Dynamic Consequences of Monetary Policy for Financial Stability

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Abstract

We theoretically investigate the state-dependent effects of monetary policy on macroeconomic instability. In the model, banks borrow using deposits and allocate resources to productive projects. Because banks do not actively issue equity, aggregate outcomes depend on the level of equity in the financial sector. Carefully targeted monetary policy can improve stability by increasing the rate of bank equity growth, and improve allocations by encouraging leverage when intermediation is needed. A fed put is generally stabilizing, but the marginal impact of a rate cut depends on the state of the economy. The effectiveness of monetary policy depends on the extent to which rate cuts pass through to bank returns. When banks are relatively well-capitalized, rate cuts primarily decrease banks' returns. In terms of welfare, the costs of "leaning against the wind" generally outweigh the benefits, but a fed put can improve outcomes if the costs of deviating from the inflation target are sufficiently small.

Keywords: Monetary policy, Leaning against the wind, Financial stability, Macroeconomic instability, Banks, Liquidity.

JEL classification: E44, E52, E58, G01, G12, G20, G21.

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1 Introduction

Economists increasingly debate the extent to which monetary policy should be used to stabilize the financial system. On one side is the "BIS view" that monetary authorities should systematically "lean against the wind" ("LAW"), raising rates to mitigate overheating in the the financial sector.[1] The argument for LAW is that raising the cost of intermediation in good times, thus curtailing credit growth or asset bubbles, can decrease the probability of (extremely costly) financial crises (e.g., Adrian and Liang, 2016; Adrian and Duarte, 2016). However, there are prevailing doubts that the benefits of LAW outweigh the costs (notably Svensson (2017)). To improve financial stability, LAW causes a weaker economy in good times, and if a crisis hits, the economy may enter a worse downturn because it started from a weaker position. As economists debate the costs and benefits of LAW, it is also widely recognized that central bankers will likely pursue aggressive policies to stabilize the financial system during downturns—i.e., enacting a "fed put" to cut borrowing costs. Critics worry that it encourages moral hazard in the form of excessive risk-taking and leverage because participants in financial markets expect that the central bank will step in during downturns (see Blinder and Reis (2005)). By causing riskier behavior throughout the business cycle, the fed put may backfire and heighten the probability of financial crisis.

But because LAW and the fed put occur at different points in the cycle, they are not opposing policies. Does considering the joint policies of LAW and fed put change the effects of monetary policy on financial stability? Should cost-benefit analyses consider state-dependent variations in effectiveness?

To answer these questions, we use a continuous-time stochastic general equilibrium model in which banks allocate resources to productive projects, and bank deposits provide liquidity services, following Phelan (2016). Banks can invest in certain projects more efficiently than households can directly, but banks can issue only risk-free debt and not equity. As a result, banks invest more when they have more equity, and the economy's resources are allocated more efficiently when financial-sector net worth is high. Importantly, in crisis times households do invest directly in bank-dependent projects. The model builds on Brunnermeier and Sannikov (2014), which demon-

[1]Proponents of this view include BIS (2014, 2016), Borio (2014); Borio et al. (2018) and Juselius et al. (2017).

strates the inherent instability of economies with financial sectors and the pecuniary externalities caused by equity constraints. To this model we add the model of monetary policy transmission from Drechsler et al. (2018)[2], in which monetary policy determines the liquidity premia on banks' investments, with implications for risk premia. In our model, monetary policy can affect the return on banks' investments, the rate at which banks build up equity, and the amount of leverage banks use. Importantly, however, how changes in monetary policy affect these variables varies systematically with the state of the economy. Accordingly the implications of LAW for stability depends on whether the central bank also pursues a fed put.

To understand the intuition for our results, it is helpful to consider two extreme benchmarks for how monetary policy could pass through into "prices and quantities." On the one hand, in a frictionless economy, a decrease in nominal rates would merely decrease the nominal rate of return on banks' investment exactly one-for-one, so that the profitability of investing would not change. In this case, there would be no change in banks' leverage, the rate of equity growth, or the volatility in the economy. On the other hand, if a decrease in nominal rates did not pass through to returns at all, then the profitability of banking would increase (funding is cheaper relative to investment return), banks would increase their leverage, and the changes in funding costs and leverage would have implications for the rate of growth and volatility of bank equity.

By affecting the risk premium of banks' investment, monetary policy changes the frequency and duration of good and bad outcomes. We solve for the global dynamics of the economy to demonstrate the stability consequences of various monetary policies across the state space. We show that whether the effect of monetary policy is more like the first or second extreme depends on the state of the economy in which monetary policy is changing. In particular, how bank leverage varies over the cycle primarily depends on how monetary policy *changes* over the cycle, and whether monetary policy becomes accommodating "early" or "late." In our model, when banks are well-capitalized, banks are the marginal investors in bank-dependent investments, and changes

[2]We choose this mechanism rather than assuming nominal rigidities because Van der Ghote (2018) shows that, in a New Keynesian version of the model in Brunnermeier and Sannikov (2014), it is still optimal for monetary policy to mimic the natural rate. The reason is that monetary policy in that model affects price dispersion, not banks' risk-taking incentives. Because we want to study the impact of monetary policy on macroeconomic instability, we need an alternative to nominal rigidities.

in monetary policy primarily affect bank returns, with little effect on bank leverage. However, when banks have very low levels of capital, households may be the marginal investors in bank-dependent investments, in which case changing banks' funding costs will not have a large effect on their returns but will instead encourage banks to use more leverage and enable banks to more quickly rebuild equity. Thus, our analysis allows us to consider how monetary policy endogenously changes the probability of a crisis—the primary proposed benefit of LAW—and whether a fed put increases macroeconominic instability.

Our analysis has positive and normative implications for the dynamic consequences of monetary policy for financial stability. First, a policy of leaning against the wind without a fed put *decreases* financial stability, while a combination of LAW and put increases stability. The key reason is that the fed put bolsters the expected rate at which banks recapitalize in bad times and reduces asset price volatility nearly everywhere. However, the effects of monetary policy on leverage, volatility, and stability are state-dependent, so the power of monetary policy is contingent on the timing of interest rate movements. Second, the normative implications of LAW depend on when welfare is measured (i.e., the initial condition). In contrast, for a given level of rates during good times, a fed put generally increases welfare so long as such a policy does not create large inflation losses.

Opponents of LAW emphasize the importance of macroprudential regulations as alternatives to interest rate policy for reducing financial fragility. To evaluate how significant these measures are relative to monetary policy, we compare our results with a modified variant of the model in Phelan (2016). In that paper, macroprudential regulations took the form of fixed leverage constraints, which were shown to improve welfare by trading off flow outcomes for greater stability. We find that while macroprudential regulation is much more successful at reducing the probability of crisis, monetary policy has much larger quantitative effects on welfare.[3] Therefore, although our analysis suggests that LAW is inadvisable without a fed put or if inflation costs are too high, the "BIS view" may correctly emphasize the importance of considering monetary policy and macroprudential policy together to address macroeconomic stability and the effects of financial crises.

[3]Moreover, leverage constraints may not be time-consistent since they worsen flow outcomes in bad times. In contrast, the fed put lowers borrowing costs and bolsters asset prices when banks are poorly capitalized.

Related Literature Methodologically, our paper follows the stochastic continuous-time macro literature, pioneered by Brunnermeier and Sannikov (2014, 2015, 2016) and He and Krishnamurthy (2012, 2013, 2014), who analyze the nonlinear global dynamics of economies with financial frictions, building on seminal results from Bernanke and Gertler (1989); Kiyotaki and Moore (1997); Bernanke et al. (1999). Within this literature, we combine the models of Phelan (2016) and Drechsler et al. (2018) to study how monetary policy affects global dynamics.[4]

The macroeconomic framework in Phelan (2016) is most closely related to Brunnermeier and Sannikov (2014) and makes three modifications to their framework. First, the model contains two goods, so that policy may affect the allocation of resources and the returns on different investments. Second, banks are owned by households (banks are not competing agents) and have a comparative advantage at investing in one sector ("bank-dependent"). Third, bank deposits provide liquidity value, which motivates steady state leverage. Phelan (2016) studies how macroprudential policies (i.e., leverage limits) can improve welfare by increasing stability. We study how monetary policy affects financial stability.

Drechsler et al. (2018) develop a dynamic asset pricing model in which monetary policy affects the risk premium component of the cost of capital. They consider a model in which risk-tolerant agents (banks) take deposits from risk-averse agents to buy an asset. Lower nominal rates make liquidity cheaper and raise leverage, resulting in lower risk premia and higher asset prices and volatility. Drechsler et al. (2017) empirically confirm that this mechanism exists by showing increases in the nominal rate induce big inward shifts in the supply of retail bank deposits, a large and important class of liquid assets. We embed their model of monetary policy transmission into the Phelan (2016) model to illustrate the state-dependent effects of monetary policies driven by the role of banks in allocating resources to bank-dependent investments. Importantly, monetary policy has different effects on stability depending on the extent to which the returns on different types of investments are affected.

In our model, monetary policy affects financial stability by changing the endogenous evolution of banks' equity levels. The key assumption that equity is "sticky" or "slow-moving" is closely

[4]There is a rich asset pricing literature within this methodology. As examples, see Adrian and Boyarchenko (2012), Moreira and Savov (2014), Gârleanu and Panageas (2015), Gârleanu et al. (2015).

related to He and Krishnamurthy (2012, 2013). The assumption that bank equity is "sticky" is empirically supported by Acharya et al. (2011), which shows that the capital raised by banks during the crisis was almost entirely in the form of debt and preferred stock and not in the form of common equity. Adrian and Shin (2010, 2011) provide evidence that the predetermined balance sheet variable for banks and other financial banks is equity, not assets. Our results generalize so long as banks do not issue equity too frequently. Relatedly, Gambacorta and Shin (2016) provide evidence that bank capital matters for monetary policy transmission.

Stein (2012) provides a model in which monetary policy affects financial stability by affecting private money creation. Farhi and Tirole (2012) consider how time-inconsistent monetary policy can provide incentives for maturity mismatch and correlated portfolios. In our model we take maturity mismatch and correlated risks as given and then ask, in light of these features, how changes in monetary policy affects stability. Diamond and Rajan (2012) emphasize that low interest rate policies may encourage excessive leverage. Interestingly, because of the general equilibrium effects in our model, a fed put increases leverage when rates are low but not before. Furthermore, the increase in leverage when rates are low is stability improving, even though our model leads to excessive leverage (see Phelan, 2016).

2 The Baseline Model

The economy is populated by households and banks, which are owned by households. There is a single factor of production that can be used to produce two intermediate goods. Banks have an advantage for producing one intermediate good and households for the other. As a result, output and growth depend endogenously on capital ownership. The financial friction is that equity issuance is costly, and thus outcomes will depend on the level of equity in the banking sector. The model combines, with modifications, elements of the models in Brunnermeier and Sannikov (2014), Drechsler et al. (2018), and Phelan (2016).

2.1 Technology, Environment, and Markets

Time is continuous and infinite, and there are aggregate productivity shocks that follow a Wiener process. One factor of production, capital, can be used to produce two types of intermediate goods at unit rate. The effective capital quantity y_t evolves according to equation (1),

$$\frac{dy_t}{y_t} = g_y dt + \sigma dW_t, \tag{1}$$

where dW_t is an exogenous standard Brownian motion and g_y depends on who manages capital and what it is used to produce. The values of g_y, given in Table 1, imply that banks are comparatively better at managing good-1 production and households are better at managing good-2 production. We interpret good-1 production as bank-dependent investments. We define the parameter restriction on g_B more clearly later in this section.

Table 1: Expected capital productivity growth rates by agent and good produced.

g_y	Good 1	Good 2
Households	$g-\ell$	g
Banks	g_B	g_B

Denote by Y_t the stock of effective capital at time t, which is also the flow production of goods at time t. The consumption good is produced using goods 1 and 2 according to

$$C_t = Y_{1t}^{\frac{1}{2}} Y_{2t}^{\frac{1}{2}},$$

where C_t is the quantity of the consumption good, Y_{jt} is the quantity of good j (equivalently the quantity of capital used to produce good j). Standard static optimization implies that the equilibrium prices of intermediates are given by

$$p_{1t} = \frac{1}{2}\left(\frac{Y_{2t}}{Y_{1t}}\right)^{\frac{1}{2}} p_t, \quad p_{2t} = \frac{1}{2}\left(\frac{Y_{1t}}{Y_{2t}}\right)^{\frac{1}{2}} p_t,$$

where p_t is the price of consumption. Let $\lambda_t = \frac{Y_{1t}}{Y_t}$ be the fraction of capital cultivating good 1.

Then the real prices of intermediate goods ($P_{jt} = p_{jt}/p_t$) are

$$P_{1t} = \frac{1}{2}\left(\frac{1-\lambda_t}{\lambda_t}\right)^{\frac{1}{2}}, \quad P_{2t} = \frac{1}{2}\left(\frac{\lambda_t}{1-\lambda_t}\right)^{\frac{1}{2}}.$$

Capital is traded in a perfectly competitive market at a real price Q_t. We postulate that the real capital price (the "asset price") follows the process

$$\frac{dQ_t}{Q_t} = \mu_{Q,t}\,dt + \sigma_{Q,t}\,dW_t, \tag{2}$$

which will be determined endogenously in equilibrium. The return to owning capital includes the value of the output produced and the capital gains on the value of the capital. By Ito's Lemma, the rate of return to agent i using capital to produce good j is

$$dr_t^{ij} = \left(\frac{P_{jt}}{Q_t} + g_y + \mu_{Q,t} + \sigma\sigma_{Q,t}\right)dt + (\sigma + \sigma_{Q,t})dW_t,$$

where g_y is appropriately defined for agent i. The volatility of returns on investments is $\sigma + \sigma_{Q,t}$, which includes fundamental risk σ and endogenous price risk $\sigma_{Q,t}$. Denote by dr_t^{bj} and dr_t^{hj} the returns respectively to banks and households from owning capital to produce good j. To simplify notation, denote the expected returns as: $\mathbb{E}[dr_t^{bj}] = \bar{r}_t^{bj}dt$, $\mathbb{E}[dr_t^{hj}] = \bar{r}_t^{hj}dt$.

Finally, there is a market for risk-free deposits, which are in zero net-supply with endogenous return r_t.

2.2 Households

There is a continuum of risk-neutral households denoted by $h \in [0,1]$ with initial wealths $n_{h,0}$. Households have the discount rate r, may consume positive and negative amounts (though in equilibrium their consumption will always be positive), and have liquidity in the utility function with constant marginal utility over bank deposits. Lifetime utility is given by

$$V_\tau = \mathbb{E}_\tau\left[\int_\tau^\infty e^{-r(t-\tau)}(c_{h,t} + \phi_L\delta_{h,t})\,dt\right],$$

where $c_{h,t}$ is household flow consumption, $\delta_{h,t}$ are bank deposits, and $\phi_L > 0$ is the liquidity preference parameter. It follows that households require an expected return of r on any real investment and a return of $r - \phi_L$ on deposits due to liquidity value.[5]

We model the liquidity value of bank deposits directly in the utility function as a modeling convenience. Deposits have liquidity values for a variety of reasons outside of the model, which we leave out to simplify exposition.[6] Because our objective is not to study liquidity provision or demand, we impose constant marginal utility for deposits to simplify the model and so that no endogenous value affects liquidity value or deposit rates. As we discuss below, this also allows the interpretation that changes in monetary policy represent deviations from a baseline desired interest rate, perhaps determined based on a standard Taylor rule.

2.3 Banks

There is a continuum of banks, denoted by $b \in [0,1]$, with book value ("equity") $n_{b,0}$. Banks invest in capital and issue deposits. Banks are owned by households, who choose dividend payouts, the level of deposits, the level of liquid reserves, and the portfolio weights on capital used by banks to produce goods 1 and 2. Because of un-modeled financial frictions, banks are subject to two constraints. First, equity issuance is infinitely costly (i.e., dividends must be positive). Second, the value of banks' assets minus liabilities $n_{b,t}$ cannot become negative (bankruptcy).

The banks' objective is to maximize the present value of dividends discounted at rate r (households' time preference) subject to its constraints. Because banks can borrow using debt at a real interest rate $r^L = r - \phi_L < r$, banks will never choose a capital structure that is completely equity. And because banks cannot have negative equity, to avoid bankruptcy banks will never choose a capital structure that is completely debt. To reduce the advantage of banks, we assume that $g_B = g - \phi_L$ so that banks have a net advantage at cultivating good 1 but not at cultivating good 2.

Following Drechsler et al. (2018) we assume deposits are subject to funding shocks, which are

[5] In Appendix 4.2 we suppose that households may also suffer welfare losses from deviations of inflation from its target level. Adding ad-hoc inflation losses in the way we do does not change any of the positive implications of the model.

