposite case where $g_V = 0$ and the increase in real energy prices is entirely due to the convexity of the extraction cost curve.

To explain the data, the cost curve must be highly convex, $\psi = 3.64$. In this case, environmental policy interventions significantly reduce extraction costs. As a result, higher taxes are needed to meet the policy goals. Specifically, $g_\tau = 0.133$ (1.3%/year) in the CD model. In this case, the DTC model misses the 2055 policy target by 16 percentage points. Cumulative energy use is 14% higher in the DTC model. Figure B.4 in the appendix presents more detailed results. In the CD case, the policy actually increases consumption in the long run. This occurs because environmental policy lowers energy prices so dramatically Policy does not increase long-run consumption in the DTC case.



(a) Baseline

(b) $\lambda = 0.21$

(c) Exogenous p_E

(d) $\psi = 3.64$

Figure 8: Robustness Results. Comparison of energy use in the DTC and CD models when using an identical path of taxes. The policy goal is a 15% reduction in energy use between 2005 and 2055, which is achieved in the CD model. In baseline and $\lambda = 0.21$ cases, $g_\tau = 0.100$ (1.0%/year). In the exogenous energy price scenario, $g_\tau = 0.088$ (0.9%/year). When $\psi = 3.64$, $g_\tau = 0.133$ (1.3%/year). The policy is announced in 2005 and takes effect in 2015.

5.2 Research Subsidies

Many policymakers favor approaches, such as research subsidies or energy efficiency mandates, that try to reduce energy use without raising prices (Gillingham et al., 2009; Allcott and Greenstone, 2012). A large academic literature, however, suggests that rebound effects could undermine the effectiveness of these approaches (Gillingham et al., 2016). Rebound occurs when economic behavior following improvements in energy efficiency leads to increases in energy use, at least partially undoing the initial reduction. For example, people might drive more when cars get better gas mileage. I use the DTC model to examine the effectiveness of these policies in reducing cumulative energy use in the long-run, an important goal for climate change mitigation. Policies that increase energy efficiency could be desirable for other reasons if they are correcting a different externality (Chan and Gillingham, 2015).

5.2.1 Existing Literature

The study of rebound has a long history in economics, dating back at least to Jevons (1865). A large microeconomic literature examines the degree of rebound, usually in static, partial equilibrium settings (Gillingham, 2014; Gillingham et al., 2016).[40] If energy mandates or subsidies are to have a meaningful impact on climate change mitigation, however, they will necessarily have dynamic, general equilibrium consequences. As stressed by Gillingham et al. (2016), the exclusion of endogenous and directed technical change is a particularly important shortcoming in the existing literature on rebound. Indeed, the existing literature abstracts from dynamic considerations almost entirely.[41] Thus, I contribute to the existing literature by examining the long-run consequences of such policies, while paying special attention to the role of endogenous and directed technical change. The existing literature stresses the importance of the elasticity of substitution between energy and other inputs (e.g., Saunders, 1992; Wei, 2010). The DTC model is explicitly designed to capture this aggregate elasticity, which varies over time.

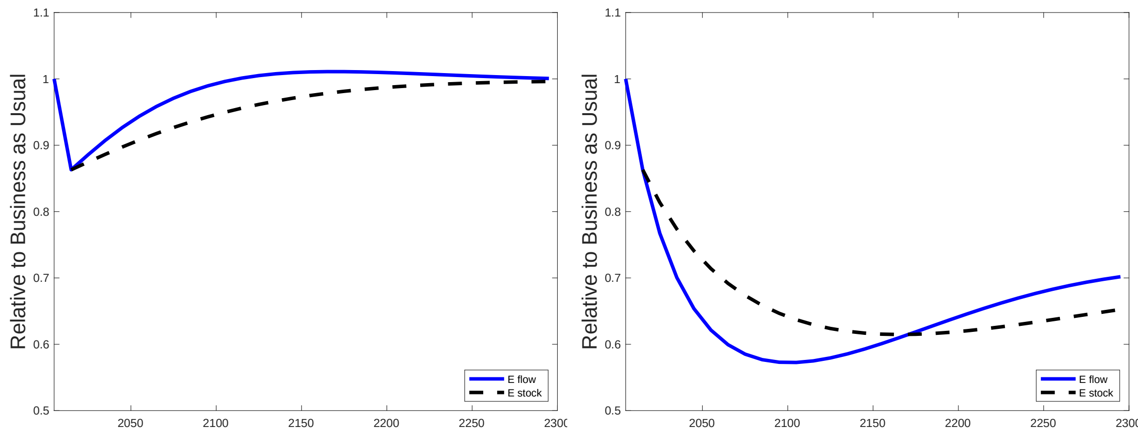
Before continuing, it is worth briefly reviewing existing evidence on macroeconomic rebound, which is closely related to the data on energy use and efficiency presented in Section 2. Descriptive evidence from Davis (2017) suggests that new energy efficient technologies in lighting have recently reduced household energy use in the United States. As stressed in that paper, however, the results only demonstrate a short-run impact, and it is not yet clear whether long-run rebound will undo the short-run decline. Longer-term evidence on lighting suggests that the introduction of new technologies has led to enormous improvements in efficiency, but these improvements have coincided with increased energy usage from lighting (e.g., Fouquet, 2008). Section 2 also demonstrates that rapid increases in energy efficiency do not prevent aggregate energy usage from rising. Thus, the aggregate evidence suggests that long-run rebound is likely to undo short-run reductions in energy use, which is consistent with the theoretical results presented in Section 3.5. As shown below, the quantitative model is consistent with all of these facts.

5.2.2 Results

Figure 9 presents the results. Panel (a) considers a single period research subsidy of 52% in 2015. This is a useful exercise for two reasons. First, this is analogous to the setting in most of the existing literature, which examines one-off efficiency improvements. Second, it highlights the mechanisms of the model in a simple and transparent manner. In the short-run, energy use decreases considerably, due to the low short-run elasticity of substitution between energy and non-energy inputs. After energy efficiency increases in the short-run, the incentive for further investment

[40] A recent exception is work by Lemoine (2016) who provides an analytic framework for examining rebound in a static, general equilibrium setting, but does provide any quantitative analysis.

[41] Rausch and Schwerin (forthcoming) perform a growth accounting exercise assuming the usual CD production function. They find that increases in energy efficiency have led to higher energy use.



(a) Single Period Subsidy

(b) Permanent Subsidy

Figure 9: Research Subsidies. All results are shown relative to a 'business as usual' case with no policy intervention. Panel (a) demonstrates the effects of a single period research subsidy of 52%. Panel (b) demonstrates the effects of a permanent subsidy of 52%. This policy achieves a 15% reduction in energy use by 2055, compared to 2005 levels. $E\ flow$ refers to per period energy use. $E\ stock$ refers to cumulative energy use since 2015, the first year the policy takes effect.

in energy-saving technology decreases, and the economy converges back to the original BGP. By the end of the century, energy use is actually higher than in the business as usual case. This is known as 'backfire' in the literature. Long-run per period and cumulative energy use are identical in the policy and baseline cases. In the literature, this is known as 'full rebound' (Wei, 2010). Full rebound occurs because one-off policy interventions do not change the long-run incentives of capital good producers. Thus, the DTC model is consistent with the evidence presented in Davis (2017), but suggests that endogenous R&D allocations will undo the short-run reductions in energy use, limiting the effectiveness of these policies as tools for climate change mitigation.

While the existing literature often focuses on one-off shocks to estimate the degree of rebound, attempts to reduce long-run energy use need not be constrained to temporary interventions. In panel (b), I consider a permanent subsidy of 52% to energy efficiency research. This subsidy is sufficient to achieve the 15% reduction in energy use discussed in the previous section. Permanent interventions reduce long-run energy use relative to a business as usual scenario. As demonstrated theoretically in Section 3.5, however, R&D subsidies are not sufficient to generate absolute long-run declines in energy use. On the BGP, the economy will still have investment in both types of technology. Given that energy prices are driven by increasing extraction costs, the incentives for investment in energy efficient technologies come from increasing energy use.

Overall, the model suggests that policy interventions cannot achieve long-run reductions in energy use without increasing prices. As a result, research subsidies are unlikely to be sufficient tools for achieving climate change mitigation goals. Given the long transition paths, however, they may be helpful as part of a larger set of interventions.

5.3 Long-run Growth in the Absence of Policy Intervention

The main purpose of this paper is to develop a decentralized model of final-use energy consumption and examine its implications for climate change mitigation policy. The model also delivers novel predictions about long-run growth in the absence of policy intervention. Hassler et al. (2012, 2016b) use a DTC model to examine how a social planner would manage a finite energy resource and study the implications of the model for future consumption growth. They use the observation of a trendless energy expenditure share to motivate their model, but find that the energy expenditure share cannot be constant at its current level due to Hotelling (1931) forces (see also, André and Smulders, 2014). To generate a long-run decline in energy use, their model requires greater investment in energy efficiency and less investment in non-energy technology, implying that output and consumption growth will not continue at current levels.

As discussed in Section 2.3, however, the U.S. data are inconsistent with the Hotelling (1931) approach to energy prices, because energy use has been increasing while the expenditure share has been constant. The data, therefore, are more consistent with a model where prices are driven by increasing extraction costs. In a model with increasing extraction costs, current energy use and expenditure patterns will continue in the absence of a shock that pushes the economy away from the BGP. Thus, in the absence of policy or negative consequences energy use, the new DTC model predicts a higher long-run growth rate of consumption.

Given that it predicts increasing energy use on the BGP, the new DTC is also more likely to lead to an 'environmental disaster'[42] when compared to the existing literature. Such a disaster could occur when energy inputs are exhausted, or when the climate consequences of fossil energy use hit a 'tipping point' (Stern, 2008; Lemoine and Traeger, 2014). Existing evidence suggests that the latter is a greater concern and that it is virtually impossible that all available sources of fossil fuels will eventually be used (Rogner, 1997; Rogner et al., 2012; Covert et al., 2016).