[6] See for example Diamond and Dybvig (1983), Gorton and Pennacchi (1990), or Lagos and Wright (2005).

modeled as a Poisson process J_t with constant intensity χ. J_t is an aggregate shock, and when J_t realizes, banks must immediately redeem a fraction $\frac{\kappa}{1+\kappa}$ of their deposits, where $\kappa > 0$. Only a fraction $1-\rho \in (0,1)$ of capital's value can be recovered quickly enough to absorb a funding shock. Banks may self-insure by holding liquid assets, which can be liquidated without causing fire sales when a funding shock realizes.

2.4 Monetary Policy: Nominal Rates, Inflation, and Liquidity Premia

We model monetary policy transmission following Drechsler et al. (2018). Monetary policy determines the opportunity cost of holding liquid assets rather than capital (i.e., determines the liquidity premium). Two types of liquid assets exist: government bonds and central bank reserves. To capture the money multiplier of reserves, we assume that each dollar of reserves yields $m > 1$ effective liquid assets. The central bank can create and withdraw reserves by exchanging government bonds through open market operations.

Let G_t and M_t be the total dollar value of government bonds and reserves in the economy (what matters is the total value of all bonds and reserves rather than the actual quantity of their supply), and let s_t be the value of a dollar in consumption units. Letting reserves be the numeraire, s_t becomes the inverse price level, and the real value of liquid assets held by banks, measured in units of government bonds and scaled by aggregate wealth, is

$$S_t = \frac{s_t(G_t + (m-1)M_t)}{Q_t Y_t}.$$

The remaining liquidity is given by the value of government bonds held by the central bank, $s_t M_t$.

Inflation and the Nominal Rate We assume inflation is locally deterministic, i.e.

$$-\frac{ds_t}{s_t} = \frac{dp_t}{p_t} = \pi_t \, dt.$$

Since the rate on deposits pins down the risk-free interest rate, the nominal interest rate is

$$i_t = r - \phi_L + \pi_t.$$

The central bank targets i_t, and we assume that i_t depends on a state variable η_t (i.e., $i_t = i(\eta_t)$) to be described later.

We interpret changes in monetary policy in this model as deviations from a desired rate $\bar{i} = r - \phi_L + \bar{\pi}$, where $\bar{\pi}$ is the inflation target and r may be the desired real interest rate coming out of a New Keynesian model. Thus, deviations in i_t from $\bar{i}$ represent changes in the inflation target in response to the state of the financial sector. That said, in our model the state of the financial sector will determine an "output gap" due to misallocation of resources, a distinct mechanism from the standard New Keynesian model.

Liquidity Premia The liquidity premium on reserves is the opportunity cost of holding liquid assets. In equilibrium, this premium is determined by forgone returns from holding liquid assets rather than deposits, the next best investment. Because reserves pay no interest, their return is equal to their capital gain, so the liquidity premium that deposits earn is

$$r_t - \frac{ds_t}{s_t} = r_t + \pi_t = i_t,$$

which is precisely the nominal interest rate. The premium on government bonds is similarly determined. Let $r_{g,t}$ be the real interest rate on government bonds. Since these bonds produce liquid assets at a supply $1/m$ of reserves, the liquidity premium on government bonds is

$$r_t - r_{g,t} = \frac{1}{m} i_t$$

We assume that government bonds are backed by deposits that banks hold with the Treasury and earn the rate $r_{g,t}$. Thus, the spread $r_t - r_{g,t}$ reflects the returns per bond forgone by banks in order to hold liquidity.

Through the conduct of monetary policy and the management of government bonds, seigniorage profits are earned. Scaled by aggregate wealth, the seigniorage accrues at the rate

$$\frac{s_t G_t}{Q_t Y_t}(r_t - r_{g,t}) + \frac{s_t M_t}{Q_t Y_t}\left(r_{t,g} - \frac{ds_t}{s_t}\right) = \frac{s_t(G_t + (m-1)M_t)}{Q_t Y_t}(r_t - r_{g,t}) = \frac{S_t}{m} i_t.$$

On the left, we have the returns on Treasury and central bank accounts. For each government bond, the Treasury earns r_t and pays out $r_{g,t}$. Since central bank reserves are government bonds, they earn $r_{g,t}$ but pay out the capital gains on reserves to banks. In the middle, we have the seigniorage profits earned by the government due to banks holding liquid assets rather than deposits. The $(m-1)M_t$ term captures the fact that for each dollar of reserves, the government earns seigniorage on $(m-1)$ liquid assets held by banks. To close the model, we assume the seigniorage is distributed to agents according to their wealth, so the distribution of wealth is unaffected by seigniorage, and the government maintains zero net worth.

2.5 Banks' Problem

Let $x_t = (x_{k1,t}, x_{k2,t}, x_{G,t}, x_{M,t})$ be portfolio weights (summing to one) on capital used for good 1, capital used for good 2, government bonds, and reserves. We can simplify the banks' problem by noting that in equilibrium government bonds and reserves are perfectly substitutable. Therefore, $x_{L,t} \equiv x_{G,t} + m x_{M,t}$ describes the effective share of wealth held in government bonds. We will also use the shorthand $x_{k,t} \equiv x_{k1,t} + x_{k2,t}$ to refer to banks' share of wealth invested in capital.

Formally, banks solve the problem

$$\max_{\{x_t, d\zeta_t\}} U_\tau = \mathbb{E}_\tau \left[\int_\tau^\infty e^{-r(t-\tau)}\, d\zeta_t \right],$$

subject to

$$\begin{aligned} \frac{dn_{b,t}}{n_{b,t}} = &\left(r_t - \frac{x_{L,t}}{m} i_t + \frac{S_t}{m} i_t\right) dt + (x_{k1,t}\, dr_t^{b1} + x_{k2,t}\, dr_t^{b2} - x_{k,t} r_t\, dt) - \frac{d\zeta_t}{n_{b,t}} \\ &- \frac{\rho}{1-\rho} \max\left\{ \frac{\kappa}{1+\kappa}(x_{k,t} + x_{L,t} - 1) - x_{L,t}, 0 \right\} dJ_t, \end{aligned} \tag{3}$$

$$n_{b,t}, x_{k1,t}, x_{k2,t}, x_{L,t}\, d\zeta_t \geq 0. \tag{4}$$

Banks earn the deposit rate, pay the liquidity premium on government bonds, receive seigniorage payments from the government, earn the risk premium on capital holdings, and pay dividends at rate $d\zeta_t$.The second line of (3) reflects the exposure of banks to funding shocks given their portfolio.

By homogeneity and price-taking, the maximized value of a bank with equity $n_{b,t}$ can be written as

$$\theta_t n_{b,t} \equiv \max_{\{x_t \geq 0, d\zeta_t\}} \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} d\zeta_s \right], \tag{5}$$

where θ_t is the marginal value of equity, i.e., the proportionality coefficient that summarizes how market conditions affect the value of the bank's value function per dollar of equity. The marginal value of equity equals 1 plus the multiplier on the equity-issuance constraint and reflects the aggregate condition of the financial sector.

Given the following conditions, it is optimal for banks to fully self-insure in equilibrium.

Lemma 1. *Suppose χ and ρ are such that, for all i_t,*

$$\chi \frac{\rho}{1-\rho} \frac{\kappa}{1+\kappa} \geq \frac{\kappa}{m} i_t \equiv LP_t, \tag{6}$$

where $LP_t \equiv \frac{\kappa}{m} i_t$ is the liquidity premium at interest rate i_t. Then banks fully self-insure, and their liquidity demand is given by

$$x_{L,t} = \max\left\{\kappa(x_{k,t}-1), 0\right\}.$$

We can further characterize the optimality conditions in the following way.

Proposition 1. *Consider a finite process*

$$\frac{d\theta_t}{\theta_t} = \mu_{\theta,t}\, dt + \sigma_{\theta,t}\, dW_t, \tag{7}$$

with $\sigma_{\theta,t} \leq 0$. Then $\theta_t n_t$ represents the maximal future expected payoff that a bank with book value n_t can attain, and $\{x_t, d\zeta_t\}$ is optimal if and only if

1. *$\theta_t \geq 1\ \forall t$, and $d\zeta_t > 0$ only when $\theta_t = 1$,*
2. *$\mu_{\theta,t} = \phi_L - \left(\frac{\kappa + S_t}{m}\right) i_t$,*

3. $\mathbb{E}[dr_t^{bj}] - r_t - LP_t \leq -\sigma_{\theta,t}(\sigma + \sigma_{Q,t})$, *with strict equality when* $x_{jk,t} > 0$,

4. *The transversality condition* $\mathbb{E}[e^{-rt}\theta_t n_t] \to 0$ *holds under* $\{x_t, d\zeta_t\}$.

Hence, $RP_t \equiv -\sigma_{\theta,t}(\sigma + \sigma_{Q,t})$ represents the bank's required risk premium (or instantaneous level of risk aversion), which must at least equal the expected excess return over the liquidity premium. Cutting interest rates increases the drift of θ_t, and banks will not pay dividends when $\theta_t \geq 1$.[7]

2.6 Equilibrium Asset Pricing

Since we look for an equilibrium where the risk premium and liquidity premium are both non-negative, banks will never produce good 2. Thus, households will always produce good 2 and sometimes produce good 1. Because households require a return of r on real investments, it follows that household returns satisfy

$$\frac{P_{2t}}{Q_t} + g + \mu_{Q,t} + \sigma\sigma_{Q,t} = r, \tag{8}$$

$$\frac{P_{1t}}{Q_t} + g - \ell + \mu_{Q,t} + \sigma\sigma_{Q,t} \leq r, \tag{9}$$

where the inequality reflects that households may not always produce good 1 in equilibrium. From the banks' investment in good 1 we have

$$\frac{P_{1t}}{Q_t} + g + \mu_{Q,t} + \sigma\sigma_{Q,t} = r + RP_t + LP_t. \tag{10}$$

When households do not produce good 1, then banks and households specialize in their respective sectors. Taking the difference between the equations (8) and (10), we obtain a market-clearing condition for capital allocations:

$$RP_t + LP_t = \frac{P_{1t} - P_{2t}}{Q_t}. \tag{11}$$

When this is the case, changes in the liquidity premium may affect the difference in returns between

[7] Furthermore, θ_t can never be less than one because banks can always pay out the full value of equity, guaranteeing a value of at least $n_{b,t}$.

goods 1 and 2. In contrast, when households produce good 1, we have $\frac{P_{1t}-P_{2t}}{Q_t} = \ell$, which implies

$$RP_t + LP_t = \ell. \tag{12}$$

In other words, the sum of the banks' risk premium and the liquidity premium equals the household efficiency loss from investing in good 1. The differences between equations (11) and (12) provide a crucial insight. When households do not produce good 1, banks are the marginal investors in the intermediation sector. Accordingly, a decrease in interest rates might, all else equal, decrease the relative return between sectors 1 and 2 ($P_1 - P_2$ decreases). However, when households produce good 1, households are the marginal investor in the intermediation sector. In this case, a decrease in interest rates must increase banks' equilibrium risk-premium, which will occur through higher leverage.

2.7 Equilibrium

A competitive equilibrium is characterized by market price for the risky asset, together with portfolio allocations and consumption decisions such that given prices, agents optimize and markets clear. Since banks are subject to equity issuance frictions, equilibrium will depend on banks' equity levels and monetary policy will have scope to affect equilibrium.

We solve for the global equilibrium dynamics using the methods in Brunnermeier and Sannikov (2014). With limited ability to issue equity, banks' decisions depend on their level of equity, and so equilibrium depends on banks' aggregate level of equity. Define $N_t = \int n_{b,t} db$ as aggregate bank equity. Because capital grows geometrically and the bank problem is homogenous, the equilibrium state-variable of interest is aggregate bank equity as a fraction of total value of capital, or a variant of the "wealth distribution." We thus use the following state variable:

$$\eta_t = \frac{N_t}{Q_t Y_t}.$$

Hence, equilibrium consists of a law of motion for η_t, and asset allocations and prices as functions of η. The asset prices are $Q(\eta)$ and $\theta(\eta)$, and the flow allocations and goods prices are

$\lambda(\eta)$, $\psi(\eta)$, $P_1(\eta)$, $P_2(\eta)$. We derive the evolution of η_t using Ito's Lemma and the equations for returns and budget constraints.[8]

Lemma 2. *The equilibrium law of motion of η will be endogenously given as*

$$\frac{d\eta_t}{\eta_t} = \mu_{\eta,t}dt + \sigma_{\eta,t}dW_t + d\Xi_t, \tag{13}$$

where $d\Xi_t$ is an impulse variable creating a regulated diffusion. Furthermore,

$$\mu_{\eta,t} = \frac{P_{1t}}{Q_t} + (\lambda_t - \psi_t)\ell - (1-\psi_t)\phi_L + (\psi_t - \eta_t)\frac{\kappa}{m}i_t - \left(\frac{\psi_t}{\eta_t} - 1\right)(\sigma + \sigma_{Q,t})(\sigma_{\theta,t} + \sigma + \sigma_{Q,t}),$$
$$\sigma_{\eta,t} = \left(\frac{\psi_t}{\eta_t} - 1\right)(\sigma + \sigma_{Q,t}),$$
$$d\Xi_t = \frac{d\zeta_t}{N_t},$$

where $d\zeta_t = \int d\zeta_{b,t}db$ and $\psi_t = x_{k,t}\eta_t$ is the fraction of capital held by banks.

We solve for equilibrium by converting the equilibrium conditions into a system of differential equations ("ODE") in the asset prices Q and θ. Given $Q(\eta)$, $Q'(\eta)$, $\theta(\eta)$, and $\theta'(\eta)$ we can derive equilibrium returns and allocations then derive $Q''(\eta)$ and $\theta''(\eta)$. We solve the ODE using appropriate boundary conditions (additional details are in the appendix).

Proposition 2. *The equilibrium domain of the functions $Q(\eta), \theta(\eta)$, and $\lambda(\eta)$, $\psi(\eta)$, $P_1(\eta)$, $P_2(\eta)$ is an interval $[0,\eta^*]$. The function $Q(\eta)$ is increasing, $\theta(\eta)$ is decreasing, and the following boundary conditions hold:*

1. $\theta(\eta^*) = 1$;
2. $Q'(\eta^*) = 0$;
3. $\theta'(\eta^*) = 0$;
4. $Q(0) = \underline{q}$;
5. $\lim_{\eta \to 0^+} \theta(\eta) = \infty$.

[8] The one difference between Lemma 2 and the analogous result in Phelan (2016) is the liquidity premium term in the drift of η.

Over $[0,\eta^*]$, $\theta_t \geq 1$ *and* $d\zeta_t = 0$, *and* $d\zeta_t > 0$ *at* η^* *creating a regulated barrier for the process* η_t. *We refer to* η^* *as the* stochastic steady state. *Furthermore, there exists* $\bar{\eta} \in (0,\eta^*)$ *such that* $\psi(\eta) = \lambda(\eta)$ *for* $\eta > \bar{\eta}$ *and* $\psi(\eta) < \lambda(\eta)$ *for* $\eta \leq \bar{\eta}$.

Hence, the system ranges between 0 and η^*, at which point banks pay dividends because the marginal attractiveness of debt outweighs the marginal attractiveness of an additional unit of equity. For high levels of η, banks and households specialize in their relative investment sectors (i.e., households do not produce good 1), but below $\bar{\eta}$ households produce good 1. We say that a crisis occurs when $\eta < \bar{\eta}$. The evolution of η induces a stationary distribution (PDF) $f(\eta)$ with CDF $F(\eta)$ (the distribution $f(\eta)$ solves a Kolmogorov-Forward equation).

If the price function is twice-continuously differentiable, then the evolutions of the capital price and marginal bank value (equations (2) and (7)) are functions of η

$$\frac{dQ_t}{Q_t} = \mu_{Q,t}(\eta_t)dt + \sigma_{Q,t}(\eta_t)dW_t, \quad \frac{d\theta_t}{\theta_t} = \mu_{\theta,t}(\eta_t)dt + \sigma_{\theta,t}(\eta_t)dW_t,$$

where the drift and variance terms are determined by the derivatives of $Q(\eta)$ and $\theta(\eta)$. For the remainder of the paper, the dependence on the state-variable η_t is suppressed for notational ease.

3 Monetary Policy and Equilibrium Stability

In this section we consider the positive effects of monetary policy on macroeconomic stability. We first consider how interest rate policies affect the global dynamics of equilibrium, and we then consider the marginal impact of extending low interest rates.

We solve the model numerically using the parameters from Phelan (2016) and Drechsler et al. (2018).[9] The two most important variables are volatility and the monetary policy transmission value κ/m. The volatility of $\sigma = 2\%$ corresponds roughly to the volatility of TFP and also the typical volatility of bank assets. The value of $\kappa/m = 10.2\%$ implies the empirically plausible result that changing the nominal rate by 100bps changes the liquidity premium by roughly 10bps.

[9] In particular, $r = 4\%$, $g = 2\%$, $\sigma = 2\%$, $\ell = 1\%$, and $\phi_L = 2\%$, $m = 4$, and $\kappa = 0.4085$. Please see those papers for detail on parameter choices.