6 Conclusion

Economic analysis of climate change has benefited substantially from the study of growth models (e.g., Nordhaus, 1993, 2014; Golosov et al., 2014). This paper contributes to this ongoing effort by focusing on the consumption of final-use energy, a crucial margin for climate change mitigation. In particular, I develop a new directed technical change (DTC) model that can explain both short- and long-run patterns of energy use and energy prices in the United States. The existing climate change literature either abstracts from energy use (e.g., Nordhaus, 1993, 2014) or uses a Cobb-Douglas (CD) approach that cannot replicate the same facts (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014). The existing literature on directed technical change and the environmental focuses on substitution between energy sources (e.g., Acemoglu et al., 2012, 2016; Fried, 2018) or on the efficiency of the energy sector (e.g., André and Smulders, 2014), rather than final-use energy

[42]See Acemoglu et al. (2012) and Lemoine (2017) for recent work focusing on environmental disasters in the absence of policy.

efficiency.

The model developed in this paper provides a general framework for analyzing energy consumption. I use the model to analyze two climate change mitigation policies. First, I examine the impact of energy taxes. I find that policy conclusions based on the standard CD model overestimate policy-induced reductions in energy use and underestimate reductions in consumption. Second, I find that innovation-driven rebound effects prevent R&D subsidies from generating long-run declines in energy use, highlighting the need for policies that increase effective prices.

This paper focuses on the importance of final-use energy and abstracts from other important elements of climate change economics. One interesting extension would be to include the third margin of technological investment in clean versus dirty primary energy sources. Combined with a model of the carbon cycle, such an analysis would yield updated estimates of optimal carbon taxes and the social cost of carbon. It would also allow for the comparison of second-best policies. For example, it would be interesting to compare subsidies for renewable energy, which would limit the incentive to improve energy efficiency, and energy taxes, which provide no incentive to invest in clean energy sources.

It would also be interesting to examine the model presented here in a broader geographic scope. Existing work with exogenous technological progress suggest that unilateral policy actions among rich countries will have small impacts on overall carbon emissions (Nordhaus, 2010). With endogenous technological progress and diffusion or trade, however, unilateral policies would improve worldwide energy efficiency, leading to greater environmental benefit (Di Maria and Van der Werf, 2008; Hémous, 2016). This magnifies the difference with the standard CD approach, where substitution of capital and labor for energy in one country would have no direct impact on other countries. The positive implications of these international spillovers could potentially outweigh the more pessimistic conclusions that result from studying the DTC model in a closed economy.

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A Data Appendix

A.1 Figure 1

Primary Energy (E_p). Total energy extracted from the environment (i.e., production) plus net imports. For renewables used in electricity generation, production is equal to electricity generated. Measured in kilotonnes of oil equivalent (ktoe). Data available from 1971-2016. Source: 'IEA Headline Energy Data' at http://www.iea.org/statistics/topics/energybalances/.

Final-Use Energy (E_f). Total energy consumption: total primary energy minus losses occurring during transformation and energy industry own use. Measured in ktoe. Data available from 1971-2016. Source: 'IEA Headline Energy Data' at http://www.iea.org/statistics/topics/energybalances/.

Carbon Dioxide Emissions (CO_2). Carbon dioxide emissions from fuel combustion. Measured in megatonnes (Mt). Data available from 1971-2016. Source: 'IEA Headline Energy Data' at http://www.iea.org/statistics/topics/energybalances/.

Real GDP (Y). Real gross domestic product in 2012 chained dollars. Data available from 1949-2018. Source: NIPA Table Section 1. Accessed via 'Table C1: Population, U.S. gross domestic product, and U.S. Gross Output' at https://www.eia.gov/totalenergy/data/annual/.

Price of Solar Energy. Real levelized cost of electricity ($2005/kWh) produced from photovoltaic (PV) modules in the United States. Data available from 1977-2009. Original source: Nemet (2006). Accessed accessed via the Performance Curve Database from the Sante Fe Institute (Nagy et al., 2013), which includes updated data through 2009. See 'Photovoltaics 2' at http://pcdb.santafe.edu/index.php.

Nominal Energy Price. Nominal average price of energy paid by end users in the United States. Due to data limitations, prices for energy derived from renewable sources are not included.[43] Source: 'Total energy prices and expenditures' at https://www.eia.gov/state/seds/seds-data-complete.php.

GDP Deflator. GDP implicit price deflator with base year 2012. Data available from 1949-2018. Source: NIPA Table Section 1. Accessed via 'Table C1: Population, U.S. gross domestic product, and U.S. Gross Output' at https://www.eia.gov/totalenergy/data/annual/.

Real Energy Price. Average real price of primary energy in 2012 chained dollars. Author's

[43] Documentation is available at https://www.eia.gov/state/seds/seds-technical-notes-complete.php. Section 7 of 'Prices and expenditures' covers consumption adjustments.

calculation: *Nominal Energy Price* divided by *GDP Deflator*.

A.2 Figure 2

See **Real GDP**, **Nominal Energy Price**, and **Real Energy Price** from figure 1. Variables are detrended with OLS, assuming a constant growth rate.

Nominal Energy Expenditure. Nominal energy expenditure in the United States. Due to price data limitations, spending on final-use energy derived from renewable sources is not included. Source: 'Total energy prices and expenditures' at https://www.eia.gov/state/seds/seds-data-complete.php.

Nominal GDP (Y). Nominal gross domestic product. Data available from 1929-2018. Source: NIPA Table Section 1. Accessed via 'Table C1: Population, U.S. gross domestic product, and U.S. Gross Output' at https://www.eia.gov/totalenergy/data/annual/.

Energy Expenditure Share (E_{share}). Author's calculation: *Nominal Energy Expenditure* divided by *Nominal GDP*.

Energy Use. Final-use energy consumption. Author's calculation: *Nominal Energy Expenditure* divided by *Nominal Energy Price*. Given the limitations on price data, this is a measure of final-use energy derived from non-renewable energy sources.[44]

Energy Intensity of Output (E/Y). Total final-use energy consumption per real dollar of GDP. Author's calculation: *Energy Use* divided by *Real GDP*. By construction, this is the energy intensity of output that matches the expenditure data.

A.3 Figure 3

See **Primary Energy** and **Final-Use Energy** from figure 1.

Oil. Total primary energy from oil, natural gas liquids, and feedstocks. Measured in kilotonnes of oil equivalent (ktoe). Data available from 1971-2016. Source: 'IEA Headline Energy Data' at http://www.iea.org/statistics/topics/energybalances/.

Renewable Energy. Total primary energy from renewable sources and waste. Data available from 1971-2016. Source: 'IEA Headline Energy Data' at http://www.iea.org/statistics/topics/

[44] As of this writing, the adjusted energy consumption measures are not directly available from the EIA. See at 'Adjusted consumption for expenditure calculations' CSV file for 1960-2017 at https://www.eia.gov/state/seds/seds-data-complete.php.

energybalances/.

Non-renewable Energy. Total primary energy from non-renewable sources. Author's calculation: *Primary Energy* minus *Renewable Energy*.

Energy Extraction Costs. Estimates of available fossil fuel energy resources remaining in the environment, and the cost of extracting those resources. Costs and availability are measured in terms of final-use energy that can eventually used from primary resources. The original estimates come from McGlade and Ekins (2015), who focus on primary energy availability and corresponding extraction costs. I use conversion factors from Rogner et al. (2012) to convert heterogeneous primary energy sources into common units, and data from the IEA and EIA to estimate efficiency of transforming primary energy into final-use energy. Appendix Section B.6 provides further detail on the calculations. Further background is available in McGlade (2014). Data available at: https://www.nature.com/articles/nature14016 (see source data for table 1).

A.4 Calibration

See above for details regarding **Real GDP**, **Energy Use**, **Energy Expenditure Share**, and **Energy Extraction Costs**.

Population. Total resident population of the United States. Accessed via 'Table C1: Population, U.S. gross domestic product, and U.S. Gross Output' at https://www.eia.gov/totalenergy/data/annual/.

R&D Share. Share of GDP devoted to research and development. Data are available from 1981-2017. Source: Bureau of Economic Analysis. Accessed via OECD Science, Technology and RD Statistics: Main Science and Technology Indicators:
https://data.oecd.org/rd/gross-domestic-spending-on-r-d.htm.

B Online Appendix

B.1 Final Good Producer Problem

In this section, I derive the inverse demand functions (15) and (16). Consider the maximization of (3) subject to (4) with $\upsilon_t(i)$ as the Lagrange multiplier attached to capital good i,

$$\mathcal{L} = \int_0^1 A_{E,t}(i)E_t(i)di - w_tL_t - \int_0^1 p_{X,t}(i)X_t(i)di - \tau_t p_{E,t}\int_0^1 E_t(i)di - \int_0^1 \upsilon_t(i)\left[A_{E,t}(i)E_t(i) - \left(A_{N,t}(i)X_t(i)\right)^{\alpha}L_t^{1-\alpha}\right]di. \quad \text{(B.1)}$$

Complementary slackness implies

$$\upsilon_t(i)\left[A_{E,t}(i)E_t(i) - \left(A_{N,t}(i)X_t(i)\right)^{\alpha}L_t^{1-\alpha}\right] = 0\ \forall i. \quad \text{(B.2)}$$

I focus on the case where the constraint is always binding. This will necessarily be true in the empirical exercise, because $\delta = 1$ is a sufficient, but not necessary, condition for the constraint to bind. The first order conditions with respect to $E_t(i)$, $X_t(i)$, and L_t are given by:

$$\upsilon_t(i) = 1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}, \quad \text{(B.3)}$$

$$\upsilon_t(i) = \frac{p_{X,t}(i)}{\alpha A_{N,t}^{\alpha}(i)L_t^{1-\alpha}X_t(i)^{\alpha-1}}, \quad \text{(B.4)}$$

$$w_t = \int_0^1 \upsilon_t(i)(1-\alpha)A_{N,t}^{\alpha}(i)L_t^{-\alpha}X_t(i)^{\alpha}di. \quad \text{(B.5)}$$

Substituting (B.4) and (B.5) into (B.3), respectively, and multiplying through yields

$$p_{X,t}(i) = \alpha A_{N,t}(i)^{\alpha}\left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]L_t^{1-\alpha}X_t(i)^{\alpha-1}, \quad \text{(B.6)}$$

$$w_t = (1-\alpha)\int_0^1 \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]L_t^{-\alpha}\left(A_{N,t}(i)X_t(i)\right)^{\alpha}di. \quad \text{(B.7)}$$

Thus, we have arrived at equations (15) and (16) from the text. A key result is that inverse demand is iso-elastic, which allows for simple closed form solutions. This is shown in the next section.

B.2 Monopolist Problem

The monopolist maximizes profits subject to demand and research productivity constraints:

$$max\ \pi_{X,t}(i) = p_{X,t}(i)X_t(i) - \tau_t^K r_t X(i) - (1-\eta_t^S)p_{E,t}^R R_E(i) - p_{N,t}^R R_N(i) \quad \text{(B.8)}$$

$$\text{(B.9)}$$

subject to

$$p_{X,t}(i) \quad = \quad \alpha A_{N,t}(i)^{\alpha}\left[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right] L_t^{1-\alpha} X_t(i)^{\alpha-1}, \tag{B.10}$$

$$A_{J,t}(i) \quad = \quad \left[1+\eta_J R_{J,t}(i)^{1-\lambda}\right] A_{J,t-1}, \quad J \in \{N, E\}, \tag{B.11}$$

$$R_{J,t}(i) \quad \in \quad [0,1], \quad J \in \{N, E\}. \tag{B.12}$$

In equilibrium, the research allocation must be interior due to the decreasing returns. Thus, I ignore the last constraint for the remainder of this section. First, substitute (B.10) into (B.8) and take the first order condition with respect to $X(i)$. Constraint (B.11) is independent of the production level, $X_t(i)$. Hence, the model yields the standard first order conditions and results, adjusted for the effective cost of energy:

$$\tau_t^K r_t \quad = \quad \alpha^2 A_{N,t}(i)^{\alpha}\left[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right] L_t^{1-\alpha} X_t(i)^{\alpha-1}. \tag{B.13}$$

Applying $\tau_t^K = \alpha$, which undoes the monopoly distortion,

$$r_t \quad = \quad \alpha A_{N,t}(i)^{\alpha}\left[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right] L_t^{1-\alpha} X_t(i)^{\alpha-1}. \tag{B.14}$$

Rearranging gives

$$X_t(i) \quad = \quad \alpha^{\frac{1}{1-\alpha}} r^{\frac{-1}{1-\alpha}} A_{N,t}(i)^{\frac{\alpha}{1-\alpha}} L_t \left[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}}. \tag{B.15}$$

$$\tag{B.16}$$

Plugging in to (B.10) gives

$$p_{X,t}(i) \quad = \quad r_t. \tag{B.17}$$

Next, to find optimal profits, we can re-write the monopolist problem after substituting in results we have found so far. Noting that $\pi_{X,t}(i) = (p_{X,t}(i) - \tau_t^K r_t) X_t(i)$,

$$max\ \pi_{X,t}(i) = \tilde{\alpha} r_t^{\frac{-\alpha}{1-\alpha}} A_{N,t}(i)^{\frac{\alpha}{1-\alpha}} L_t \left[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}} - (1-\eta_t^S) p_{E,t}^R R_{E,t}(i) - p_{N,t}^R R_{N,t}(i) \tag{B.18}$$

subject to

$$A_{J,t}(i) \quad = \quad \left[1+\eta_J R_{J,t}(i)^{1-\lambda}\right] A_{J,t-1}, \quad J \in \{N, E\}, \tag{B.19}$$

where $\tilde{\alpha} = (1-\alpha)\alpha^{\frac{1}{1-\alpha}}$. Let κ_J be the Lagrange multiplier for constraint (B.19). The first order conditions for technology levels and research scientist allocations yield

$$\begin{aligned} p^R_{N,t} &= \kappa_N(1-\lambda)A_{N,t-1}\eta_N R_{N,t}(i)^{-\lambda}, & \text{(B.20)} \\ (1-\eta^S_t)p^R_{E,t} &= \kappa_E(1-\lambda)A_{E,t-1}\eta_E R_{E,t}(i)^{-\lambda}, & \text{(B.21)} \\ \kappa_N &= \frac{\alpha}{1-\alpha}\tilde{\alpha} r_t^{\frac{-\alpha}{1-\alpha}} A_{N,t}(i)^{\frac{\alpha}{1-\alpha}-1} L_t \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}}, & \text{(B.22)} \\ \kappa_E &= \frac{1}{1-\alpha}\tilde{\alpha} r_t^{\frac{-\alpha}{1-\alpha}} A_{N,t}(i)^{\frac{\alpha}{1-\alpha}} L_t \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}-1} \tau_t p_{E,t} A_{E,t}^{-2}. & \text{(B.23)} \end{aligned}$$

Putting these together, we have

$$\begin{aligned} p^R_{N,t} &= \alpha\psi_t A_{N,t}^{\frac{\alpha}{1-\alpha}-1}\left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}} \eta_N R_{N,t}(i)^{-\lambda} A_{N,t-1}, & \text{(B.24)} \\ (1-\eta^S_t)p^R_{E,t} &= \psi_t A_{N,t}^{\frac{\alpha}{1-\alpha}} \tau_t p_{E,t} A_{E,t}(i)^{-2}\left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}-1} \eta_E R_{E,t}(i)^{-\lambda} A_{E,t-1}, & \text{(B.25)} \end{aligned}$$

where $\psi_t = \alpha^{\frac{1}{1-\alpha}}(1-\lambda)r_t^{\frac{-\alpha}{1-\alpha}} L_t$ is common to both terms. In the next section, I shown the optimal research allocations resulting from these first order conditions. Taking ratios of these first order conditions yields (20) in the main text.

B.3 R&D Allocations

In this section, I derive the optimal research allocations given in equations (22) and (23). First, note that $R_{J,t}(i) = R_{J,t}$ $\forall i, t, J$. This occurs because all monopolists make identical decisions, and there are a unit mass of monopolists. This also implies that $A_{J,t}(i) = A_{J,t}$ $\forall i, t, J$. Also, factor mobility ensures that $p^R_{E,t} = p^R_{N,t}$ $\forall t$. Thus, equation (20) can be re-written as

$$(1-\eta^S_t)\frac{A_{E,t}}{A_{E,t-1}}\left[\frac{A_{E,t}}{\tau_t p_{E,t}} - 1\right] = \frac{A_{N,t}}{A_{N,t-1}}\frac{\eta_E R_E^{-\lambda}}{\alpha\eta_N R_N^{-\lambda}}. \quad \text{(B.26)}$$

Replacing growth rates and technology levels with the values given by (10) and applying the resource constraint (11) yields

$$(1-\eta^S_t)(1+\eta_E R_{E,t}^{1-\lambda})\left[\frac{(1+\eta_E R_{E,t}^{1-\lambda})A_{E,t-1}}{\tau_t p_{E,t}} - 1\right] = (1+\eta_N(1-R_{E,t})^{1-\lambda})\frac{\eta_E R_{E,t}^{-\lambda}}{\alpha\eta_N(1-R_{E,t})^{-\lambda}} \quad \text{(B.27)}$$

Dividing by $(1-\eta^S_t)$, then multiplying through on the left-hand side and isolating the term with energy prices yields

$$(1+\eta_E R_E^{1-\lambda})^2\frac{A_{E,t-1}}{\tau_t p_{E,t}} = \frac{1}{1-\eta^S_t}\left[\frac{\eta_E R_E^{-\lambda}}{\alpha\eta_N(1-R_E)^{-\lambda}}\left(1+\eta_N(1-R_E)^{1-\lambda}\right)\right] + (1+\eta_E R_E^{1-\lambda}). \quad \text{(B.28)}$$

Distributing terms on the right-hand side leaves

$$(1+\eta_E R_E^{1-\lambda})^2 \frac{A_{E,t-1}}{\tau_t p_{E,t}} = \frac{1}{\alpha(1-\eta_t^S)} \left[\frac{\eta_E R_E^{-\lambda}}{\eta_N (1-R_E)^{-\lambda}} + \eta_E R_E^{-\lambda} - \eta_E R_{E,t}^{1-\lambda} \right] + (1+\eta_E R_E^{1-\lambda}). \quad \text{(B.29)}$$

Now, (22) can be derived by multiplying through by $\frac{\tau_t p_{E,t}}{A_{E,t-1}}$, taking the square root of both sides, subtracting one, and dividing by $\eta_E R_E^{-\lambda}$.