3.1 Equilibrium and Interest Rate Policy

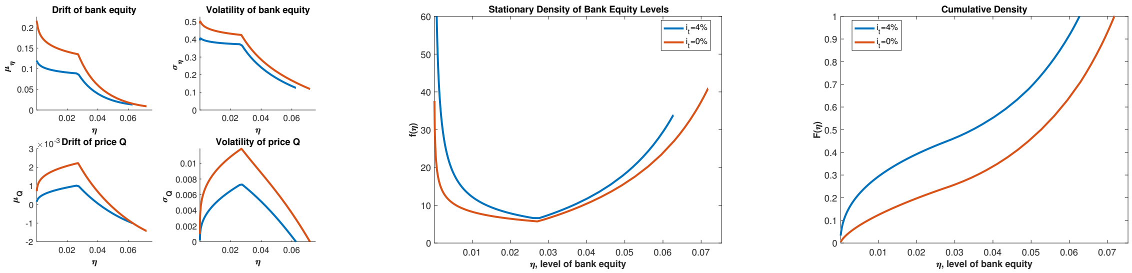
To illustrate the mechanisms of the model, we first solve for equilibrium with constant rates between 0% and 4%, and we then consider dynamic, state-dependent policies that raise or lower rates between these levels. The consequences of high or low interest rates for stability depends crucially on the global behavior of interest rate policy. The effects of interest rates on asset prices and volatility are similar to what Drechsler et al. (2018) find in their model with two agents with heterogeneous risk aversion. In contrast, in our model the stationary distribution looks quite different (bimodal rather than similar to a normal distribution), and leverage behaves slightly differently (they find that higher rates lead to lower leverage everywhere, which we do not find). Accordingly, we focus on the results concerning stability. Additional figures illustrating the properties of equilibrium are in the appendix.

Constant interest rates Figure 1(a) plots the drift and volatilities (evolutions) of Q and η. The kinks in good prices, drifts, and volatilities occur at $\bar{\eta}$, below which households produce good 1 (roughly 2.75% in these economies), thus preventing goods prices from rising if a crisis intensifies (bank equity decreases). The volatilities of the asset price and bank equity are higher with low interest rates, consistent with the standard intuition that cutting rates may be destabilizing by increasing volatility. While leverage (not shown here) is generally higher with lower rates, the difference is negligible (and leverage is actually slightly *lower* in a neighborhood around $\bar{\eta}$).

Figures 1(b) and 1(c) plot the stationary and cumulative distributions of bank equity (PDF and CDF) with high and low rates. While volatility is higher in the low-rate economy, the economy is generally more stable—spending less time at low levels of equity and more time at high levels of equity. The economy is more stable with low constant rates because the drift of bank equity is significantly higher (bank funding is cheaper owing to a lower liquidity premium). Furthermore, the steady state η^* is lower with high rates because banking is less profitable (higher liquidity costs) and thus banks pay dividends earlier.[10]

Indeed, lower constant interest rates do generally improve stability. We define stability as the

[10] However, even controlling for the change in η^*, the low-rate economy is more stable (i.e., $f(\eta)$ and $F(\eta)$ plotted against η/η^* are more stable with low rates).



(a) Evolutions of Q and η (b) PDF of η (c) CDF of η

Figure 1: Equilibrium with constant interest rates at 4% and 0%.

probability of time the economy is not in a crisis, i.e., $1-F(\bar{\eta})$.[11] Table 2 provides the stability of the economy varying interest rates from 0 to 4%. As is clear, the economy is more stable with lower rates. Taken at face value, these results with constant interest rates provide a strong argument against leaning against the wind. After all, the whole purpose of LAW is to minimize crises, but the economy is more stable with constant lower rates.[12] However, while constant low interest rates lead to a more stable distribution, volatility is higher. Perhaps policy could get the best of both world: high interest rates (at times) to generate low volatility, and low interest rates (at other times) to generate a high level of stability. Indeed, the global behavior of leverage—an important determinant of volatility and stability—depends not only on the level of rates but on how rates change. *Changes* in monetary policy across the state space can have much more significant effects on bank leverage than parallel shifts in rates. The next exercise makes clear that stability and leverage are heavily influenced by the state-dependent behavior of interest rates.

Table 2: Interest rates and stability, constant interest rates.
Higher percentage is more stable.

i_t	0%	1%	2%	3%	4%
Stability	75.05%	72.04%	68.50%	63.14%	55.32%

[11] Intuitively, a crisis occurs when banks are so constrained that they have to sell capital at fire-sale prices and households need to intermediate capital in the bank-dependent sector.

[12] As we discuss later, not only is stability higher with lower rates, but in the absence of inflation losses welfare is also higher with lower rates.

State-dependent policies We now consider the effects of state-dependent interest rate policies. We first illustrate the mechanisms by considering two dynamic policies, a "fed put" and a "fed call," each with a "strike" at $\eta^P = 2.75\%$ so that rate changes occur in the region where households produce good 1. Under the fed put, interest rates are 4% for $\eta > \eta^P$ and rates are cut to 0% for $\eta < \eta^P$; the reverse occurs under the fed call. Figure 2(a) plots the interest rate rules we consider. Importantly, while interest rates under these policies always fall between the constant levels considered above, the resulting levels of bank leverage do not fall between the levels corresponding to the constant-rate policies.

Figure 2(b) plots leverage levels across η. Notice that with constant rates leverage is hardly affected by the level of rates, and in all cases leverage levels in good times (high eta) barely change across policies. But for low levels of equity the fed put dramatically props up bank leverage and the fed call depresses leverage. These changes occur even though interest rates in each case are the same as one of the constant-rates policies.

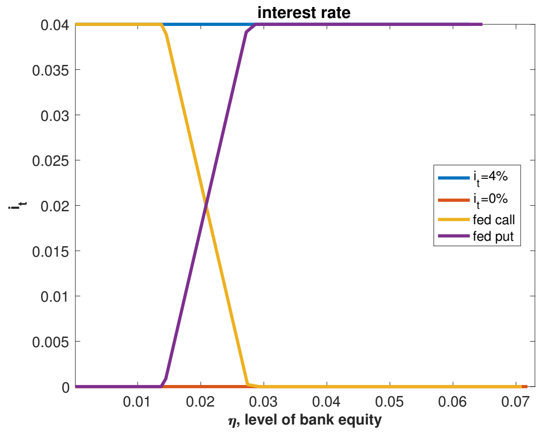
Figure 2(c) plots the drift and volatilities of the asset price and state variable. The economy is *less* volatile with a fed put. Asset price volatility is lower with the fed put—even lower than holding rates constant at 4%—and substantially lower than holding rates constant at 0%. The state variable η is only more volatile for the lowest levels of η when bank leverage increases, but this is also when the drift of the economy is the greatest, exceeding the drift when rates are held constant at 0%. The drift increases because (comparing these equilibria) bank returns are higher but funding costs are the same and banks use more leverage. Finally, as the stationary distributions in Figure 2(d) illustrate, the economy is most stable with the fed put and least stable with the fed call.[13]

Indeed, fed puts are generally stabilizing. We now consider a linear fed put policy in which $i_t = 4\%$ for $\eta > \eta^P$, but below η^P rates vary linearly to zero:

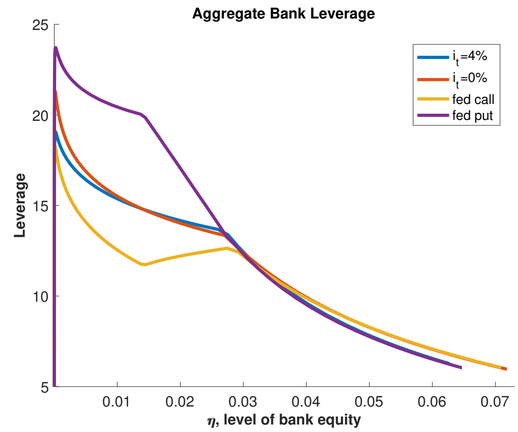
$$i(\eta) = \min\left\{4\%, \frac{4\%}{\eta^P}\eta\right\}.$$

We solve for equilibrium and then compute the average interest rate $\bar{i}(\eta^P)$ under the policy and

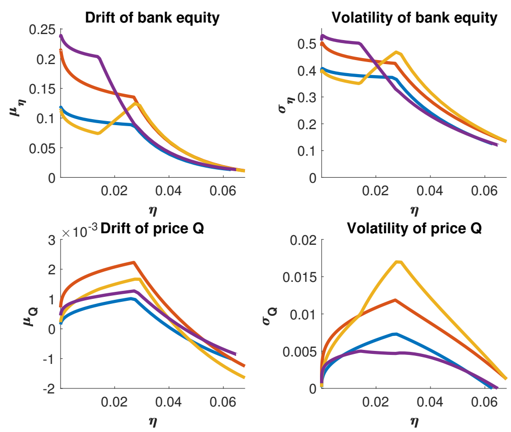
[13]Plotting probability and cumulative distributions normalizing by η^* makes the change in stability even more clear. We provide this figure in the Appendix.



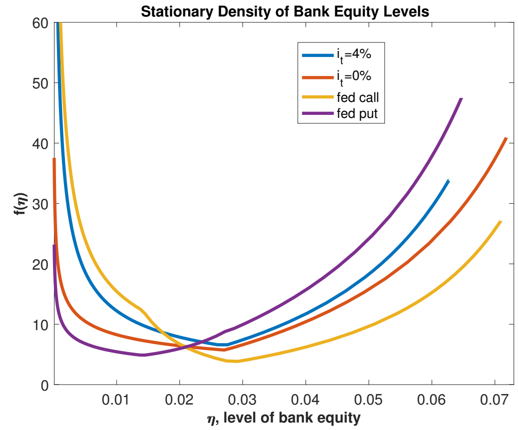
(a) Interest rates



(b) Leverage



(c) Evolutions



(d) Stationary distribution

Figure 2: Equilibrium leverage and stability with constant rates or fed put/call policies.

compare equilibria under the fed put to the constant interest rate $\bar{i}(\eta^P)$. Table 3 presents the stability of the economies, varying η^P and comparing to constant rate policies. In every case, the economy is more stable with the fed put than with constant interest rates.

Leaning against the wind When the fed intends to cut rates during crises, the effect of LAW on stability is completely different. In fact, taking the crisis policy as given, higher interest rates during good times do improve stability. We now consider piece-wise fed puts: $i_t = 0$ for $\eta < \eta^P$, and $i_t = i^{LAW}$ for $\eta > \eta^P$. We solve for equilibrium and then compute the average interest rate $\bar{i}(i^{LAW})$ under the fed put. Table 4 presents the stability of the economies, varying i^{LAW} and comparing to

Table 3: Interest rates and stability, linear fed put from 4% to zero starting at η^P, compared to constant average rates.

η^P	1.75%	2.75%	3.75%
Linear fed put	71.39%	76.69%	80.74%
Constant $\bar{i}(\eta^P)$	57.58%	60.25%	59.49%
ΔStability	13.81%	16.45%	21.24%

constant rate policies (clearly, when $i^{LAW} = 0$ the constant rate policy is the same at zero). There are two key results. First, in every case the economy is more stable with the fed put. Second, the fed put is more stabilizing the higher is i^{LAW}, implying that LAW does improve stability.

Table 4: Interest rates and stability, rates drop to zero below $\eta^P = 2.75\%$ compared to constant average rates.

i^{LAW}	0%	1%	2%	3%	4%
Fed put	75.05%	78.73%	81.76%	84.27%	86.32%
Constant $\bar{i}(\eta^P)$	75.05%	73.10%	70.11%	65.84%	60.51%

LAW is stabilizing when $\eta^P > 2.75\%$ as well, and even if interest rates in bad times are fixed at 4% instead of 0%. This suggests that increasing interest rates has a detrimental effect on stability only when higher rates increase bank funding costs during crises, precisely when increasing bank equity is most valuable (and precisely when changing funding costs has minimal effect on banks' investment returns). The effects of LAW when the fed put occurs at $\eta^P = 1.75\%$ corroborates this claim because LAW is not monotonically better for stability under this policy, as shown in Table 5. Higher rates extend into crisis times, so the effect of LAW is somewhere between the constant-rates economy and the $\eta^P = 2.75\%$-fed-put economy.

Table 5: Interest rates and stability, rates drop to zero below $\eta^P = 1.75\%$.

i^{LAW}	0%	1%	2%	3%	4%
Stability	75.05%	77.19%	77.98%	77.84%	77.24%

3.2 Marginal Impacts of Monetary Policy

While a fed put is generally stabilizing, the effects depend on when the fed put kicks in. Specifically, when the central bank cuts rates in states banks are the marginal investors in bank-dependent investments, cutting rates can decrease the Sharpe ratio for a portion of the state space. In this case, changing interest rates affects goods prices, which changes the returns banks get relative to households. However, in the region where households produce good 1, risk premium plus liquidity premium sum to ℓ, so cutting rates will in equilibrium increase banks' risk aversion. Since banks face a single balance sheet decision for risk—namely, leverage—banks' instantaneous risk aversion increases (all else equal), by increasing leverage.[14] But when households do not produce good 1, cutting rates will primarily decrease the wedge between goods prices, affecting banks' returns.

To illustrate, we first return to the fed put and call in Figure 2(a) and now also consider when the policy strike is at the higher level of $\eta^P = 4.75\%$ when only banks produce good 1. Figure 3 plots the Sharpe ratios for $\eta^P = 2.75\%$ (when households produce good 1) and $\eta^P = 4.75\%$. Importantly, the Sharpe ratio under the fed put with a high strike is substantially below what it would be with the low strike fed put, and can fall below the Sharpe with other policies (Figure 3(b)).

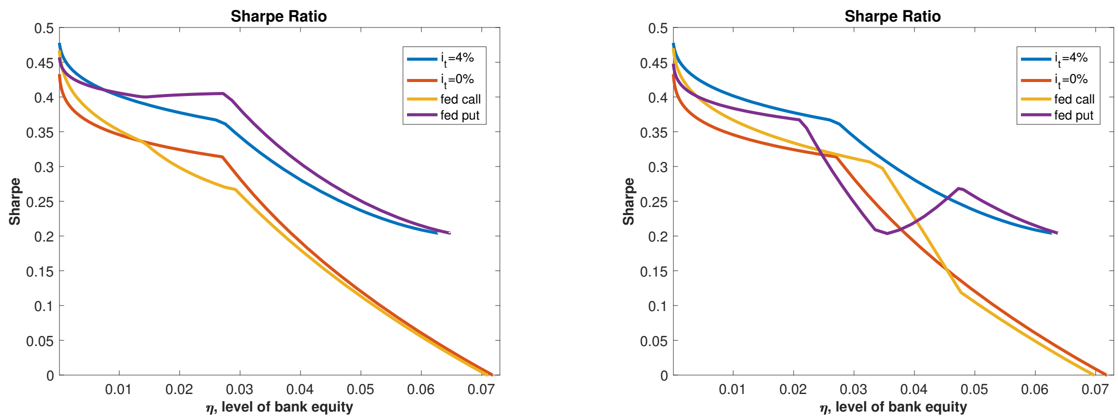
State-dependent effects of fed put We now compare in more detail the consequences of fed puts with $\eta^P = 1.75\%$, 2.75%, and 3.75% to policies of constant interest rates. Generally, compared to constant interest rates at the average level the fed put increases leverage below η^P but not otherwise; improves stability by decreasing the fraction of time the economy has low equity and increasing the fraction of time the economy has high equity; decreases price volatility; and increases Sharpe ratios, though non-monotonically depending on when the fed put kicks in.

The following figures plot the ratio of equilibrium objects under the fed put to the constant

[14]The risk premium is $-\sigma_\theta(\sigma + \sigma_Q)$. As seen in Figure 2(b), σ_Q actually decreases with the fed put, so the increase in risk aversion arises from a more negative σ_θ, whose closed form expression is

$$\sigma_\theta = \frac{\theta'(\eta)}{\theta(\eta)} \frac{(\psi - \eta)\sigma}{1 - \frac{q'(\eta)}{q(\eta)}(\psi - \eta)}.$$

The observed increase in leverage makes $\psi - \eta$ larger, which makes σ_θ more negative, as $\theta'(\cdot) < 0$.