B.4 Solving the Model

B.4.1 Intensive Form

In this section, I show how to solve the model in intensive form. This is helpful both for the quantitative exercise (see Section 4.4) and in proving the propositions in Sections 3.4 and 3.5. For any variable Z_t, I define

$$z_t \equiv \frac{Z_t}{L_t A_{R,t}}, \quad \text{(B.30)}$$

where $A_{R,t} = TFP_t^{\frac{1}{1-\alpha}}$ and $TFP_t = A_{N,t}^{\alpha}\left[1 - \frac{p_{E,t}}{A_{E,t}}\right]$. Applying (8), (9), and (12), this yields

$$y_t = k_t^{\alpha}, \quad \text{(B.31)}$$

$$k_{t+1} = \frac{y_t - c_t}{(1+g_{R,t+1})(1+n)}, \quad \text{(B.32)}$$

where $1+g_{R,t} = \frac{A_{R,t}}{A_{R,t-1}} = (1+g_{TFP,t})^{\frac{1}{1-\alpha}}$. Moreover, the Euler equation yields

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \frac{\beta r_{t+1}}{(1+g_{R,t+1})^{\sigma}}. \quad \text{(B.33)}$$

Finally, when considering the interest rate, it is also important to keep track of the energy tax rate, τ_t. Let $\tilde{A}_{R,t} = A_{N,t}^{\alpha}\left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\right]$ be TFP adjusted for energy taxes. Then, from equation (18),

$$\begin{aligned} r_t &= \alpha A_{N,t}^{\alpha}\left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\right] K_t^{\alpha-1} L_t^{1-\alpha} && \text{(B.34)} \\ &= \alpha \left(\frac{K_t}{\tilde{A}_{R,t} L_t}\right)^{\alpha-1} && \text{(B.35)} \\ &= \alpha \left(\frac{A_{R,t}}{\tilde{A}_{R,t}}\right)^{\alpha-1} \left(\frac{K_t}{A_{R,t} L_t}\right)^{\alpha-1} && \text{(B.36)} \\ &= \tilde{\tau}_t \alpha k_t^{\alpha-1}, && \text{(B.37)} \end{aligned}$$

where $\tilde{\tau}_t \equiv \left(\frac{A_{R,t}}{\tilde{A}_{R,t}}\right)^{\alpha-1} = \frac{1-\frac{\tau_t p_{E,t}}{A_{E,t}}}{1-\frac{p_{E,t}}{A_{E,t}}}$ is the interest rate wedge caused by the introduction of energy taxes.

When solving the model, I guess on a path of energy prices and then solve for the research allocations and growth rates. Then, the solution to the remainder of the model is given by (B.31), (B.32), (B.33), and (B.37). As described above, this is just the standard neoclassical growth model with a few additions. The $\tilde{\tau}_t$ term is the wedge in the interest rate caused by energy taxes, and $g_{R,t}$ may not be constant due to endogenous research allocations and energy prices.

B.4.2 Proof to Propositions 1, and 2.

Proof of items 3 – 5 of Propositions 1 and 2. To find the BGP, first note that $\tilde{\tau}_t = \bar{\tau}$, a constant. In the laissez-faire case, $\bar{\tau} = 1$. In the case of environmental policy (EP), $\bar{\tau} = \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\right]$, which is also constant. As discussed in the main text, $g_{TFP} = (1 + g_N^*)^\alpha - 1$ on the BGP because $\left[1 - \frac{p_{E,t}}{A_{E,t}}\right]$ is fixed (at 1 in the case of EP). Thus, the growth rate of output per person is given by $g_R^* = (1 + g_N^*)^{\frac{\alpha}{1-\alpha}} - 1$. With constant growth rates of technology, the BGP is given by:

$$\bar{r} = \frac{(1 + g_R^*)^\sigma}{\beta}, \tag{B.38}$$

$$\bar{k} = \left(\frac{\bar{\tau}\alpha}{\bar{r}}\right)^{\frac{1}{1-\alpha}}, \tag{B.39}$$

$$\bar{y} = \bar{k}^\alpha, \tag{B.40}$$

$$\bar{c} = \bar{y} - (1 + g_R^*)(1 + n)\bar{k}, \tag{B.41}$$

where $\bar{z}$ denotes the steady state value of z. Thus, r_t is constant, Y_t/L_t and C_t/L_t grow at rate g_R^*, and Y_t and K_t grow at rate $g_Y^* = (1 + g_R^*)(1 + n) - 1$. This proves parts (3) – (5) of Propositions 1 and 2.

Item 6 of Propositions 1 and 2. At any point in time, energy use is given by

$$E_t = \frac{A_{N,t}^\alpha}{A_{E,t}} K_t^\alpha L_t^{1-\alpha} = Y_t / A_{E,t}. \tag{B.42}$$

Using the results from the previous subsection, the BGP growth rate of flow energy use (g_M^*) given by

$$g_M^* = \frac{(1 + g_N^*)^{\frac{\alpha}{1-\alpha}}}{(1 + g_E^*)}(1 + n) - 1. \tag{B.43}$$

This proves item (6) of the propositions. Since there is a unit mass of R&D inputs, the maximum possible value for g_E^* is η_E. Thus, assumption (A1) ($\eta_E > n$) implies that $g_N^* > 0$ whenever $g_M^* > 0$. Put differently, it is possible for rapid growth in energy energy to lead to decreasing flows of energy use in the long run.

Items 1 and 2 of Propositions 1 and 2. I now turn to finding the BGP R&D allocations. I will use $g^*_{\overline{M}}$ to denote the growth rate of the cumulative stock of extracted energy on the BGP. By definition, output and technology grow at constant rates on a BGP. As a result, E_t must also grow at a constant rate, as seen in equation (B.43). If $g^*_M \leq 0$, $g^*_{\overline{M}} = 0$. If $g^*_M > 0$, $g^*_{\overline{M}} = g^*_M$.

The growth factor of the price of energy is given by

$$(1 - g_V)(1 + g^*_{\overline{M}})^{\psi}(1 + g_\tau) = (1 + g^*_P). \tag{B.44}$$

The maximum possible growth rate of the price of energy occurs when there is zero investment in energy efficiency. In this case, per period energy use grows at a positive rate, which implies that $g^*_{\overline{M}} = g^*_M$. So, the maximum growth rate of energy prices is is $(1 - g_V)\left[(1 + \eta_N)^{\frac{\alpha}{1-\alpha}}(1 + n)\right]^{\psi} - 1$. Assumption (A3) implies that this growth rate is positive.

On a BGP, energy efficiency grows at the rate of the energy price times the growth in energy taxes. Thus,

$$(1 - g_V)(1 + g^*_{\overline{M}})^{\psi}(1 + g_\tau) = (1 + g^*_E). \tag{B.45}$$

- If $g^*_{\overline{M}} = 0$, then $(1 - g_V)(1 + g_\tau) = 1 + \eta_E(R^*_E)^{1-\lambda}$, which determines the unique

$$R^*_E = \left[\frac{(1 - g_V)(1 + g_\tau) - 1}{\eta_E}\right]^{\frac{1}{1-\lambda}} \equiv \tilde{R}^*_E. \tag{B.46}$$

  There are a unit mass of R&D inputs. So, if $\eta_E > (1 - g_V)(1 + g_\tau) - 1 > 0$ (assumption (A4) in the main text), this BGP is feasible and has an interior solution. To determine when this is the endogenous outcome, we plug (B.46) into (B.43) to determine if $g^*_M \leq 0$. As a result, $R^*_E = \tilde{R}^*_E$ and $g^*_{\overline{M}} = 0$ if and only if:

$$(1 - g_V)(1 + g_\tau) \geq \left[1 + \eta_N\left(1 - \left[\frac{(1 - g_V)(1 + g_\tau) - 1}{\eta_E}\right]^{\frac{1}{1-\lambda}}\right)^{1-\lambda}\right]^{\frac{\alpha}{1-\alpha}}(1 + n). \tag{B.47}$$

  This is captured by assumption (A5) in the main text. Intuitively, the BGP has a constant stock of energy use when the exogenous component of the energy price grows sufficiently fast to incentive large investments in energy efficiency. Without environmental policy, this reduces to

$$(1 - g_V) \geq \left[1 + \eta_N\left(1 - \left[\frac{-g_V}{\eta_E}\right]^{\frac{1}{1-\lambda}}\right)^{1-\lambda}\right]^{\frac{\alpha}{1-\alpha}}(1 + n). \tag{B.48}$$

  Thus, assumption (A2) in the main text implies that energy use increase in the absence of policy.

- If $g^*_{\overline{M}} = g^*_M > 0$, then we can plug (B.43) into (B.45) to obtain:

$$(1+g^*_E)^{1+1/\psi}\left[(1+g_\tau)(1-g_V)\right]^{-1/\psi} = (1+g^*_N)^{\frac{\alpha}{1-\alpha}}(1+n). \tag{B.49}$$

  Applying (10) and (11) to this equation yields item (2) of Propositions 1 and 2. Rearranging yields item (1). The existence of an interior R&D allocation is guaranteed because both energy use and energy prices are increasing. With increasing energy prices, $g^*_E > 0$. With increasing energy use, $g^*_N > 0$, as along as assumption (A1) holds.

Item 7 of Propositions 1 and 2. All that remains to show for these two propositions is that expenditure shares are constant. I focus on income shares before taking into account taxes and transfers. To start, from equation (16) note that

$$w_t L_t = (1-\alpha)A^{\alpha}_{N,t}[1-\frac{\tau_t p_{E,t}}{A_{E,t}}]K^{\alpha}_t L^{1-\alpha}_t = \tilde{\tau}_t(1-\alpha)Y_t, \tag{B.50}$$

which implies that the share is constant on the BGP. Next, from (25) and (18),

$$r_t K_t = \alpha A^{\alpha}_{N,t}[1-\frac{\tau_t p_{E,t}}{A_{E,t}}]K^{\alpha}_t L^{1-\alpha}_t = \tilde{\tau}_t \alpha Y_t, \tag{B.51}$$

which again implies that the share is constant on the BGP. These results highlight the role of the capital subsidy τ^K_t, which undoes the monopoly distortion. Without EP, $\tilde{\tau}_t = 1$, implying that total pre-tax income paid to capital and labor is equal to GDP. The government taxes some of this income and distributes it to firms in the form of subsidies for capital purchases, which in turn creates profits for the capital good producer that can be paid to R&D inputs. Since the taxes are lump-sum, this does not distort incentives.