(a) Low strike

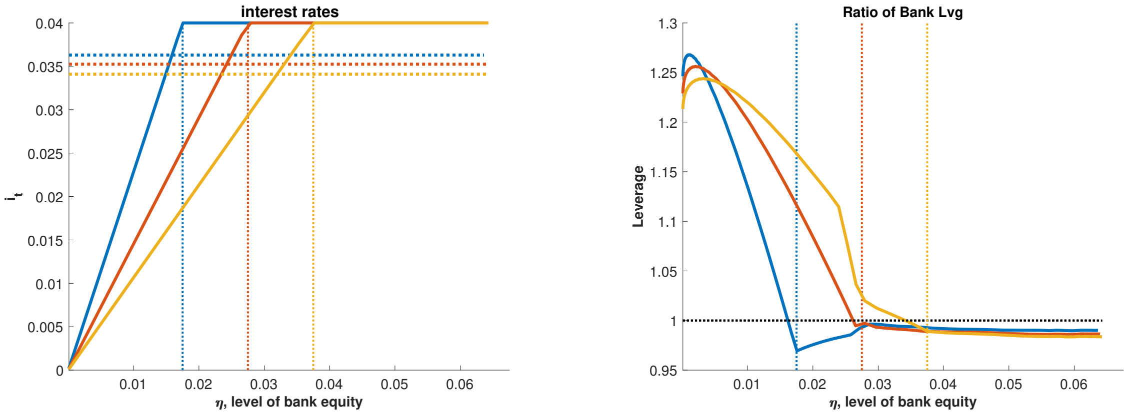
(b) High strike

Figure 3: Sharpe ratios varying the policy "strike" for fed put and call.

level. Thus, a value above 1 means that the variable of interest has a higher value with the fed put than with constant interest rates. While a fed put is generally stabilizing, some results depend on the location of η^P.

Figure 4(a) plots the interest rate rules we consider. The solid lines are the fed put and the dashed lines are the corresponding average rates. The vertical dotted lines mark the strike η^P. Figure 4(b) plots the ratio of leverage in each case. Leverage is generally higher during for $\eta < \eta^P$, but leverage can be lower for $\eta > \eta^P$ (the effect is small for high η). When η^P is very low (the blue at 1.75%), leverage before the put begins is starkly lower than would occur under constant rates. In other words, there is no "moral hazard": banks do not take on high leverage in anticipation of being "bailed out" later by a rate cut. This pattern remains in the other cases, although to a much lesser extent.

Figure 5(a) plots the non-monotonic increases in Sharpe ratios. Figure 5(b) plots the changes in the stationary distribution: as discussed, the fed put improves stability. Figure 5(c) plots the changes in price volatility and Figure 5(d) plots the changes in the evolution of η. With a fed put, price volatility is lower almost everywhere because the economy is more stable. As a result, the volatility of η falls even before the put kicks in (which contributes to economic stability) and rises only when leverage is higher. However, at this point the drift of the economy also increases, substantially so, which is why the economy is more stable with the fed put.



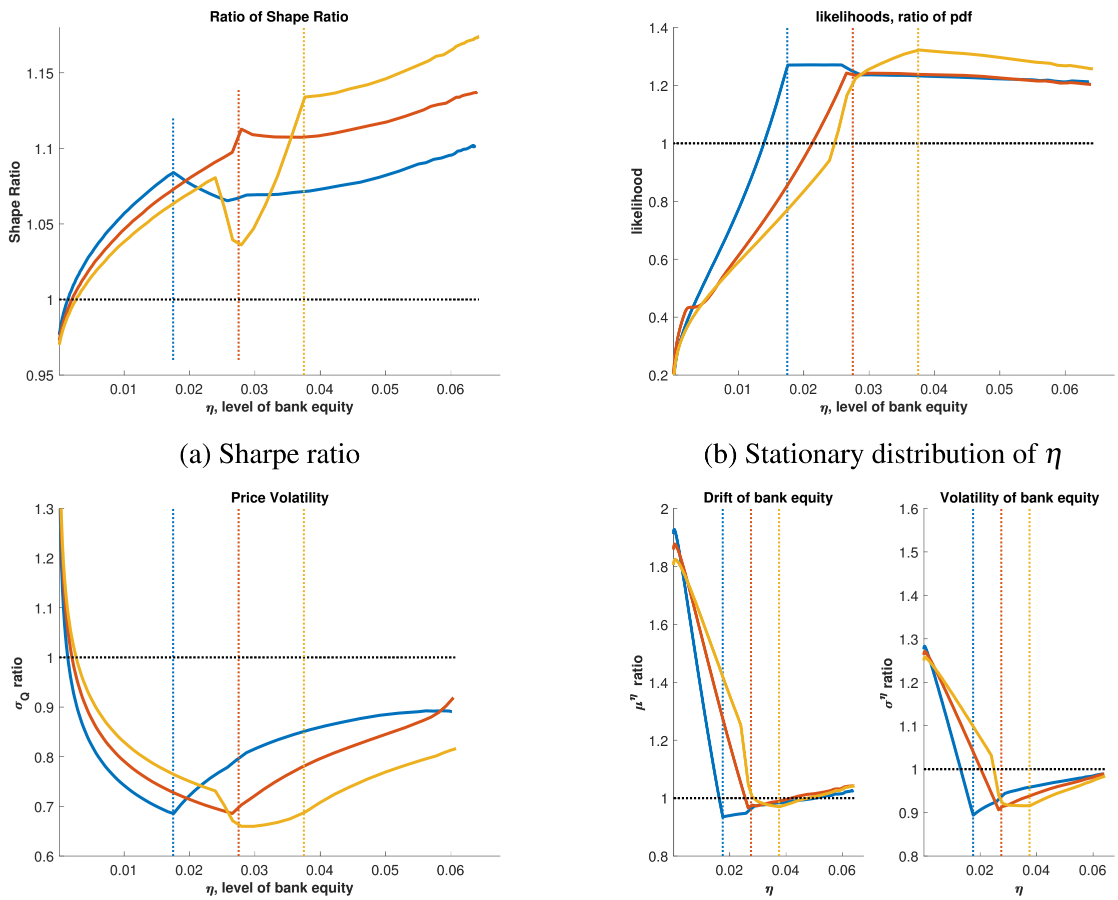
(a) Interest rate rules (b) Leverage

Figure 4: Effects of fed put on leverage compared to constant interest rates.

Marginal impacts of extending low rates We now examine the marginal impacts of changing fed policy depending on the state of the economy η. We consider interest rate rules that hold rates at zero for $\eta < \eta^P$ but set rates to 4% for high levels of η. We then compare these outcomes to when we extend the range over which rates are held at zero by the equivalent of 5% of time the economy is in that range (according to the stationary distribution). Figure 6(a) plots the interest rate rules we consider. The shaded portion corresponds to the range over which the central bank extends zero rates. (Thus, the blue exercise extends low rates from below $\eta = 1\%$ to 1.75%.) Figure 6(b) plots the ratio of leverage in each case. Leverage spikes during the extension period, with a smaller spike when extension occurs at higher η^P. Hence, the marginal impact on leverage is greatest when η^P is low.

Figure 7 plots the changes in Sharpe ratios. the stationary distribution, and evolutions. For the high-η^P policy, the marginal impact of extending dramatically decreases the Sharpe ratio over the policy extension range, with almost no increase globally. In contrast, the marginal impact of the low-η^P policy is to increase the Sharpe ratio for past η^P with a small decrease in the Sharpe over the policy extension range. Similarly the marginal impact of the low-η policy is to strictly stabilize the economy, but the marginal impact of the high-η policy can increase the likelihood of crises.[15]

[15] In fact, the support of the distribution changes substantially between those policies, so one may want to normalize the support (by dividing by η^*) to compare the stationary distribution. In this case, the likelihood of being in high



(a) Sharpe ratio

(b) Stationary distribution of η

(c) Price volatility σ_Q

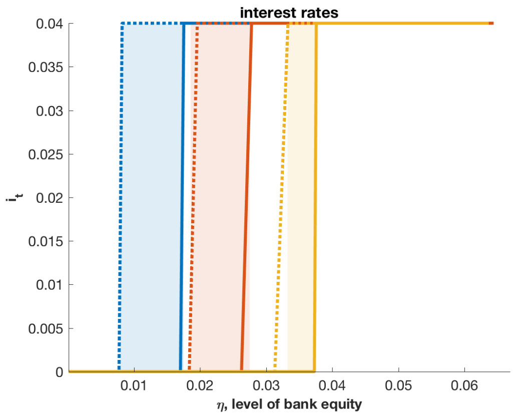
(d) η evolution

Figure 5: Effect of fed put on returns and stability.

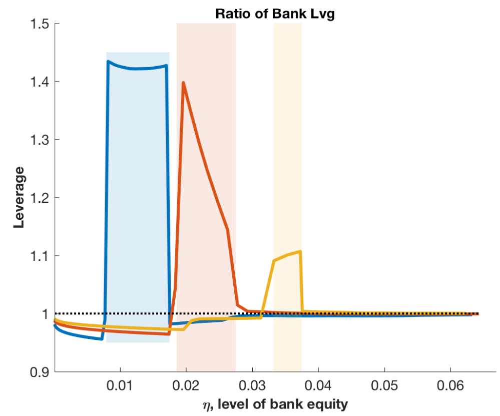
Figures 7(c) and 7(d) plot the changes in price volatility and the evolution of η. Price volatility falls for $\eta > \eta^P$ but rises once the economy enters the policy extension range below η^P. Thus, mechanically, price volatility is lower for a much larger range when the policy occurs at a lower η. Similarly, the marginal impacts on equity drift and volatility are much larger for the low-η policy, which is why the effect on stability is much greater for that policy.

Indeed the marginal impact on stability of extending rates up to $\eta^P = 3.75\%$ is bad for stability. Table 6 presents the change in stability, measured as the probability of not being in crisis $1 - F(\eta^P)$, from extending low rates up to η^P. The marginal

η regions actually decreases when the high-η policy is extended, which further supports the interpretation that the marginal impact of the high-η policy is destabilizing.



(a) Interest rate rules



(b) Change in leverage

Figure 6: Extending low interest rates and marginal impacts on changes in leverage.

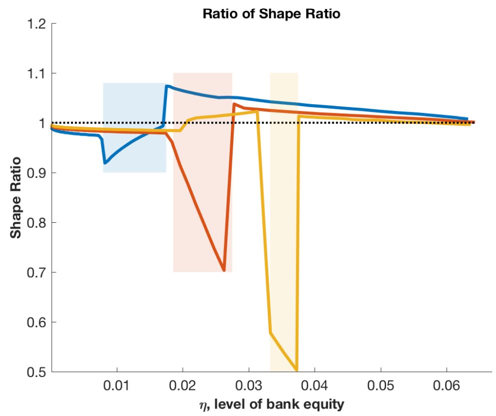
Table 6: Marginal impacts of extending low interest rates on stability.

η^P	1.75%	2.75%	3.75%
ΔStability	6.13%	8.46%	-2.20%

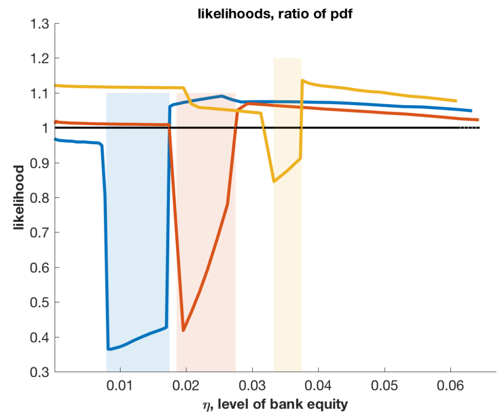
In Appendix B we do the same analysis for a measure of the marginal impact of a fed put. We consider a fed put that linearly decreases to zero beginning at some η^P, and consider the consequences of extending the aggressiveness of the fed put by increasing η^P. The results in that case are quite similar to the results in this section; however, extending the fed put changes rates for all η below the strikes, so the marginal impact will generally have more pronounced global implications.

4 Monetary Policy and Welfare

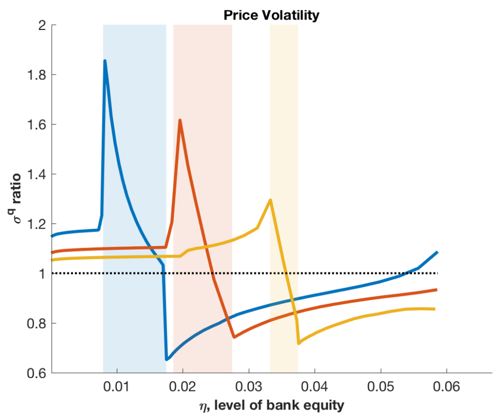
Having established the state-dependent dynamic consequences of monetary policy for financial stability (positive results), we now consider the welfare implications of these policies. Welfare in our model is easy to characterize. First, because capital grows geometrically, household welfare



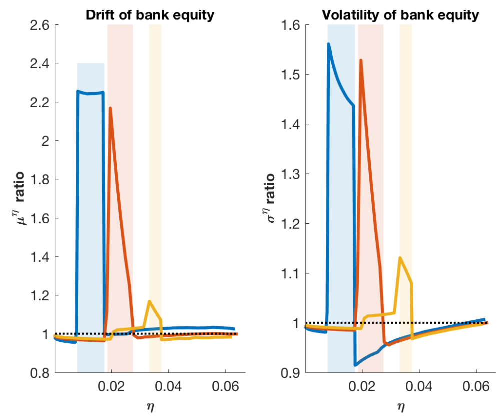
(a) Change in Sharpe ratio



(b) Change in PDF of η



(c) Change in price volatility



(d) Change in η evolution

Figure 7: Marginal impacts of extending low interest rates on returns and stability.

scales multiplicatively with capital, so we can write household welfare as

$$V_t = V(\eta)Y_t,$$

where $V(\eta)$ implicitly includes how the evolution of η affects capital growth. Second, because households are risk neutral and their investments earn expected return r and $r-\phi_L$ for deposits, expected discounted utility is equal to wealth. Household wealth includes the capital they own and the debt and equity invested in banks. The total wealth is $Q_tY_t + (\theta_t - 1)N_t = (1 + (\theta_t - 1)\eta_t)Q_tY_t$.[16]

[16] Their capital is worth $(1-\psi_t)Q_tY_t$; their debt is worth $\psi_tQ_tY_t - N_t$; bank shares are worth θ_tN_t, since the expected value of dividends from banks is $\theta N_t = \theta Q\eta_tY_t$.

Therefore we have

$$V(\eta) = (1 + (\theta - 1)\eta)Q.$$

In other words, the expected present discounted value of consumption and liquidity services is given exactly by this value, incorporating the bank value and capital price. Given the dynamics of the model, $V(\eta)$ is an increasing function, meaning that expected discounted household utility is higher when the financial sector is well-capitalized.

We first solve for welfare in the baseline model. Doing so gives us a model-specific measure of the costs and benefits of monetary policy for financial stability. In the baseline model, the only possible costs of monetary policy are decreasing the present value of banks and the capital price (which are together sufficient summaries of the expected discounted value of consumption and liquidity).

It appears that when considering constant interest rate rules, the Friedman Rule is optimal. Indeed, this is not surprising given the role of liquidity premia in the model. Low interest rates increase asset prices, which increase welfare. And while stability can be improved through state dependent interest rate policies (which could increase capital prices at any given point), capital prices are nonetheless higher with zero rates.

Because the Friedman Rule appears to be optimal, we then incorporate reduced-form inflation costs as a way of capturing mechanisms outside our model and to impose costs of lowering interest rates excessively. In this case, we suppose that any deviation of inflation from target (equivalently, of interest rates from the target $\bar{i}$) leads to flow (quadratic) inflation losses. Thus, in this extension the costs of dynamic monetary policies (e.g., LAW and fed put) are strictly higher than in the baseline model.

4.1 LAW/Fed Put in the Baseline Model

The baseline model without inflation costs provides a lower-bound on the costs of dynamic monetary policy. However, even the baseline model provides results arguably consistent with the cost-benefit analysis in Svensson (2017).

First, the effect of the fed put on welfare depends (unsurprisingly) on the initial condition:

relative to constant rates the fed put can either increase or lower welfare depending on the current state of the financial sector. In particular, for low levels of η, the fed put increases welfare, which is not surprising given the nature of the policy (and the behavior of the price Q)—when η is low, the "bailout component" of the policy is most salient (in present value terms), thus increasing welfare. Figure 8 plots the change in welfare, comparing the fed put to constant rates at the same average level, at every η. Thus, if the financial sector is currently well capitalized, then central bankers would decide against having monetary policy systematically respond to the financial sector. When the economy is presently at a high η, the costs of LAW/fed-put policies (primarily in lowering asset prices for high η), outweigh the benefits from stability.[17]

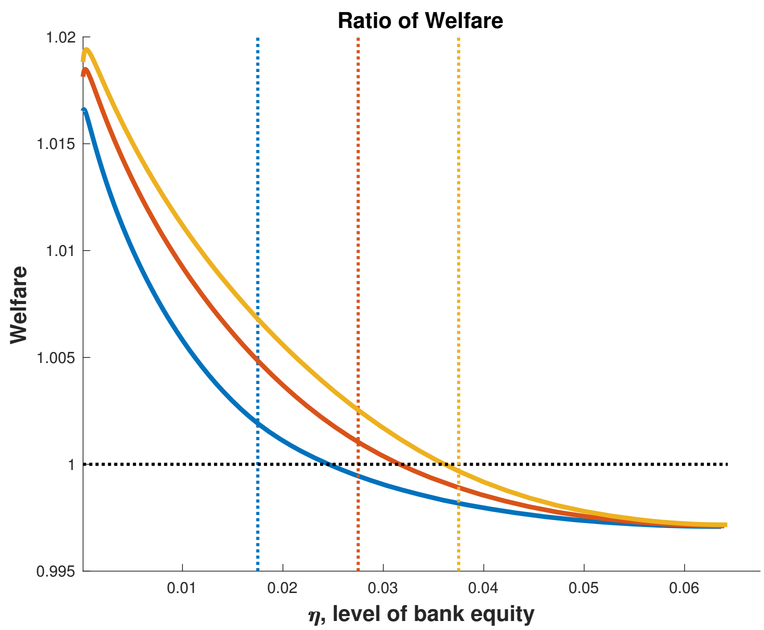


Figure 8: Effect of fed put on welfare.