All research inputs are hired at the same rate. By equations (B.24) and (18), total payments to research inputs are given by

$$p^R_t = \alpha(1-\lambda)\frac{r_t X_t}{A_{N,t}}\eta_N R^{-\lambda}_{N,t} A_{N,t-1} \tag{B.52}$$

$$= (1-\lambda)\frac{\eta_N (R_{N,t})^{-\lambda}}{1+g_{N,t}}\cdot \tilde{\tau}_t \alpha^2 Y_t, \tag{B.53}$$

noting that there is a unit mass of research inputs. Again, this share is constant on the BGP.

To get the energy expenditure share in either case, rearrange equation (22) to isolate $\frac{\tau_t p_{E,t}}{A_{E,t-1}} = \frac{\tau_t p_{E,t}(1+g^*_E)}{A_{E,t}}$, which is constant. In the laissez-faire case, $\tau_t = 1$ and $\frac{p_{E,t}}{A_{E,t}} = \frac{\theta^*_E}{1+\theta^*_E}$. In the EP case, $\lim_{t\to\infty}\frac{p_{E,t}}{A_{E,t}} = 0 \Rightarrow \lim_{t\to\infty}\theta_{E,t} - \frac{\tau_t p_{E,t}}{A_{E,t}} = 0$. Thus, item (7) of the propositions is proven.

B.5 The Cobb-Douglas Model

In this section, I derive the dynamics, BGP, and calibration procedure for the Cobb-Douglas model. To start, I note that, due to perfect competition, aggregate energy use is given by

$$E_t = \left(\frac{\nu}{\tau_t p_{E,t}}\right)^{\frac{1}{1-\nu}} (A_t^{CD})^{\frac{1}{1-\nu}} K_t^{\frac{\gamma}{1-\nu}} L_t^{\frac{1-\gamma-\nu}{1-\nu}}. \tag{B.54}$$

This, in turn, yields

$$\begin{aligned} Q_t &= \left(\frac{\nu}{p_{E,t}\cdot\tau_t}\right)^{\frac{\nu}{1-\nu}} (A_t^{CD})^{\frac{1}{1-\nu}} K_t^{\frac{\gamma}{1-\nu}} L_t^{\frac{(1-\gamma-\nu)}{1-\nu}}, && \text{(B.55)}\\ Y_t &= \left(1-\frac{\nu}{\tau}\right) Q_t. && \text{(B.56)} \end{aligned}$$

To analyze the model in intensive form, I define

$$z_t = \frac{Z_t}{L_t (A_t^{CD})^{\frac{1}{1-\gamma-\nu}} (\tau_t\cdot p_{E,t})^{\frac{-\nu}{1-\gamma-\nu}}}, \tag{B.57}$$

for any variable Z_t. This notation is specific to Appendix Section B.5.

The Euler equation is the same as in the DTC case. In intensive form,

$$\frac{c_{t+1}}{c_t} = \frac{\beta r_{t+1}}{(1+g_{CD})^{\frac{1}{1-\gamma-\nu}} (1+\tilde{g}_{P,t+1})^{\frac{-\nu}{1-\gamma-\nu}}}, \tag{B.58}$$

where $1+\tilde{g}_{P,t+1} = (1+g_{\tau,t+1})(1+g_{P,t+1})$, $1+g_{\tau,t} = \frac{\tau_t}{\tau_{t-1}}$, and I have imposed $\sigma = 1$. The rest of the dynamics are given by

$$\begin{aligned} k_{t+1} &= \frac{y_t - c_t}{(1+g_{CD,t+1})^{\frac{1}{1-\gamma-\nu}} (1+\tilde{g}_{P,t+1})^{\frac{-\nu}{1-\gamma-\nu}} (1+n)}, && \text{(B.59)}\\ y_t &= (1-\frac{\nu}{\tau})\nu^{\frac{\nu}{1-\nu}} k_t^{\frac{\gamma}{1-\nu}}, && \text{(B.60)}\\ r_t &= \gamma\nu^{\frac{\nu}{1-\nu}} k_t^{\frac{\gamma-(1-\nu)}{1-\nu}}. && \text{(B.61)} \end{aligned}$$

As in the case of the DTC model, I solve the CD model by first guessing a path of energy taxes and then solving the growth model with equations (B.58) – (B.61).

I consider the BGP in a *laissez-faire equilibrium*. This gives

$$\begin{aligned} \bar{r} &= \frac{(1+g_{CD}^*)^{\frac{1}{1-\gamma-\nu}} (1+g_P^*)^{\frac{-\nu}{1-\gamma-\nu}}}{\beta}, && \text{(B.62)}\\ \bar{k} &= \nu^{\frac{-\nu}{\gamma-(1-\nu)}} (\bar{r}/\gamma)^{\frac{1-\nu}{\gamma-(1-\nu)}}, && \text{(B.63)}\\ \bar{y} &= (1-\nu)\nu^{\frac{\nu}{1-\nu}} \bar{k}^{\frac{\gamma}{1-\nu}}, && \text{(B.64)}\\ \bar{c} &= \bar{y} - (1+g_{CD}^*)^{\frac{1}{1-\gamma-\nu}} (1+g_P^*)^{\frac{-\nu}{1-\gamma-\nu}} (1+n)\bar{k}. && \text{(B.65)} \end{aligned}$$

As a result, r_t is constant, Y_t/L_t and C_t/L_t grow at rate $(g_R^*)^{CD} = (1+g_{CD}^*)^{\frac{1}{1-\gamma-\nu}}(1+g_P^*)^{\frac{-\nu}{1-\gamma-\nu}} - 1$, and Y_t and K_t grow at rate $g_Y^{CD} = (1+g_R^*)^{CD}(1+n) - 1$.

I calibrate the CD model to the BGP using the same data as employed for the DTC model, leading to observationally equivalent paths for output and energy use. To match the energy expenditure share, I set

$$\frac{\nu}{1-\nu} = \theta_E^* \tag{B.66}$$

and

$$\gamma = \alpha - \nu. \tag{B.67}$$

All that remains is to ensure that total output grows at the same rate in the two models, which implies that energy use will also grow at the same rate. Since the energy sector is equivalent in the two models, this further implies that the price of energy will grow at the same rate. Thus, I set $(g_R^*)^{CD} = g_R^*$, where the latter comes from the DTC model in Section B.4.1. This implies that

$$g_R^* = (1+g_{CD}^*)^{\frac{1}{1-\gamma-\nu}}(1+g_P^*)^{\frac{-\nu}{1-\gamma-\nu}} - 1 \;\Rightarrow \tag{B.68}$$

$$g_{CD}^* = (1+g_R^*)^{1-\gamma-\nu}(1+g_E^*)^{\nu} - 1. \tag{B.69}$$

B.6 Extraction Cost Estimates

B.6.1 Data

Extraction Costs. Extraction cost and energy availability estimates for coal, oil, and natural gas are taken from McGlade and Ekins (2015).[45] Estimates of energy availability include known energy reserves in or scheduled to be in production, known resources not currently in production, estimates of reserve growth within known sources of energy, and estimates of undiscovered resources. Quantities correspond to *remaining ultimately recoverable resources* (RURR), the amount of energy that could be profitably extracted from the environment at some point in the future, even if it is not currently profitable to do so. This definition requires assumptions about future energy prices and technology. 'Additional occurrences' of coal, oil, and natural gas that fall outside the definition RURR are likely to be quite large (Rogner et al., 2012). I use the McGlade and Ekins (2015) data to estimate the shape of the extraction cost curve, but not to estimate a limit to total energy availability.

Extraction costs in McGlade and Ekins (2015) are estimated for current technology and include operating expenses, capital expenditure, and capital costs necessary to bring primary sources of energy to the market. They include exploration costs and exclude taxes. Costs were estimated separately for sub-categories of energy within the three broad types of fossil fuel.

[45] Data available at: https://www.nature.com/articles/nature14016 (see source data for table 1). See McGlade (2014) for further details.

Both cost and energy availability estimates are uncertain. McGlade and Ekins (2015) construct a range of extraction cost curves at the country level for each fossil fuel. The publicly available global data are the median values from a Monte Carlo procedure that aggregates these country-level estimates. I combine their estimates to construct a cost curve for an aggregate fossil fuel energy composite.

Aggregation. Energy availability is measured in different units for each broad type of energy. Oil is measured in barrels, natural gas is measured in cubic meters, and coal is measured by energy content. I use data from Rogner et al. (2012) to estimate the energy content of oil and natural gas.[46] Energy conversion factors are presented in Table B1. Figure B.1a plots the primary energy supply curves for each type of fossil fuel. Once availability is measured in terms of energy content for all sources, the data can be aggregated to derive a single primary energy cost curve, which is shown in figure B.1b.

Different types of energy are converted from primary to final-use energy at different rates. To measure primary-to-final energy conversion factors, I take data on primary and final-energy use in the United States from the International Energy Agency (IEA). These are the same data used in figure 1a, except that I now focus on the E_p/E_f ratio for individual types of fossil fuel energy, rather that country-level aggregates. Details on these calculations are provided below.

- **Oil.** I take total final-use energy consumption from oil products and subtract net imports of final-use oil products. Then, I divide this difference by the supply of primary energy from crude oil, natural gas liquids and refinery feedstocks. Data are average over the period 1971-2016.