Second, however, the results can be quite different when using an agnostic perspective on the initial condition. Rather than taking a particular η as given, we consider calculating welfare by letting the stationary distribution determine the ex-ante distribution of initial conditions and then compute $\mathbb{E}[V(\eta)]$ using the stationary distribution $f(\eta)$ occurring in equilibrium. One can think

[17]These asset pricing implications are similar to those in Drechsler et al. (2018); however, their model with heterogeneous agents does not include the same tight relationship between asset prices and welfare.

of this exercise as adopting a prior on the capitalization of the financial sector and computing the expected welfare, given this prior. Table 7 shows the percentage gain in welfare, according to this ex-ante measure, comparing the fed put to constant rates. In each case the fed put increases welfare, and the effects are non-monotonic in the strike η^P. This occurs because the higher strike policy has a greater effect on stability, thus increasing the fraction of time in states with higher welfare, but it is precisely in those high states that the fed put hurts welfare relative to constant rates.

Table 7: Effect of fed put on welfare using ex-ante measure.

η^P	1.75%	2.75%	3.75%
Gain in $\mathbb{E}[V(\eta)]$	0.488%	0.479%	0.612%

Finally, Table 8 calculates the marginal impact on ex-ante welfare of extending low rates. As one would expect given the positive results, the marginal impact is state-dependent and non-monotonic.

Table 8: Marginal impacts of extending low interest rates on welfare using ex-ante measure

η^P	1.75%	2.75%	3.75%
Gain in $\mathbb{E}[V(\eta)]$	0.440%	0.339%	0.361%

Thus, whether the benefits of monetary policy for stability can outweigh the costs depends on the perspective one takes. However, one credible conclusion is that even without a cost for deviating from $\bar{i}$, the benefits of LAW/fed-put policies are unlikely to outweigh the costs.

4.2 Welfare Losses From Inflation

As already discussed, in our model the Friedman Rule appears to be optimal in this model since asset prices respond significantly to nominal rates. Because the model does not include any New Keynesian feedback between output gaps and inflation (or vice versa), the central bank has no reason to avoid inflation far below target. To mitigate this result, we attempt to capture some of the costs of having inflation deviate from target in a very reduced-form way: we assume that households suffer quadratic utility loss when inflation deviates from the target. Household utility is now

$$V_\tau = \mathbb{E}_\tau \left[\int_\tau^\infty e^{-r(t-\tau)} (c_{h,t} + \phi_L \delta_{h,t} - \varepsilon_\pi (\pi_t - \bar{\pi})^2 Y_t) \, dt \right], \tag{14}$$

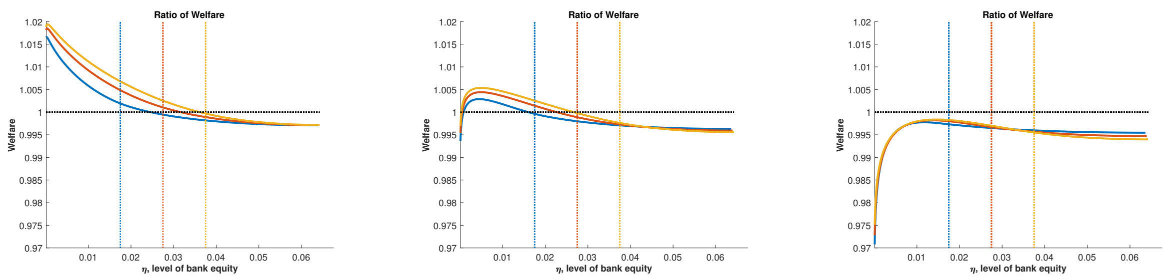
where ε_π is the measure of inflation costs and we scale by aggregate capital to maintain homogeneity in welfare. We let $L(\eta)$ denote the discounted expected inflation loss, and so household welfare is now given by $V(\eta) - L(\eta)$. The inflation loss $L(\eta)$ solves the following HJB equation:

$$rL(\eta) = \varepsilon_\pi (\pi(\eta) - \bar{\pi})^2 + L'(\eta) \eta \mu^\eta + \frac{1}{2} L''(\eta)(\eta \sigma^\eta)^2. \tag{15}$$

Adding the inflation loss does not have any consequences for prices or quantities in equilibrium, but only affects household welfare. (Figure 16 in the appendix plots welfare with high and low constant rates and with a fed put, incorporating two sets of inflation costs.)

Comparing fed put and constant rates We now more carefully consider the welfare implications of a fed put relative to constant interest rates. However, importantly, we now suppose that the constant interest rate is the target interest rate $\bar{i}$ such that no inflation losses occur. Thus, the fed put will incur inflation costs at high η when leaning against the wind (interest rates are above target), and also at low η when interest rates are below target.

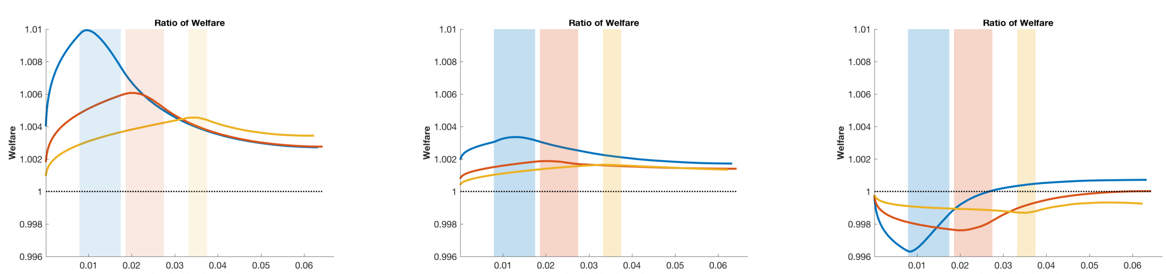
In general, the higher are inflation costs, the less likely is welfare to increase with a fed put relative to constant rates at the average level. With low inflation costs, welfare is higher with a fed put, but with high inflation costs welfare is higher with constant rates. Figure 9 shows how inflation costs affect the welfare implications of a fed put. The figure shows the ratio of welfare gains under the fed put compared to constant interest rates at the average level, with each panel considering different levels of inflation costs. For low inflation costs, a fed put is welfare improving for low η but not for high η (this is driven entirely by the change in the asset price Q, which we saw earlier). However, as inflation costs increase, the potential welfare gains from the fed put decrease because the fed put incurs higher inflation costs. With higher inflation costs, the likelihood that the fed put is welfare improving decreases, and for high enough inflation costs the fed put is not welfare improving for any η. A fed put incurs at times large deviations interest rates, while constant rates lead to smaller deviations throughout. With quadratic costs, the losses from the fed put can be larger.



(a) $\varepsilon_\pi = 0$ (b) $\varepsilon_\pi = 20$ (c) $\varepsilon_\pi = 40$

Figure 9: Relative welfare with inflation costs comparing fed put to constant rates.

Figure 10 shows the marginal impact on welfare when rates are held at zero, varying the level of inflation costs. The figure plots the ratio of welfare with extended low rates compared to welfare under the baseline levels of low rates. With low inflation costs the marginal impact is positive for welfare in every case (the asset price increases). However, for high inflation costs, the marginal impact can be generally negative, although there are state-dependent effects. The low-η policy can have a positive marginal impact for welfare when η is high, but negative impact when η is low, whereas the high-η policy has negative impact throughout the state space. With even higher inflation costs, the marginal impact of all policies is negative throughout the state space.



(a) $\varepsilon_\pi = 0$ (b) $\varepsilon_\pi = 20$ (c) $\varepsilon_\pi = 40$

Figure 10: Marginal impact of low rates on welfare with inflation costs.

Table 9 presents the gains in ex-ante welfare from a fed put for various levels of inflation costs. As discussed, we set the target central bank rate to the average interest rate over the fed put so that there are no inflation costs with constant interest rates (hence, constant interest rates corresponds to regular policy $\bar{i}$). Interestingly, even when welfare at each particular η can be lower with the fed put, the ex-ante welfare with the fed put can still be higher because of the change in stability. In other words, because the fed put economy is more stable, integrating over welfare using $f(\eta)$ means summing over high-η states with higher welfare. Thus, the ex-ante measure can still lead to welfare gains from a fed put even when the initial conditions all suggest welfare losses.

Table 9: Effect of fed put on welfare using ex-ante measure with inflation costs.

η^P	1.75%	2.75%	3.75%
$\varepsilon_\pi = 20$	0.290%	0.235%	0.326%
$\varepsilon_\pi = 40$	0.091%	-0.011%	0.038%
$\varepsilon_\pi = 60$	-0.110%	-0.259%	-0.249%

Table 10 presents the marginal impacts on welfare of extending low interest rates using ex-ante measure for various levels of inflation costs. Again, the marginal impacts are non-monotonic in the policy level η^P, and can be negative if inflation costs are sufficiently high. For this exercise we maintain $\bar{i} = 4\%$ throughout. Importantly, the marginal impact on inflation losses from extending low interest rates is greatest when the policy occurs at low levels (see Figure 18 in the Appendix).

Table 10: Marginal impacts on welfare of extending low interest rates using ex-ante measure.

η^P	1.75%	2.75%	3.75%
$\varepsilon_\pi = 20$	0.279%	0.157%	0.090%
$\varepsilon_\pi = 40$	0.116%	-0.027%	-0.183%
$\varepsilon_\pi = 60$	-0.047%	-0.212%	-0.461%

Optimal fed puts In the previous exercise we compared the fed put to a constant rate, where the target rate was the constant rate. Thus, the aggressiveness of the fed put, which changes the average rate, would change both how much rates are cut in bad times and also how much higher rates are kept during good times; this leads to inflation losses in both good and bad times. However, one may also want to consider more flexibility by holding fixed the target rate $\bar{i}$ and considering how a fed put changes welfare when $\bar{i}$ is held fixed.

Given the Friedman Rule result we've already seen, in the absence of inflation costs, lowering rates—anywhere or everywhere—increases welfare by increasing asset prices. However, as inflation costs rise, an aggressive fed put leads to larger inflation losses. We investigate how the optimality of a fed put changes with inflation costs by considering fed puts with two degrees of freedom: above η^P interest rates are held at $\bar{i}$, but below η^P interest rates decline linearly to i_0. Thus, a policy consists of a pair (η^P, i_0). We calculate welfare at η^* given these pairs, and we calculate how welfare varies with the fed put even for high inflation costs. Specifically, we search over $\eta^P \in (0, 5\%)$ and $i_0 \in [0, 4\%]$. We find that even for very large inflation costs, i.e., $\varepsilon_\pi = 100$, the optimal strike is $\eta^P = 5\%$, beginning to cut rates as soon as possible. However, as inflation costs rise, the rate at which the central bank cuts rates declines, so that i_0 increases as ε_π increases. For low inflation costs, the optimal put cuts rates to zero, but if $\varepsilon_\pi = 100$ then the optimal put cuts rates to 2%, thus decreasing the inflation losses when the central bank cuts.

Finally, this exercise provides additional evidence against LAW. Raising rates in good times would improve stability, but the welfare costs would not justify such a policy (supposing a high initial condition). Instead, the optimal policy is to maintain the desired target level $\bar{i}$ and to systematically deviate below that target when bank capitalization falls. This policy cuts banks' funding costs, allowing them to rebuild equity more quickly. Given the state-dependent consequences of monetary policy, the costs and benefits of cutting rates in bad times can be quite different from the costs and benefits of raising rates in good times.

4.3 Comparison with Macroprudential Policy

In the debate about whether monetary policy should be used to address financial stability, one of the key considerations is the extent to which macroprudential policy measures ("MaP") can be used instead. Informed by the analysis in Phelan (2016), we briefly discuss how the results with monetary policy would compare to the effects with MaP. Specifically, Phelan (2016) considers the effects of leverage limits on financial stability.

First, the mechanics of how MaP and monetary policy affect stability are quite different. When leverage limits bind, intermediation falls and so flow outcomes suffer. At the same time, leverage limits increase banks' investment returns (when they bind), and so banks rebuild equity faster, thus improving stability. Thus, MaP provides a tradeoff between current outcomes (worsened) and dynamic stability (improved). Furthermore, since Phelan (2016) finds that leverage limits are actually likely to bind following losses (i.e., balance sheet leverage is countercyclical), MaP provides a time-inconsistency problem because regulators would be tempted to relax leverage limits following bad shocks.

In contrast, the monetary policy can increase the rate of equity growth for banks by decreasing funding costs and liquidity premia. Crucially, lower rates encourage higher leverage (when rates are lower) and so accommodative monetary policy improves flow allocations. Thus, using monetary policy to target financial stability is actually time-consistent, given the transmission channel and mechanism in our model.

Second, the quantitative implications of MP and MaP appear to be quite different. Leverage limits have quantitatively larger effects on financial stability, more effectively shifting mass toward high η states. Indeed, very stringent leverage limits (assuming they are effective) can lead to extremely stable financial sectors almost without bound, whereas there appears to be a bound to how much monetary policy can improve stability.

Nonetheless, monetary policy appears to have quantitatively much larger effects on welfare. Decreasing interest rates improves stability and flow outcomes at the same time, while MaP improves stability at the cost of flow outcomes. Thus, using MaP to improve welfare is more difficult, and so it appears that the potential welfare gains from MaP are negligible compared to the potential gains from monetary policy. As an example, consider when interest rates are 0% and imposing leverage limits of 12. The welfare gain evaluated at η^* is roughly 0.04%, and evaluated under the ex-ante measure it is 0.35%. In contrast, compared to holding rates at 4% everywhere, cutting rates to zero below 2.75% improves welfare by 0.87% evaluated at η^* and by 1.9% evaluated under the ex-ante measure. Even with large inflation costs of $\varepsilon_\pi = 40$, this fed put improves welfare by 0.17% evaluated at η^* and by 0.69% evaluated under the ex-ante measure.[18]

Finally, MaP does not incur inflation losses arising from deviating from the target rate. Indeed, since MaP can substantially improve stability, it appears that using MaP in conjunction with monetary policy could decrease the inflation losses from deviations in monetary policy. Since flow inflation costs occur predominantly at low η, and since MaP decreases the likelihood of the economy entering low η regions, the expected discounted inflation losses decrease with MaP is also used. However, the quantitative significance of these changes appear to be negligibly small.

5 Conclusion

By affecting risk and liquidity premia, monetary policy can affect the stability of the financial sector and potentially improve the stability of the macroeconomy. We provide a macroeconomic model with a financial sector in which monetary policy endogenously determines the stability of the economy, and therefore determines the probability of crises. Policies combining leaning against the wind with accommodating rates during financial distress can substantially improve stability. Importantly, the effectiveness of monetary policy depends on the extent to which decreasing rates affects banks' investment returns, and we find that during times of financial crisis the effect on returns is low. Hence, the consequences of monetary policy for financial stability are state-dependent, and so the stability benefits of monetary policy depend critically on the timing, with the greatest potential benefits coming when rate cuts occur during financial crises. However, it is less clear that the costs of using monetary policy to target financial stability outweigh the potential benefits.

References

ACHARYA, V., I. GUJRAL, N. KULKARNI, AND H. SHIN (2011): "Dividends and bank capital in the financial crisis of 2007-2009," Tech. rep., National Bureau of Economic Research.

[18] As an extreme example, with no inflation costs lowering interest rates from 4% to 0% increases welfare by 7.16% evaluated at η^* and by 7.92% under the ex-ante measure.

ADRIAN, T. AND N. BOYARCHENKO (2012): "Intermediary leverage cycles and financial stability," Tech. Rep. 567, Staff Report, Federal Reserve Bank of New York.

ADRIAN, T. AND F. DUARTE (2016): "Financial vulnerability and monetary policy," .

ADRIAN, T. AND N. LIANG (2016): "Monetary policy, financial conditions, and financial stability," .

ADRIAN, T. AND H. SHIN (2010): "Liquidity and Leverage," *Journal of Financial Intermediation*, 19, 418–437.

——— (2011): "Procyclical Leverage and Value at Risk," Federal Reserve Bank of New York Staff Reports, N 338.

BERNANKE, B. AND M. GERTLER (1989): "Agency Costs, Net Worth, and Business Fluctuations," *The American Economic Review*, 79, pp. 14–31.

BERNANKE, B. S., M. GERTLER, AND S. GILCHRIST (1999): "The financial accelerator in a quantitative business cycle framework," in *Handbook of Macroeconomics*, ed. by J. B. Taylor and M. Woodford, Elsevier, vol. 1 of *Handbook of Macroeconomics*, chap. 21, pp. 1341–1393.

BIS (2014): "84th Annual Report," *Bank for International Settlements*.

——— (2016): "86th Annual Report," *Bank for International Settlements*.

BLINDER, A. AND R. REIS (2005): "Understanding the Greenspan standard," *Proceedings - Economic Policy Symposium - Jackson Hole*, 11–96.

BORIO, C. (2014): "Monetary policy and financial stability: what role in prevention and recovery?" *Bank for International Settlements Working Papers*.

BORIO, C. E., P. DISYATAT, M. JUSELIUS, AND P. RUNGCHAROENKITKUL (2018): "Monetary policy in the grip of a pincer movement," *Bank for International Settlements Working Papers*.

BRUNNERMEIER, M. K. AND Y. SANNIKOV (2014): "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 104, 379–421.

——— (2015): "International Credit Flows and Pecuniary Externalities," *American Economic Journal: Macroeconomics*, 7, 297–338.

——— (2016): "The I theory of money," Tech. rep., National Bureau of Economic Research.

DIAMOND, D. W. AND P. H. DYBVIG (1983): “Bank Runs, Deposit Insurance, and Liquidity,” *The Journal of Political Economy*, 91, pp. 401–419.

DIAMOND, D. W. AND R. G. RAJAN (2012): “Illiquid banks, financial stability, and interest rate policy,” *Journal of Political Economy*, 120, 552–591.

DRECHSLER, I., A. SAVOV, AND P. SCHNABL (2017): “The deposits channel of monetary policy,” *The Quarterly Journal of Economics*, 132, 1819–1876.

——— (2018): “A model of monetary policy and risk premia,” *The Journal of Finance*, 73, 317–373.

FARHI, E. AND J. TIROLE (2012): “Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts,” *American Economic Review*, 102, 60–93.

GAMBACORTA, L. AND H. S. SHIN (2016): “Why bank capital matters for monetary policy,” *Journal of Financial Intermediation*.

GÂRLEANU, N. AND S. PANAGEAS (2015): “Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing,” *Journal of Political Economy*, 123, 670–685.

GÂRLEANU, N., S. PANAGEAS, AND J. YU (2015): “Financial entanglement: A theory of incomplete integration, leverage, crashes, and contagion,” *American Economic Review*, 105, 1979–2010.

GORTON, G. AND G. PENNACCHI (1990): “Financial Intermediaries and Liquidity Creation,” *Journal of Finance*, 45, 49–71.

HE, Z. AND A. KRISHNAMURTHY (2012): “A Model of Capital and Crises,” *The Review of Economic Studies*, 79, 735–777.

——— (2013): “Intermediary Asset Pricing,” *American Economic Review*, 103, 732–70.

——— (2014): “A Macroeconomic Framework for Quantifying Systemic Risk,” Tech. rep., National Bureau of Economic Research.

JUSELIUS, M., C. E. BORIO, P. DISYATAT, AND M. DREHMANN (2017): “Monetary policy, the financial cycle and ultralow interest rates,” *International Journal of Central Banking*, 13.

KIYOTAKI, N. AND J. MOORE (1997): “Credit Cycles,” *Journal of Political Economy*, 105, pp. 211–48.

LAGOS, R. AND R. WRIGHT (2005): "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113, 463–484.

MOREIRA, A. AND A. SAVOV (2014): "The macroeconomics of shadow banking," *The Journal of Finance*.

PHELAN, G. (2016): "Financial intermediation, leverage, and macroeconomic instability," *American Economic Journal: Macroeconomics*, 8, 199–224.

STEIN, J. C. (2012): "Monetary policy as financial stability regulation," *The Quarterly Journal of Economics*, 127, 57–95.

SVENSSON, L. E. (2017): "Cost-benefit analysis of leaning against the wind," *Journal of Monetary Economics*, 90, 193–213.

VAN DER GHOTE, A. (2018): "Coordinating Monetary and Financial Regulatory Policies," .

Appendices

A Proofs and Additional Equations

A.1 Equilibrium System of Differential Equations

By (20), we have

$$\mu_{\theta,t} = \phi_L - \frac{\kappa}{m}(1 + \psi_t - \eta_t)i_t.$$

Since $\eta_t < 1$, we have that $\mu_{\theta,t}$ is always less than or equal to the equivalent drift in the economy with zero liquidity shocks. Since θ_t decreases with η, a smaller drift fo θ_t implies that θ_t, in expectation, moves toward larger values and bad times at a slower rate, reflecting the partial equilibrium intuition that higher interest rates should disincentivize excessive risk-taking. By Ito's lemma, we also have that

$$\mu_{\theta,t} = \frac{\theta'}{\theta}\eta_t\mu_{\eta,t} + \frac{1}{2}\frac{\theta''}{\theta}\eta_t^2\sigma_{\eta,t}^2.$$

Setting these two equations equal to each other yields a second-order ODE in η_t for θ. Similarly, using equation (9) and Ito's lemma, we have the following equations

$$\mu_{Q,t} = r - \frac{P_{2t}}{Q_t} - g - \sigma\sigma_{Q,t},$$
$$\mu_{Q,t} = \frac{Q'}{Q}\eta_t\mu_{\eta,t} + \frac{1}{2}\frac{Q''}{Q}\eta_t^2\sigma_{\eta,t}^2.$$

Hence, we obtain a coupled system of second-order ODEs:

$$\theta'' = \frac{2\theta}{(\eta_t\sigma_{\eta,t})^2}\left(\phi_L - \frac{\kappa}{m}(1+\psi_t-\eta_t)i_t - \frac{\theta'}{\theta}\eta_t\mu_{\eta,t}\right),$$
$$Q'' = \frac{2Q}{(\eta_t\sigma_{\eta,t})^2}\left(r - \frac{P_{2t}}{Q_t} - g - \sigma\sigma_{Q,t} - \frac{Q'}{Q}\eta_t\mu_{\eta,t}\right).$$

We solve for $\sigma_{Q,t}, \sigma_{\theta,t}$ in closed form using Ito's lemma, and these terms remain the same as in Phelan (2016).

A.2 Proofs

Proof Lemma 1. Taking the first derivative w.r.t. $x_{L,t}$ and multiplying through by κ, we have that an optimal choice of $x_{L,t}$ satisfies

$$0 = -\frac{\kappa}{m}i_t - \chi\kappa\frac{\rho}{1-\rho}\max\left\{\frac{\kappa}{1+\kappa}-1, 0\right\} \geq -\frac{\kappa}{m}i_t + \chi\frac{\rho}{1-\rho}\frac{\kappa}{1+\kappa}$$

Using our hypothesis, we have

$$0 \geq -\frac{\kappa}{m}i_t + \frac{\kappa}{m}i_t = 0,$$

so the FOC is always satisfied. Since i_t in equilibrium depends on η_t, a bounded variable, there always exist χ, ρ sufficiently large to ensure that (6) always holds.

Finally, to ensure that the max function returns zero, we need

$$0 = \frac{\kappa}{1+\kappa}(x_{k,t} - 1) - \frac{1}{1+\kappa} x_{L,t},$$

which implies the desired demand function. □

Proof of Proposition 1. Homogeneity and price-taking imply that banks' value function takes the form $U_t = \theta_t n_{b,t}$, where θ_t is the marginal value of banks' equity. The HJB can be written as

$$r\theta_t n_{b,t} = \max_{x_{k1,t}, x_{k2,t}, x_{L,t}, d\zeta_t} d\zeta_t + \mathbb{E}[d(\theta_t n_{b,t})], \tag{16}$$

subject to the constraints (3) and (4).

By Ito's product rule,

$$\begin{aligned}\frac{d(\theta_t n_{b,t})}{\theta_t n_{b,t}} &= (\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t})\, dt + (\sigma_{\theta,t} + \sigma_{nb,t})\, dW_t \\ &\quad - \frac{\rho}{1-\rho} \max\left\{\frac{\kappa}{1+\kappa}(x_{k,t} + x_{L,t} - 1) - x_{L,t}, 0\right\} dJ_t.\end{aligned}$$

Suppressing the controls and dropping the differential dt, equation (16) simplifies to

$$r\theta_t n_{b,t} = \max d\zeta_t + \theta_t n_{b,t}\left(\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t}\right) - \chi \frac{\rho}{1-\rho} \max\left\{\frac{\kappa}{1+\kappa}(x_{k,t} + x_{L,t} - 1) - x_{L,t}, 0\right\}$$

Under the assumption of full self-insurance, we may ignore the Poisson term. Because banks provide liquidity services, they will always be levered, so we may directly substitute $x_{L,t} = \kappa(x_{k,t} - 1)$ into the Bellman equation. Using the dynamic budget constraint (3),

$$\begin{aligned} r\theta_t n_{b,t} &= \max d\zeta_t + \theta_t n_{b,t}(\mu_{\theta,t} + \sigma_{\theta,t}\sigma_{nb,t}) \\ &\quad + \theta_t n_{b,t}\left(r_t - \frac{\kappa(x_{k,t}-1)}{m} i_t + \frac{S_t}{m} i_t + x_{k1,t}(\mathbb{E}[dr_t^{b1}] - r_t\, dt) + x_{k2,t}(\mathbb{E}[dr_t^{b2}] - r_t\, dt) - \frac{d\zeta_t}{n_{b,t}}\right) \\ &= \max\, (1-\theta_t) d\zeta_t + \theta_t n_{b,t}(\mu_{\theta,t} + \sigma_{\theta,t}\sigma_{nb,t}) \\ &\quad + \theta_t n_{b,t}\left(r_t - \frac{\kappa(x_{k,t}-1)}{m} i_t + \frac{S_t}{m} i_t + x_{k1,t}(\mathbb{E}[dr_t^{b1}] - r_t\, dt) + x_{k2,t}(\mathbb{E}[dr_t^{b2}] - r_t\, dt)\right). \end{aligned}$$

In real terms, bank returns on capital holdings satisfy:

$$dr_t^{bj} = \left(\frac{P_{jt}}{Q_t} + g - \phi_L + \mu_{Q,t} + \sigma\sigma_{Q,t}\right) dt + (\sigma + \sigma_{Q,t})\, dW_t, \tag{17}$$

where $j = 1,2$ and $P_{jt} = p_{jt}/p_t$ is the real price of intermediate good j. Using these returns, we may write $\sigma_{nb,t} = x_{k,t}(\sigma + \sigma_{Q,t})$. Our remaining controls are $x_{k1,t}, x_{k2,t}, d\zeta_t$. The linearity in $d\zeta_t$ implies that banks use consumption to create a reflecting barrier whenever $\theta_t \leq 1$. Taking FOCs w.r.t. portfolio shares, we obtain, with a slight abuse of notation, the asset-pricing condition

$$\mathbb{E}[dr_t^{bj}] - r_t \leq -\sigma_{\theta,t}(\sigma + \sigma_{Q,t}) + \frac{\kappa}{m} i_t, \tag{18}$$

where $j = 1,2$. Intuitively, this expression says that in equilibrium the excess (real) returns on capital holdings equal the sum of a risk premium and a fixed multiple of the liquidity premium. □

A.3 Policy Implementation

Proposition 3. *To implement the nominal interest rate rule i_t, the nominal supply of reserves M_t and government bonds G_t must grow according to*

$$\begin{aligned}\varepsilon_t \frac{dM_t}{M_t} + (1-\varepsilon_t)\frac{dG_t}{G_t} = (i_t - r_t)\, dt + \frac{dS_t}{S_t} + \frac{dQ_t}{Q_t} + \left(\frac{dS_t}{S_t}\right)\left(\frac{dQ_t}{Q_t}\right) \\ + \frac{dY_t}{Y_t} + \left(\frac{dS_t}{S_t} + \frac{dQ_t}{Q_t}\right)\left(\frac{dY_t}{Y_t}\right),\end{aligned} \tag{19}$$

where $\varepsilon_t = (m-1)s_t M_t/(S_t Q_t Y_t)$ is the net contribution of reserves to aggregate liquidity, and the real value of liquidity as a share of wealth satisfies

$$S_t = \eta_t \kappa (x_{k,t} - 1). \tag{20}$$

Proof. We can express the real value of all liquid assets held by the public as

$$S_t Q_t Y_t = s_t G_t + (m-1) s_t M_t.$$

By Ito's product rule,

$$\frac{d(S_tQ_tY_t)}{S_tQ_tY_t} = \frac{dS_t}{S_t} + \frac{dQ_t}{Q_t} + \left(\frac{dS_t}{S_t}\right)\left(\frac{dQ_t}{Q_t}\right) + \frac{dY_t}{Y_t} + \left(\frac{dS_t}{S_t} + \frac{dQ_t}{Q_t}\right)\left(\frac{dY_t}{Y_t}\right).$$

We can write the evolution of liquid assets as

$$\begin{aligned}\frac{d(G_t+(m-1)M_t)}{G_t+(m-1)M_t} &= \frac{1}{S_tQ_tY_t}\left(\frac{dG_t}{G_t}G_t + (m-1)M_t\frac{dM_t}{M_t}\right)\\ &= \frac{G_t}{S_tQ_tY_t}\frac{dG_t}{G_t} + (m-1)\frac{M_t}{S_tQ_tY_t}\frac{dM_t}{M_t}\\ &= \frac{s_tG_t}{S_tQ_tY_t}\frac{dG_t}{G_t} + (m-1)\frac{s_tM_t}{S_tQ_tY_t}\frac{dM_t}{M_t}\end{aligned}$$

Recall that $S_tQ_tY_t = s_tG_t + (m-1)s_tM_t$. Thus, $1-\varepsilon_t = s_tG_t$. By Ito's product rule and the fact that inflation is locally deterministic, we obtain (19) after re-arranging.

To obtain (20), recall that $\eta_t = n_{b,t}/(Q_tY_t)$ and $S_tQ_tY_t$ is the total real value of liquidity. Therefore,

$$\eta_t\kappa(x_{k,t}-1) = \frac{n_{b,t}x_{L,t}}{Q_tY_t} = \frac{S_tQ_tY_t}{Q_tY_t} = S_t,$$

as desired. □

A.4 Evolution of η

It remains to derive the evolution of η_t. The net worth of banks is scale invariant, and banks only use capital in the production of good 1. Define $d\Xi_t \equiv d\zeta_t/n_{b,t}$. Assuming full self-insurance, the law of motion becomes

$$\frac{dn_{b,t}}{n_{b,t}} = \left(r_t - \frac{\kappa(x_{k,t}-1)}{m}i_t + \frac{S_t}{m}i_t\right)dt + x_{k,t}(dr_t^{b1} - r_t\,dt) - d\Xi_t.$$

Substituting in $r_t = r - \phi_L$, (10), and (20) yields

$$\frac{dn_{b,t}}{n_{b,t}} = \left(r - \phi_L - (1-\eta_t)(x_{k,t}-1)\frac{\kappa}{m}i_t + x_{k,t}\left(-\sigma_{\theta,t}(\sigma+\sigma_{Q,t}) + \frac{\kappa}{m}i_t\right)\right)dt - d\Xi_t + x_{k,t}(\sigma+\sigma_{Q,t})\,dW_t$$

Define ψ_t to be the share of capital held by banks. Then $x_{k,t} = \psi_t/\eta_t$, and we have

$$\frac{dn_{b,t}}{n_{b,t}} = \left(r - \phi_L + (1+\psi_t-\eta_t)\frac{\kappa}{m}i_t - \frac{\psi_t}{\eta_t}\sigma_{\theta,t}(\sigma+\sigma_{Q,t})\right)dt - d\Xi_t + \frac{\psi_t}{\eta_t}(\sigma+\sigma_{Q,t})\,dW_t.$$

By Ito's product rule,

$$\frac{d(Q_tY_t)}{Q_tY_t} = (\mu_{Q,t}+\mu_{Y,t}+\sigma\sigma_{Q,t})\,dt + (\sigma+\sigma_{Q,t})\,dW_t.$$

We may write $\mu_{Y,t}$ and $\mu_{Q,t}$ as:

$$\begin{aligned}
\mu_{Y,t} &= \psi_t(g-\phi_L) + (\lambda_t-\psi_t)(g-\ell) + (1-\lambda_t)g = g - \psi_t\phi_L - (\lambda_t-\psi_t)\ell \\
\mu_{Q,t} &= r - \sigma_{\theta,t}(\sigma+\sigma_{Q,t}) + \frac{\kappa}{m}i_t - \frac{P_{1t}}{Q_t} - g - \sigma\sigma_{Q,t} \\
&= r - \phi_L - \sigma_{\theta,t}(\sigma+\sigma_{Q,t}) + \frac{\kappa}{m}i_t - \frac{P_{1t}}{Q_t} - (g-\phi_L) - \sigma\sigma_{Q,t}
\end{aligned}$$

Plugging this in and applying Ito's quotient rule implies

$$\begin{aligned}
\frac{d(1/(Q_tY_t))}{1/(Q_tY_t)} = &\left((\sigma+\sigma_{Q,t})^2 - (r-\phi_L) + \sigma_{\theta,t}(\sigma+\sigma_{Q,t}) - \frac{\kappa}{m}i_t + \frac{P_{1t}}{Q_t} + g - \phi_L + \sigma\sigma_{Q,t}\right)dt \\
&- \left(g - \psi_t\phi_L - (\lambda_t-\psi_t)\ell + \sigma\sigma_{Q,t}\right)dt - (\sigma+\sigma_{Q,t})\,dW_t.
\end{aligned}$$

By Ito's product rule,

$$\begin{aligned}
\frac{d\eta_t}{\eta_t} &= \left((1+\psi_t-\eta_t)\frac{\kappa}{m}i_t - \frac{\psi_t}{\eta_t}\sigma_{\theta,t}(\sigma+\sigma_{Q,t}) \right) dt - d\Xi_t + \frac{\psi_t}{\eta_t}(\sigma+\sigma_{Q,t})\,dW_t \\
&\quad + \left((\sigma+\sigma_{Q,t})^2 + \sigma_{\theta,t}(\sigma+\sigma_{Q,t}) - \frac{\kappa}{m}i_t + \frac{P_{1t}}{Q_t} + g - \phi_L + \sigma\sigma_{Q,t} \right) dt \\
&\quad - \left(g - \psi_t\phi_L - (\lambda_t-\psi_t)\ell + \sigma\sigma_{Q,t} \right) dt - (\sigma+\sigma_{Q,t}), dW_t - \frac{\psi_t}{\eta_t}(\sigma+\sigma_{Q,t})^2\,dt \\
&= \left(\frac{P_{1t}}{Q_t} + (\lambda_t-\psi_t)\ell - (1-\psi_t)\phi_L + (\psi_t-\eta_t)\frac{\kappa}{m}i_t - \left(\frac{\psi_t}{\eta_t}-1\right)(\sigma+\sigma_{Q,t})(\sigma_{\theta,t}+\sigma+\sigma_{Q,t}) \right) dt \\
&\quad - d\Xi_t + \left(\frac{\psi_t}{\eta_t}-1\right)(\sigma+\sigma_{Q,t})\,dW_t
\end{aligned}$$

Compared to Phelan (2016), the only new term is the liquidity premium, which affects the drift of η_t. Thus, many of the properties in Phelan (2016) should hold true here as well.