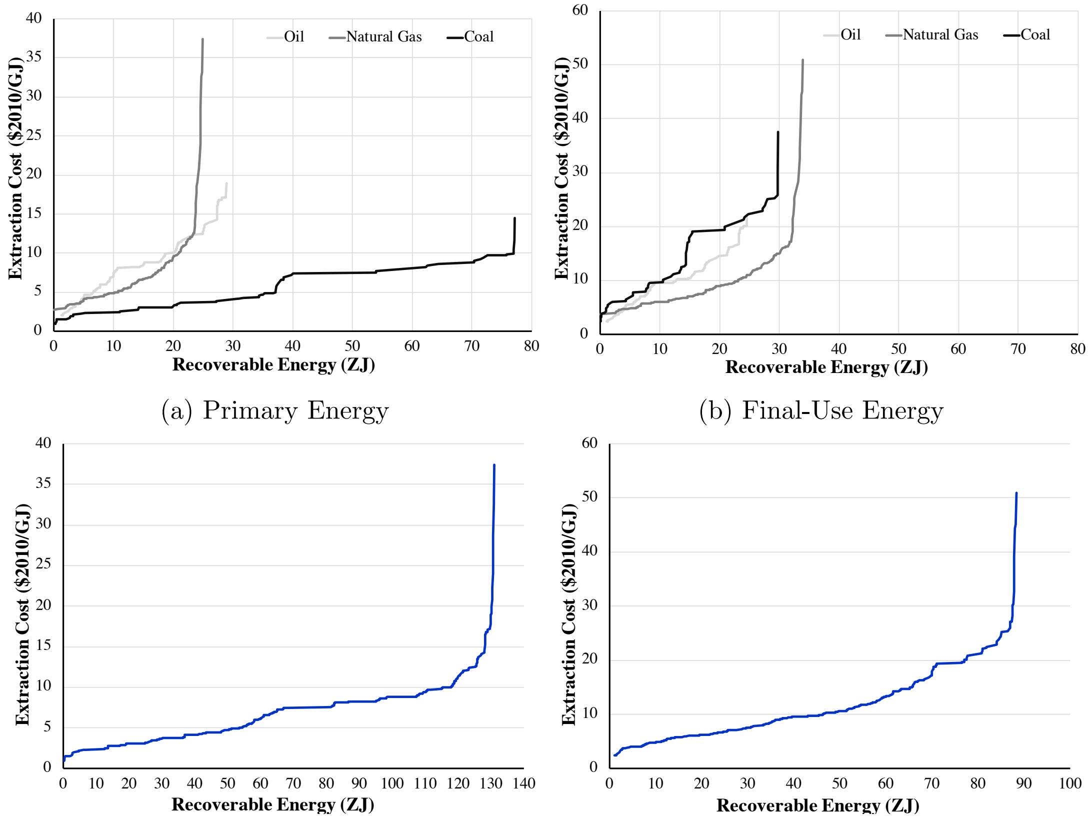
- **Natural Gas.** Using the IEA data, I break natural gas usage into two categories: gas used directly as a final energy source and gas used to generate electricity. Data on natural gas usage is available from 1971-2016. By definition, the transformation efficiency for gas used as final energy is one. For electricity, the average transformation efficiency is taken from the EIA.[47] Due to data limitations on the heat rate of electricity production, the average efficiency is calculated using data from 2001-2016.

- **Coal.** The calculation for the transformation efficiency of coal is identical to the calculation for natural gas.

Figure B.1c plots the final-use energy supply curves separately for each source of energy, and figure B.1d plots the aggregated curve.

[46] See table 7.3 in Rogner et al. (2012).

[47] Transformation efficiency is calculated as the heat rate of electricity generation divided by 3,412 Btu, which is the heat content of a kWh of electricity (`https://www.eia.gov/tools/faqs/faq.php?id=107&t=3`). Heat rate data are taken from Table A6, 'Approximate heat rates for electricity, and heat content of electricity,' of the Annual Energy Review (`https://www.eia.gov/totalenergy/data/annual/`).



(a) Primary Energy (b) Final-Use Energy

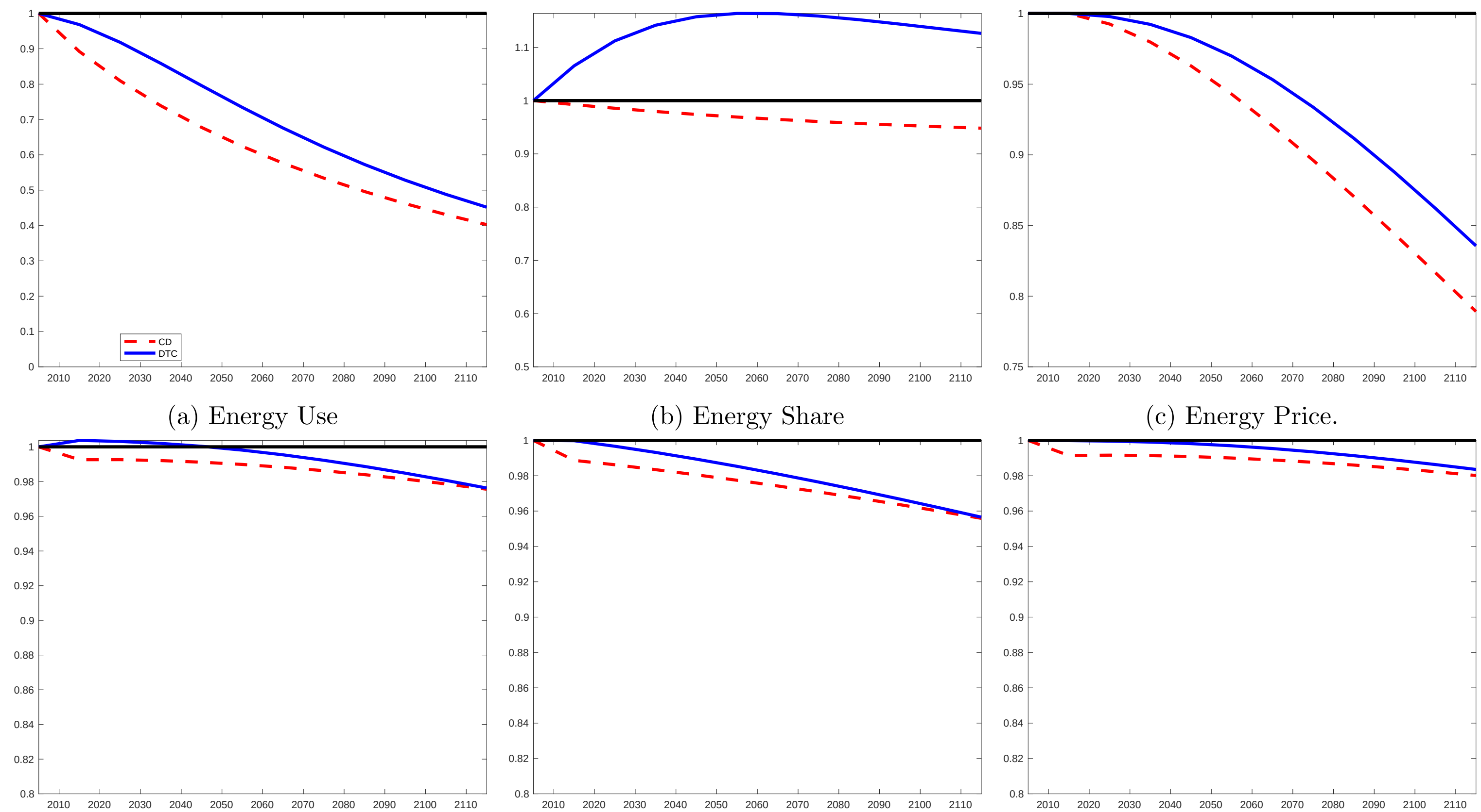
(c) Primary Energy (d) Final-Use Energy

Figure B.1: Extraction Cost Curves. Estimates are originally from McGlade and Ekins (2015). Conversion factors given in table B1.

Primary Type	Original Unit	Unit Conversion	Final/Primary
Oil	Barrels	5.71 GJ/bbl	85%
Nat. Gas	Cubic meters	0.04 GJ/m^3	73%
Coal	Joules	1 GJ/GJ	39%

Table B1: Energy Conversion. Unit conversion factors for primary energy taken from Rogner et al. (2012). 'Primary/final' is the efficiency of transforming primary sources of energy (e.g., coal, oil) into final sources of energy (e.g., electricity, gasoline). Efficiency data are taken from the IEA and EIA.