A.5 Numerical Algorithm: No Constraints

Assuming that the government implements (19), the only significant change in the numerical algorithm from Phelan (2016) is the additional term representing the liquidity premium in the first equation of (10). Thus, we can use the exact same algorithm with only mild adjustments to asset pricing conditions and equilibrium expressions. For clarity, we derive these expressions here.

The Bellman equation is given by

$$\begin{aligned}
r\theta_t n_{b,t} = \max_{x_{k,t},d\zeta_t} &(1-\theta_t)\,d\zeta_t + \theta_t n_{b,t}(\mu_{\theta,t} + \sigma_{\theta,t}\sigma_{nb,t}) \\
&+ \theta_t n_{b,t}\left(r_t + \left(\frac{\kappa+S_t}{m}\right) i_t + x_{k,t}(\mathbb{E}[dr_t^{b1}] - r_t - \frac{\kappa}{m}i_t) \right).
\end{aligned}$$

Plugging in our first-order conditions and using the bang-bang control in $d\zeta_t$ imply

$$r\theta_t n_{b,t} = \theta_t n_{b,t}\mu_{\theta,t} + \theta_t n_{b,t}\left(r - \phi_L + \left(\frac{\kappa+S_t}{m}\right) i_t \right) \Rightarrow \mu_{\theta,t} = \phi_L - \left(\frac{\kappa+S_t}{m}\right) i_t.$$

By (20), we have

$$\mu_{\theta,t} = \phi_L - \frac{\kappa}{m}(1+\psi_t-\eta_t)i_t.$$

Since $\eta_t < 1$, we have that $\mu_{\theta,t}$ is always less than or equal to the equivalent drift in the economy with zero liquidity shocks. Since θ_t decreases with η, a smaller drift fo θ_t implies that θ_t, in expectation, moves toward larger values and bad times at a slower rate, reflecting the partial equilibrium intuition that higher interest rates should disincentivize excessive risk-taking. By Ito's lemma, we also have that

$$\mu_{\theta,t} = \frac{\theta'}{\theta}\eta_t\mu_{\eta,t} + \frac{1}{2}\frac{\theta''}{\theta}\eta_t^2\sigma_{\eta,t}^2.$$

Setting these two equations equal to each other yields a second-order ODE in η_t for θ.

We can similarly derive a second-order ODE for q. Using the second equation in (10), we may write

$$\mu_{Q,t} = r - \frac{P_{2t}}{Q_t} - g - \sigma\sigma_{Q,t}.$$

By Ito's lemma, we also have

$$\mu_{Q,t} = \frac{Q'}{Q}\eta_t\mu_{\eta,t} + \frac{1}{2}\frac{Q''}{Q}\eta_t^2\sigma_{\eta,t}^2.$$

In this way, we obtain a coupled system of second-order ODEs:

$$\theta'' = \frac{2\theta}{(\eta_t\sigma_{\eta,t})^2}\left(\phi_L - \frac{\kappa}{m}(1+\psi_t-\eta_t)i_t - \frac{\theta'}{\theta}\eta_t\mu_{\eta,t}\right), \tag{21}$$

$$Q'' = \frac{2Q}{(\eta_t\sigma_{\eta,t})^2}\left(r - \frac{P_{2t}}{Q_t} - g - \sigma\sigma_{Q,t} - \frac{Q'}{Q}\eta_t\mu_{\eta,t}\right). \tag{22}$$

By Ito's lemma, we can solve for $\sigma_{Q,t}, \sigma_{\theta,t}$ in closed form, and they remain the same as in Phelan (2016).

Specialization Households do not produce good 1, so $\psi = \lambda$. Taking the difference between the two equations in (10), we obtain a market-clearing condition for capital allocations:

$$P_{1t} - P_{2t} = -\sigma_{\theta,t}(\sigma + \sigma_{Q,t})Q_t + \frac{\kappa}{m}i_tQ_t \tag{23}$$

Non-Specialization Households produce good 1, so we must have

$$\frac{P_{1t}}{Q_t} + g - \ell + \mu_{Q,t} + \sigma\sigma_{Q,t} - r = \frac{P_{2t}}{Q_t} + g + \mu_{Q,t} + \sigma\sigma_{Q,t} - r \Rightarrow P_{1t} - P_{2t} = lQ_t.$$

However, since $\psi < \lambda$, we need an additional condition to pin down ψ. In this case, we use the fact that households' asset pricing condition for good 1 is satisfied, hence

$$\frac{P_{1t}}{Q_t} + g + \mu_{Q,t} + \sigma\sigma_{Q,t} - \frac{P_{1t}}{Q_t} - (g - \ell) - \mu_{Q,t} - \sigma\sigma_{Q,t} = r - \sigma_{\theta,t}(\sigma + \sigma_{Q,t}) + \frac{\kappa}{m} i_t - r$$

$$\Rightarrow \ell = -\sigma_{\theta,t}(\sigma + \sigma_{Q,t}) + \frac{\kappa}{m} i_t \tag{24}$$

In other words, the household efficiency loss from investing in the "intermediation sector" equals the sum of the banks' risk premium and the liquidity premium.

Plugging in, we obtain

$$\ell = -\frac{\theta'}{\theta} \frac{Q(\psi - \eta)\sigma}{Q - Q'(\psi - \eta)} \left(\frac{\sigma Q}{Q - Q'(\psi - \eta)} \right) + \frac{\kappa}{m} i_t.$$

Let $x = \psi - \eta$ and $y = \ell - \frac{\kappa}{m} i_t$. Then

$$\begin{aligned} y\left(Q - Q'x\right)^2 &= -\frac{\theta'}{\theta}\sigma^2 Q^2 x \\ Q^2 - 2QQ'x + (Q'x)^2 &= -\frac{\theta'}{\theta}\frac{\sigma^2}{y} Q^2 x \\ x^2 (Q')^2 + x\left(\frac{\theta'}{\theta}\frac{\sigma^2}{y} Q^2 - 2QQ'\right) + Q^2 &= 0. \end{aligned}$$

By the quadratic formula, we may solve for x and obtain using that $\psi = x + \eta$.

A.6 Leverage Constraints

Suppose that banks are subject to a leverage constraint L. When constraints do not bind, the evolution of η is as above. If they do bind, then the first equation in (10) no longer holds. We leave asset-pricing conditions undetermined and leave the excess returns terms as they are. Instead, we simply note that

$$\mu_{Q,t} = \mathbb{E}[dr_t^{b1}] - \frac{P_{1t}}{Q_t} - g - \sigma\sigma_{Q,t},$$

so

$$\begin{aligned}
\frac{dQ_tY_t}{Q_tY_t} &= \left(g - \psi_t\phi_L - (\lambda_t - \psi_t)\ell + \mathbb{E}[dr_t^{b1}] - \frac{P_{1t}}{Q_t} - g\right)dt + (\sigma + \sigma_{Q,t})\,dW_t \\
\frac{d(1/(Q_tY_t))}{1/(Q_tY_t)} &= \left((\sigma + \sigma_{Q,t})^2 + \psi_t\phi_L + (\lambda_t - \psi_t)\ell - \mathbb{E}[dr_t]^{b1} + \frac{P_{1t}}{Q_t}\right)dt - (\sigma + \sigma_{Q,t})\,dW_t.
\end{aligned}$$

Therefore, by Ito's product rule,

$$\begin{aligned}
\frac{d\eta_t}{\eta_t} &= \left(r - \phi_L - (1-\eta_t)\left(\frac{\psi_t}{\eta_t} - 1\right)\frac{\kappa}{m}i_t + \frac{\psi_t}{\eta_t}(\mathbb{E}[dr_t^{b1}] - (r - \phi_L))\right)dt - d\Xi_t + \frac{\psi_t}{\eta_t}(\sigma + \sigma_{Q,t})\,dW_t \\
&\quad + \left((\sigma + \sigma_{Q,t})^2 + \psi_t\phi_L + (\lambda_t - \psi_t)\ell - \mathbb{E}[dr_t]^{b1} + \frac{P_{1t}}{Q_t}\right)dt - (\sigma + \sigma_{Q,t})\,dW_t - \frac{\psi_t}{\eta_t}(\sigma + \sigma_{Q,t})^2\,dt \\
&= \left(\frac{P_{1t}}{Q_t} - (1-\psi_t)\phi_L + (\lambda_t - \psi_t)\ell + \left(\frac{\psi_t}{\eta_t} - 1\right)\left(\mathbb{E}[dr_t^{b1}] - (r - \phi_L) - (\sigma + \sigma_{Q,t})^2\right) - (1-\eta_t)\frac{\kappa}{m}i_t\right)dt \\
&\quad - d\Xi_t + \left(\frac{\psi_t}{\eta_t} - 1\right)(\sigma + \sigma_{Q,t})^2\,dW_t.
\end{aligned}$$

A.7 Numerical Algorithm: Leverage Constraints

Subject to a leverage constraint, the Bellman equation is given by

$$\begin{aligned}
r\theta_t n_{b,t} = \max_{x_{k,t},d\zeta_t}\ & (1-\theta_t)\,d\zeta_t + \theta_t n_{b,t}(\mu_{\theta,t} + \sigma_{\theta,t}\sigma_{nb,t}) \\
& + \theta_t n_{b,t}\left(r_t + \left(\frac{\kappa + S_t}{m}\right)i_t + x_{k,t}(\mathbb{E}[dr_t^{b1}] - r_t - \frac{\kappa}{m}i_t)\right).
\end{aligned}$$

Using a bang-bang control, we can remove the first term. Substituting in $x_{k,t} = (1+L)$ and $S_t = \kappa(x_{k,t}-1)/m$, we have

$$\begin{aligned} r\theta_t n_{b,t} &= \theta_t n_{b,t}(\mu_{\theta,t} + \sigma_{\theta,t}(1+L)(\sigma+\sigma_{Q,t})) \\ &\quad + \theta_t n_{b,t}\left(r - \phi_L + \frac{\kappa}{m} i_t + \frac{\eta_t \kappa(1+L-1)}{m} i_t + (1+L)(\mathbb{E}[dr_t^{b1}] - (r-\phi_L) - \frac{\kappa}{m} i_t)\right) \end{aligned}$$

Simplifying, we acquire

$$\begin{aligned} r\theta_t n_{b,t} = \theta_t n_{b,t} &\left(\mu_{\theta,t} + r - \phi_L + (1+\eta_t L)\frac{\kappa}{m} i_t\right) \\ &+ (1+L)\left(\mathbb{E}[dr_t^{b1}] - (r-\phi_L) - \frac{\kappa}{m} i_t + \sigma_{\theta,t}(\sigma+\sigma_{Q,t})\right) \end{aligned}$$

Dividing through by $\theta_t n_{b,t}$ and re-arranging, we have

$$\mu_{\theta,t} = \phi_L - (1+\eta_t L)\frac{\kappa}{m} i_t - (1+L)(\mathbb{E}[dr_t^{b1}] - (r-\phi_L) - \frac{\kappa}{m} i_t + \sigma_{\theta,t}(\sigma+\sigma_{Q,t}))$$

We still use that

$$\mu_{Q,t} = r - \frac{P_{2t}}{Q_t} - g - \sigma\sigma_{Q,t}$$

as this still holds in equilibrium. Therefore, our ODEs become

$$\begin{aligned} \theta'' &= \frac{2\theta}{(\eta_t\sigma_{\eta,t})^2}\left(\phi_L - (1+\eta_t L)\frac{\kappa}{m} i_t\right) \\ &\quad + \frac{2\theta}{(\eta_t\sigma_{\eta,t})^2}\left(-(1+L)\left(\mathbb{E}[dr_t^{b1}] - (r-\phi_L) + \sigma_{\theta,t}(\sigma+\sigma_{Q,t}) - \frac{\kappa}{m} i_t\right) - \frac{\theta'}{\theta}\eta_t\mu_{\eta,t}\right) \end{aligned} \tag{25}$$

$$Q'' = \frac{2Q}{(\eta_t\sigma_{\eta,t})^2}\left(r - \frac{P_{2t}}{Q_t} - g - \sigma\sigma_{Q,t} - \frac{Q'}{Q}\eta_t\mu_{\eta,t}\right) \tag{26}$$

B Marginal Impacts of Extending Fed Put

Figure 11 plots the marginal impact of extending the fed put. Most importantly, the marginal impact on the Sharpe ratio can be very negative if the fed put occurs at high η when banks are the marginal investors in bank-dependent investments.

C Additional Figures

This section provides additional figures.

Figure 13 plots PDFs and CDFs for constant rates and fed put and fed call with normalized support (dividing by η^*). Fed put provides most stable economy, followed by low constant rates and high constant rates, and the fed call is the least stable economy.

Figure 14 plots the effects of leaning against the wind. Rates are zero below $\eta^P = 2.75\%$ and $i = 4, 5, 6\%$ above. With LAW, welfare decreases, leverage is mostly unchanged, though higher for low η, asset price volatility drops, and the economy is slightly more stable, adjusting for the stochastic stead state.