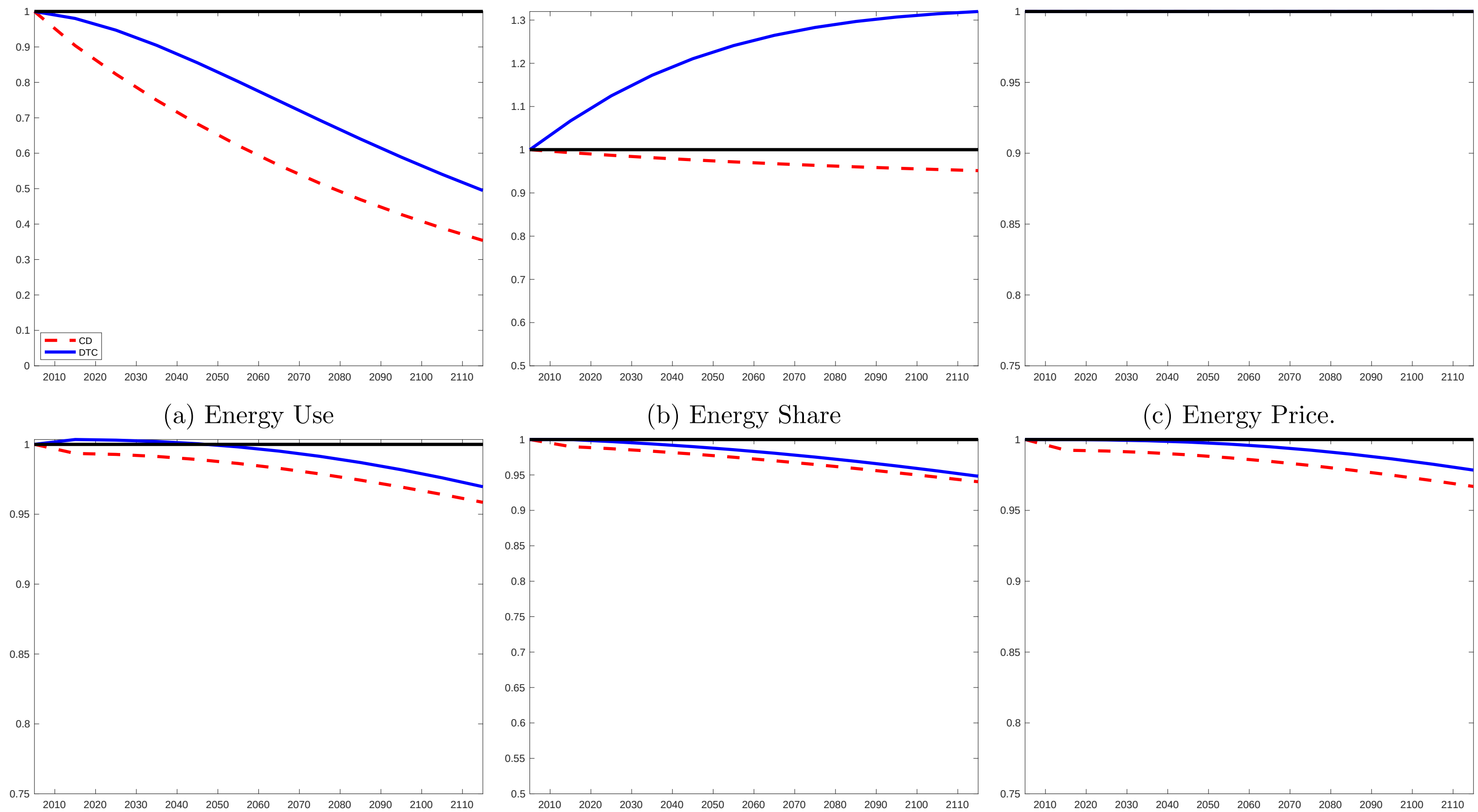
B.7 Robustness Results



(a) Energy Use (b) Energy Share (c) Energy Price.

(d) Consumption (e) GDP (f) TFP

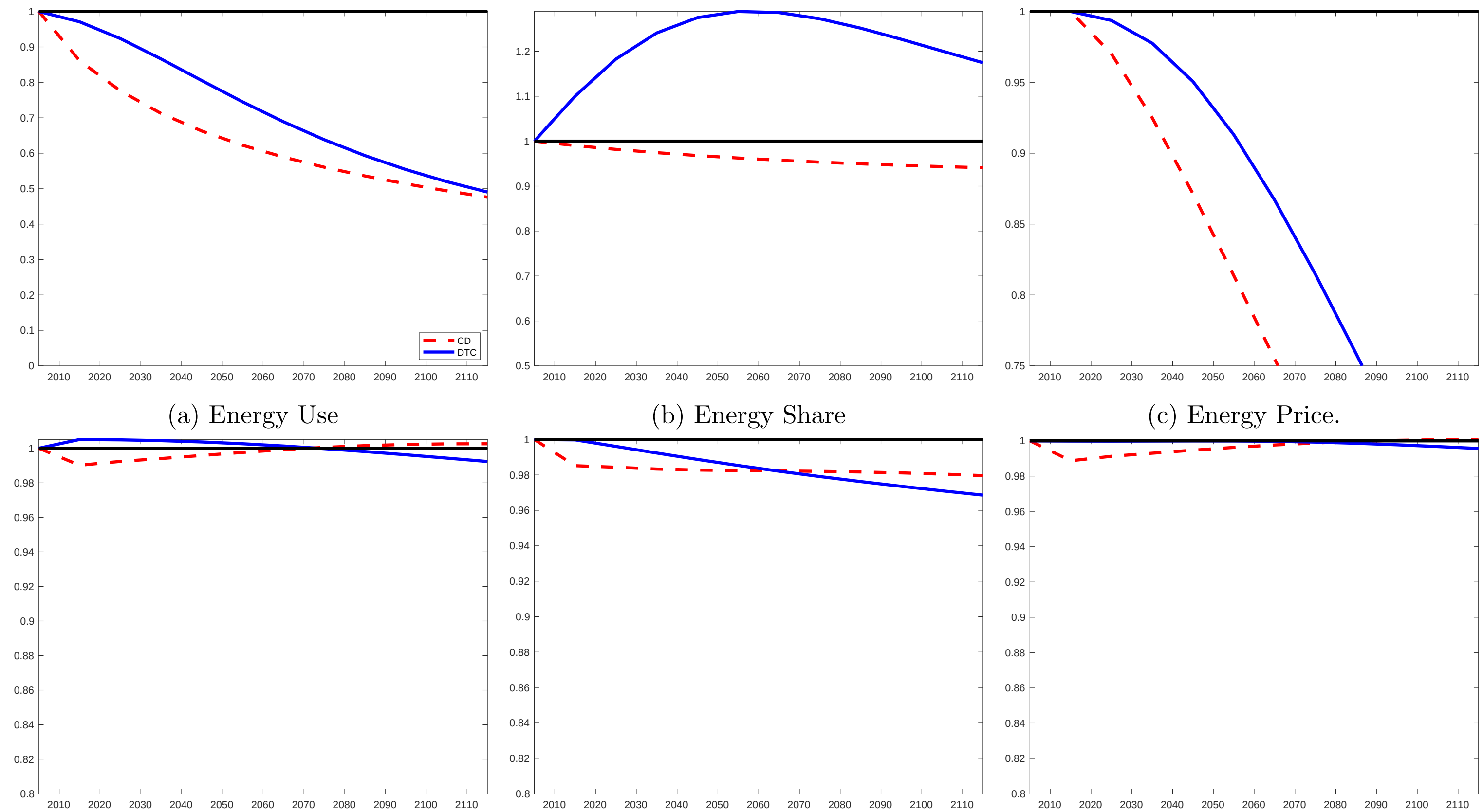
Figure B.2: **Robustness with $\lambda = 0.21$.** Comparison of the DTC and CD models when using an identical path of taxes. In particular, $g_\tau = .100$ (1.0%/year), which meets the policy goal in the CD model. The policy goal is that energy use in 2055 is 15% lower than energy use in 2005. The policy is announced in 2005 and takes effect in 2015. All results are shown relative to a 'business as usual' case where the economy remains on the original BGP. Panel (a) compares flow energy use. Panel (b) gives the energy expenditure share of final output. Panel (c) shows the pre-tax price of energy, which is equal to the energy extraction cost. Panel (d) gives consumption by the representative household. Panel (e) gives final output. Panel (f) gives TFP. For ease of comparison between the models, TFP is calculated as $Y_t/(K_t^\alpha L_t^{1-\alpha})$ in both cases.



(a) Energy Use (b) Energy Share (c) Energy Price.

(d) Consumption (e) GDP (f) TFP

Figure B.3: **Robustness with exogenous energy prices.** Comparison of the DTC and CD models when using an identical path of taxes. In particular, $g_\tau = 0.088$ (0.9%/year), which meets the policy goal in the CD model. The policy goal is that energy use in 2055 is 15% lower than energy use in 2005. The policy is announced in 2005 and takes effect in 2015. All results are shown relative to a 'business as usual' case where the economy remains on the original BGP. Panel (a) compares flow energy use. Panel (b) gives the energy expenditure share of final output. Panel (c) shows the pre-tax price of energy, which is equal to the energy extraction cost. Panel (d) gives consumption by the representative household. Panel (e) gives final output. Panel (f) gives TFP. For ease of comparison between the models, TFP is calculated as $Y_t/(K_t^\alpha L_t^{1-\alpha})$ in both cases.



(a) Energy Use (b) Energy Share (c) Energy Price.

(d) Consumption (e) GDP (f) TFP

Figure B.4: **Robustness with $\psi = 3.64$.** Comparison of the DTC and CD models when using an identical path of taxes. In particular, $g_\tau = 0.135$ (1.3%/year), which meets the policy goal in the CD model. The policy goal is that energy use in 2055 is 15% lower than energy use in 2005. The policy is announced in 2005 and takes effect in 2015. All results are shown relative to a 'business as usual' case where the economy remains on the original BGP. Panel (a) compares flow energy use. Panel (b) gives the energy expenditure share of final output. Panel (c) shows the pre-tax price of energy, which is equal to the energy extraction cost. Panel (d) gives consumption by the representative household. Panel (e) gives final output. Panel (f) gives TFP. For ease of comparison between the models, TFP is calculated as $Y_t/(K_t^\alpha L_t^{1-\alpha})$ in both cases.

B.8 Extension of Base Model

In this section, I consider an extension of the DTC model that allows for labor reallocation between production and research, as well as the entry of new capital good producers. This extended model incorporates insights from 'second wave' endogenous growth theory (e.g., Peretto, 1998; Young, 1998; Howitt, 1999). As a result, it eliminates scale effects that are present in existing models of directed technical change (e.g., Acemoglu, 1998, 2002; Hassler et al., 2012, 2016b). I show that the extended model continues to explain the key patterns observed in U.S. data.

Consider the following extension of the aggregate production function:

$$Q_t = \int_0^{M_t} \min\left[(L_t/M_t)^{1-\alpha}\left(A_{N,t}(i)X_t(i)\right)^{\alpha},\ A_{E,t}(i)E_t(i)\right] di, \tag{B.70}$$

where M_t gives the mass of (atomistic) capital good producers in operation at time t. This particular functional form is standard in the existing literature and eliminates the 'love of variety' in production. I will refer to L_t as production workers. To operate in period t, a capital good producer must hire φ^{-1} workers to cover fixed costs. This yields

$$M_t = \varphi F_t, \tag{B.71}$$

where F_t is the total number of workers hired to cover fixed costs. There is free entry into capital good production.

As in the main text, I assume

$$A_{J,t}(i) \quad = \quad \left[1 + \eta_J R_{J,t}(i)^{1-\lambda}\right] A_{J,t-1}, \quad J = N, E, \tag{B.72}$$

where $A_{J,t} = \frac{1}{M_t}\int_0^{M_t} A_{J,t}(i)\, di$,[48] except that research is now conducted by workers. The new labor market clearing condition is given by

$$N_t = L_t + F_t + R_t \ \forall t, \tag{B.73}$$

where N_t is the size of the aggregate workforce. The key difference with the main text is that workers are now fully mobile across production sectors. As before, N_t grows at rate n. The Euler equation is still standard, but is now written in terms of N_t to accommodate the broader notion of population,

$$\{C_t\}_{t=0}^{\infty} \quad = \quad \operatorname{argmax} \sum_{t=0}^{\infty} \beta^t N_t \frac{\tilde{c}_t^{1-\sigma}}{1-\sigma}, \tag{B.74}$$

where $\tilde{c}_t = C_t/N_t$. The remainder of the model is unchanged.

[48] As explained in footnote 21, this is equivalent to $A_{J,t} = \max_i\{A_{J,t-1}(i)\}$ in the baseline model.

Using the same steps outlined in earlier appendix sections, it is straightforward to derive the following key expressions:

$$Y_t = \left[1 - \frac{p_{E,t}}{A_{E,t}}\right] (A_{N_t} K_t)^{\alpha} L_t^{1-\alpha}, \tag{B.75}$$

$$w_t = (1-\alpha) \left[1 - \frac{p_{E,t}}{A_{E,t}}\right] L^{-\alpha} (A_{N_t} K_t)^{\alpha}, \tag{B.76}$$

$$\bar{\pi}_{X,t} \propto r_t^{\frac{-\alpha}{1-\alpha}} A_{N,t}^{\frac{\alpha}{1-\alpha}} (L_t/M_t) \left[1 - \frac{p_{E,t}}{A_{E,t}}\right]^{\frac{1}{1-\alpha}}, \tag{B.77}$$

$$p_t^R \propto r_t^{\frac{-\alpha}{1-\alpha}} (L_t/M_t) A_{N,t}(i)^{\frac{\alpha}{1-\alpha}} \left[1 - \frac{p_{E,t}}{A_{E,t}}\right]^{\frac{1}{1-\alpha}} \eta_N \bar{R}_{N,t}^{-\lambda} \frac{1}{1+g_{N,t}}, \tag{B.78}$$

where $\bar{R}_{J,t} = M_t^{-1} \int_0^{M_t} R_{J,t}(i)di$. Also, $\bar{R}_t = \bar{R}_{N,t} + \bar{R}_{E,t}$. I have applied the relevant market clearing conditions, and the fact that all capital good producers face an identical problem and make identical decisions. The latter implies that $\bar{R}_{J,t} = R_{J,t}(i)\ \forall i, J$. Again, $\bar{\pi}_{X,t}$ is the profits of the capital good producer before taking into account R&D costs. Compared to the equations in the main text, the only difference is the inclusion of M_t in the last two equations.

I will show that there exists a balanced growth path with constant expenditure shares, matching the data. On this BGP, a constant fraction of workers will be allocated to each occupation. I show this in two steps. First, I show that, conditional on constant labor allocations, the extended model reduces to the model presented in the main text. Thus, it has a BGP with constant expenditures shares for all factors including energy and is consistent with the Kaldor facts. Second, I show that this BGP is compatible with free mobility between occupations.

BGP assuming constant allocations. — Conditional on constant labor allocations (in shares), the BGP of the extended model is almost identical to the version presented in the main text. With constant allocations, L_t, M_t, and R_t grow at the same constant rate. Since R_t and M_t grow at the same rate, the average number of R&D workers per firm $\bar{R}_t$ is constant over time. This is also true in the base model, where both the number of firms and number of researchers are fixed. In terms of the return to R&D, the only difference with the main text is that L_t/M_t is constant, whereas the baseline model had only L_t, which was growing. The size of the labor force, however, has no impact on the relative return to improving the two types of technology.[49] Taking $\bar{R}$ as given, therefore, the incentives for R&D are essentially the same as in the main text,

$$\bar{R}_{E,t} = \frac{\sqrt{\frac{\tau_t p_{E,t}}{A_{E,t-1}}} \sqrt{\frac{1}{\alpha(1-\eta_t^S)} \left[\frac{\eta_E \bar{R}_{E,t}^{-\lambda}}{\eta_N (\bar{R} - \bar{R}_{E,t})^{-\lambda}} + \eta_E \bar{R}_{E,t}^{-\lambda} - \eta_E \bar{R}_{E,t}^{1-\lambda}\right] + (1 + \eta_E \bar{R}_E^{1-\lambda})} - 1}{\eta_E \bar{R}_{E,t}^{-\lambda}}, \tag{B.79}$$

$$\bar{R}_{N,t} = \bar{R} - R_{E,t}, \tag{B.80}$$

with the only difference being that $\bar{R}$ replaces the normalized value of one. This immediately

[49] It will matter for the overall return to R&D, which I discuss in the next subsection.

implies that $p_{E,t}/A_{E,t}$ is constant on the BGP, which gives a constant energy expenditure share.

With a constant energy expenditure share and growth rates of technology, the rest of the model is identical to the baseline case (which, in turn, a version of the neoclassical growth model with time-varying TFP growth off of the BGP). Appendix section B.4.2 demonstrates that this model has a standard BGP that matches the usual Kaldor facts.

Wage growth on the BGP. — Now, I show that constant allocations are consistent with free mobility between sectors. In particular, I will show that wages in each sector grow at rate $(1+g_N^*)^{\frac{\alpha}{1-\alpha}}-1$, which is also the growth rate of output per capita. Equations (B.75) and (B.81) are identical to the baseline model. They imply that the wages of production workers grow at the rate of output per capita. The intuition is straightforward as these are standard equations from the neoclassical growth model once $p_{E,t}/A_{E,t}$ is constant. On a BGP, research allocations, technology growth rate, the return to capital investment will all be constant. With constant allocations, L_t/M_t is constant. Putting these together, it is immediate that the wage paid to scientists, p_t^R also grows at $(1+g_N^*)^{\frac{\alpha}{1-\alpha}}-1$ on the BGP. Finally, payments to fixed cost workers are determined by the free entry condition. In particular, wages paid to fixed cost workers are proportional to the value of operating a capital good firm conditional on technology ($\bar{\pi}_{X,t}$) minus R&D costs,

$$w_t^F = \frac{1}{\varphi}\left(\bar{\pi} - p_t^R\right) \tag{B.81}$$

$$\propto A_{N,t}(i)^{\frac{\alpha}{1-\alpha}}. \tag{B.82}$$

So, the growth rate of wages is the same for all occupations. The labor allocation is then determined by the market clearing condition (B.73) and the free mobility condition, $w_t = w_t^F = p_t^R$. There is no closed form solution for the resulting allocation.

So, the extended model explains the same BGP facts while allowing for labor reallocation between production and research. Following the existing quantitative literature on the macroeconomics of climate change – e.g., Acemoglu et al. (2016) and Fried (2018) – the analyses conducted in this paper use the baseline model, which does not allow for this reallocation. The extended model demonstrates that the core intuition of the baseline model holds even in this richer setting.

B.9 R&D Spillovers and Differential Productivity Growth

Data on energy use and productivity growth indicate that technological progress in energy efficiency is slower than overall technology growth ($g_E^* < g_N^*$). Matching this fact places restrictions on the functional form for equation (10), the law of motion for technology. In particular, it rules out R&D spillovers between technologies and semi-endogenous growth specifications.

To see this, consider the following alternate R&D specification

$$A_{E,t}(i) = \left[1+\eta_E R_{E,t}(i)^{1-\lambda}\right] A_{E,t-1}^{1-\phi} A_{N,t-1}^{\phi}, \tag{9'}$$

where $\phi \in (0,1)$ and the law of motion for $A_{N,t}(i)$ has a symmetric form. The degree of spillovers is decreasing in ϕ, and $\phi = 0$ corresponds to the baseline case of no spillovers.

Following the same steps as in Appendix Section B.2, the research arbitrage condition is

$$1 = \frac{\alpha\left[1 - \frac{p_{E,t}}{A_{E,t}}\right]}{\frac{p_{E,t}}{A_{E,t}}} \cdot \frac{\eta_N R_N^{-\lambda}}{\eta_E R_E^{-\lambda}} \cdot \frac{A_{E,t} A_{N,t-1}^{1-\phi} A_{E,t-1}^{\phi}}{A_{N,t} A_{E,t-1}^{1-\phi} A_{N,t-1}^{\phi}}. \tag{B.83}$$

Clearly, the LHS of this equation is constant. The first two terms on the RHS are the same as in the baseline model. To match the trendless energy expenditure share, the first term on the RHS must be constant on the BGP. Similarly, for productivities to grow at a constant rate, research allocations must be constant on the BGP, which implies that the second term is constant.

It is straightforward to re-write the last term as $\frac{1+g_{E,t}}{1+g_{N,t}} \cdot \frac{A_{E,t-1}^{2\phi}}{A_{N,t-1}^{2\phi}}$. Since technological progress is constant on the BGP, this term could only be constant if $\frac{A_{E,t-1}}{A_{N,t-1}}$ is constant or $\phi = 0$. As explained above, the former option is inconsistent with the data. So, the data are inconsistent with a model that includes research spillovers.

It is also straightforward to show that the same limitation holds for a semi-endogenous specification. In this case,

$$A_{E,t}(i) = \left[1 + \eta_E R_{E,t}(i)^{1-\lambda}\right] A_{E,t-1}^{\phi}, \quad \phi \leq 1 \tag{9''}$$

and

$$1 = \frac{\alpha\left[1 - \frac{p_{E,t}}{A_{E,t}}\right]}{\frac{p_{E,t}}{A_{E,t}}} \cdot \frac{\eta_N R_N^{-\lambda}}{\eta_E R_E^{-\lambda}} \cdot \frac{A_{E,t} A_{N,t-1}^{\phi}}{A_{N,t} A_{E,t-1}^{\phi}}. \tag{B.84}$$

Following the same steps as above rules out the case where $\phi < 1$.