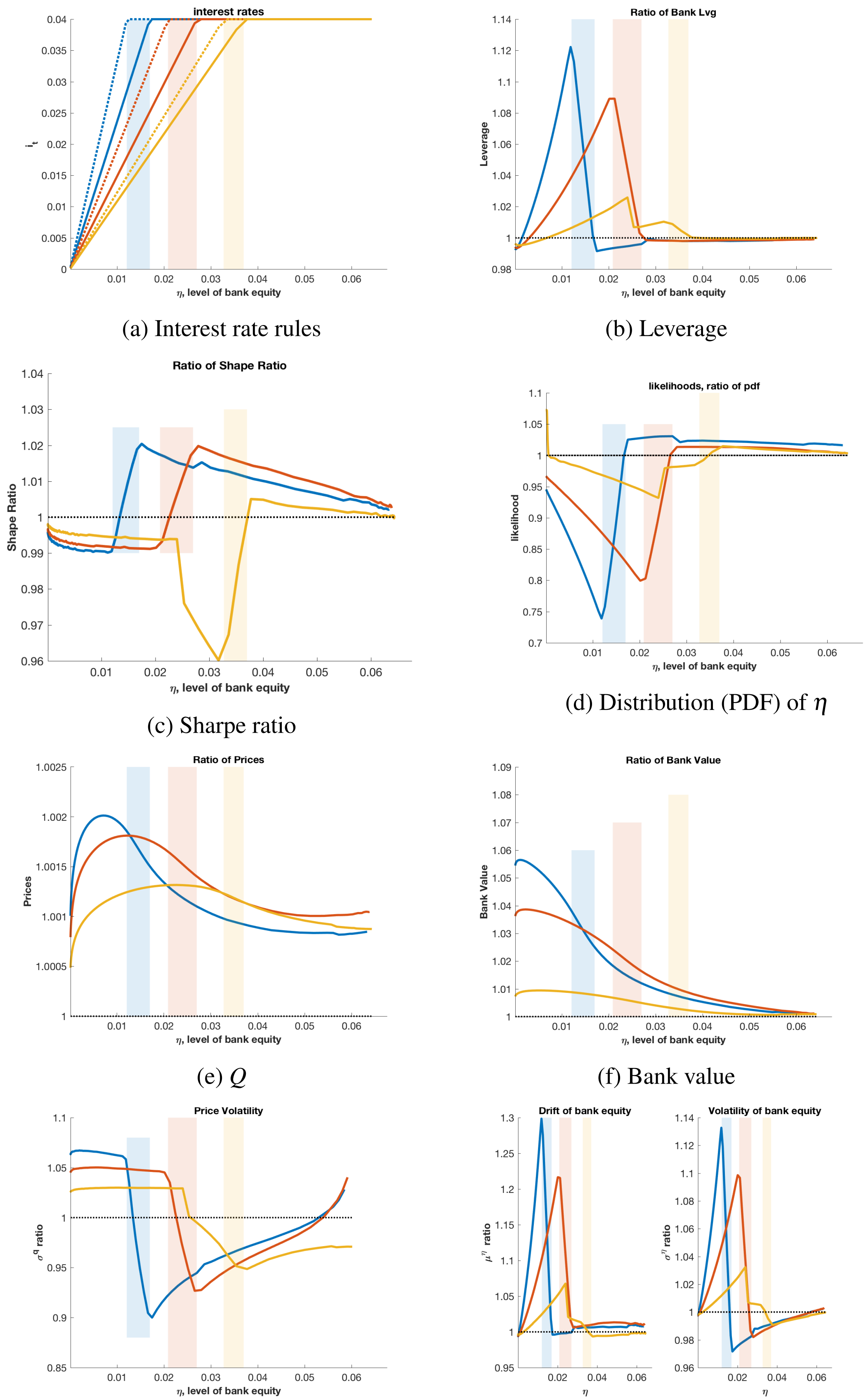
Figure 15 plots the changes in the asset price and bank value. With the fed put, prices are higher in bad times and lower in good times, but the highest effectiveness to raise bad-time prices occurs for low strike puts. Banks always like the fed put. In the model, banks are owned by households, so there is no conflict of interest regarding how households would value the fed put relative to a policy of constant rates. However, household welfare is not everywhere higher with the fed put (because the asset price falls), and so if households and banks had competing interests, banks would lobby for a fed put.

We first illustrate how inflation costs can affect welfare by looking at a simple example. Figure 16 plots welfare with high and low constant rates and with a fed put, incorporating two sets of inflation costs with $\varepsilon_\pi = 40$ and $\varepsilon_\pi = 20$ and supposing that the baseline rate is $\bar{i} = 4\%$. Hence, welfare with rates at constant 4% is always the same since this is the case without inflation losses. To give a sense of the quantitative significance, with $\varepsilon_\pi = 20$, as a fraction of the welfare level without inflation losses, the welfare losses from inflation with constant rates of 0% are roughly 3.5% (i.e., $L(\eta)/V(\eta) \approx 3.5\%$). In contrast, the fed put policy leads to relative inflation losses varying from 2-3% (highest when calculated starting at very low η).

With low inflation costs, welfare is still highest with $i_t = 0\%$ (consistent with the Friedman Rule) and hence welfare is maximized for zero interest rates whenever $\varepsilon_\pi \leq 20$. However, for $\varepsilon_\pi = 40$, the fed put delivers highest welfare, and the lowest welfare occurs when rates are constant at zero. With costs at this level, the inflation loss from setting rates at zero outweigh the benefits from higher asset prices. With inflation cost $\varepsilon_\pi \geq 40$, welfare is highest for constant rates at 4% (i.e., no deviations in policy and therefore no inflation losses).

Figure 17 plots the changes in the asset price and bank value. The marginal impact of increasing prices varies over the state space similarly to the effect of "forward guidance" in Drechsler et al. (2018). Additionally, for low η, the marginal impact on the asset price is greatest when the central bank policy occurs for low η^P. However, the marginal impact on bank value is always greater when the central bank policy occurs for low η^P. When the central bank cuts rates at a high η^P, the marginal impact of extending the rate cut has almost no effect on bank value, and even decreases bank value in the lowest-capitalized states. In essence, the effectiveness of monetary policy to support the economy (including bank values) wears off in this case, and extending the range of easing means that monetary policy becomes ineffective more quickly.

Figure 18 plots the marginal impacts on inflation loss (since it plots a ratio, the figure applies for all $\varepsilon_\pi > 0$).



(a) Interest rate rules

(b) Leverage

(c) Sharpe ratio

(d) Distribution (PDF) of η

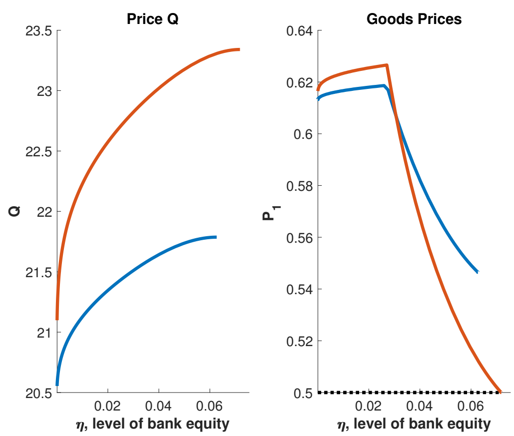
(e) Q

(f) Bank value

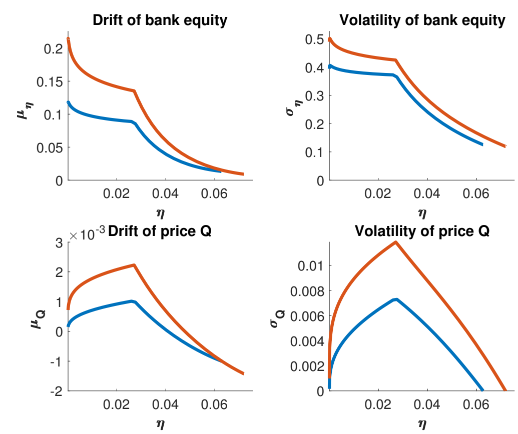
(g) Price volatility

(h) η evolution

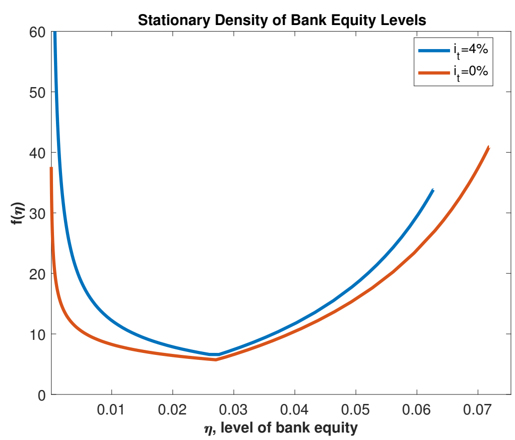
Figure 11: Marginal impact of extending fed put.



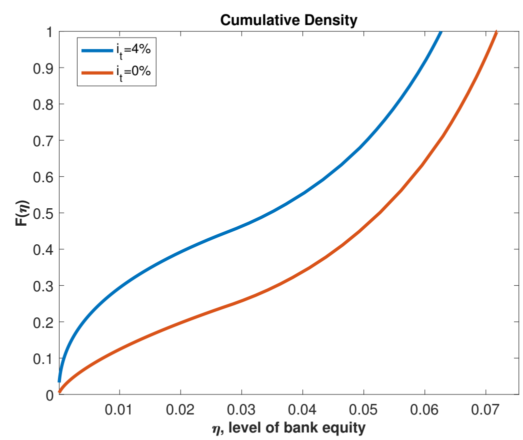
(a) Capital and goods prices



(b) Evolutions of Q and η

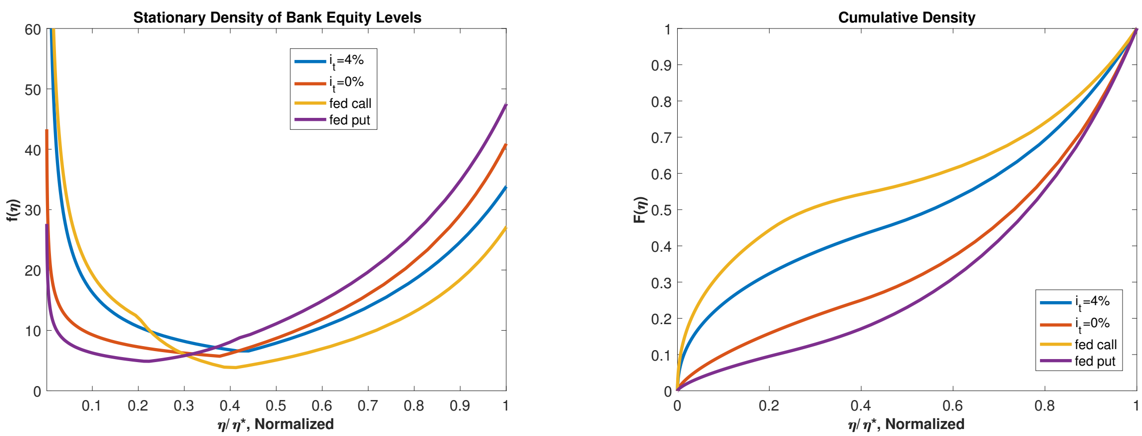


(c) PDF of η



(d) CDF of η

Figure 12: Equilibrium with high and low constant interest rates.



(a) PDF

(b) CDF

Figure 13: Stability (normalized) with constant rates or fed put/call policies.

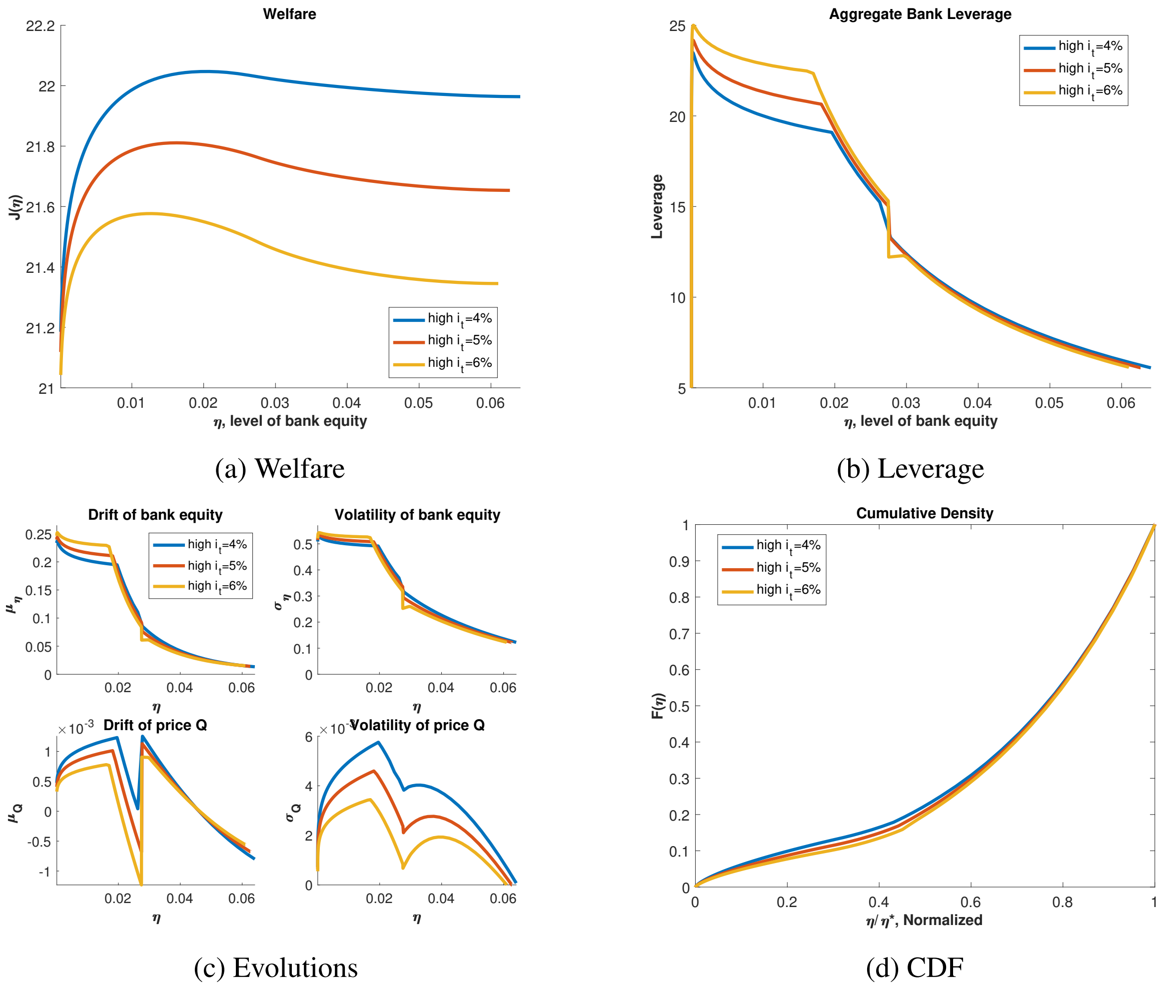
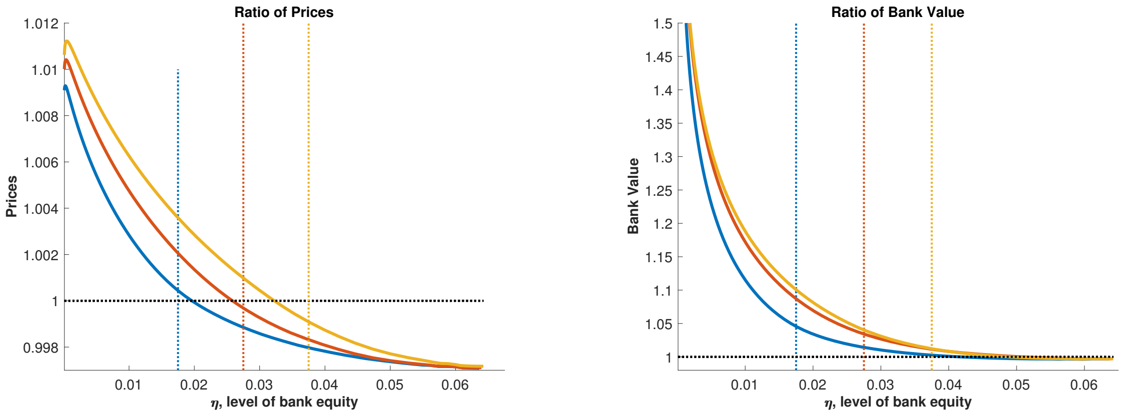


Figure 14: Equilibrium and leaning against the wind.



(a) Asset price Q

(b) Bank Value

Figure 15: Effect of fed put on asset price and bank value.

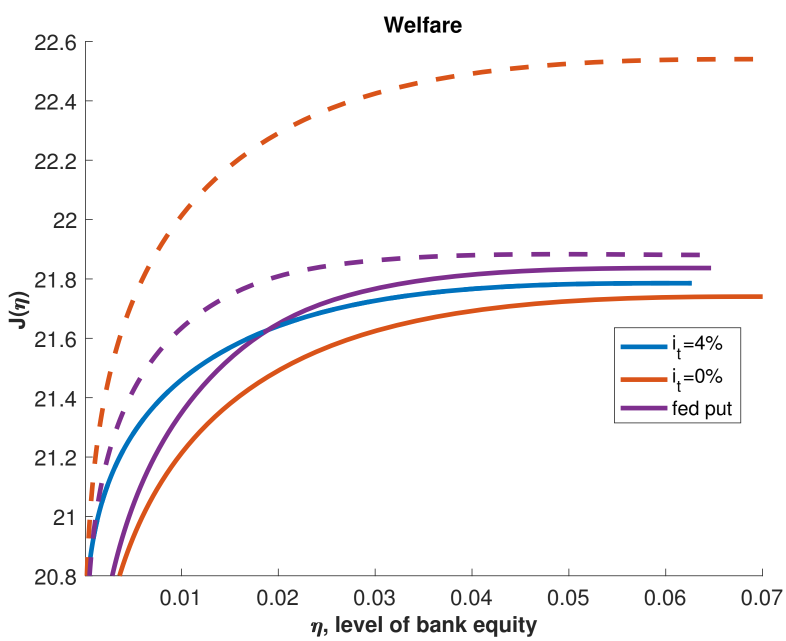
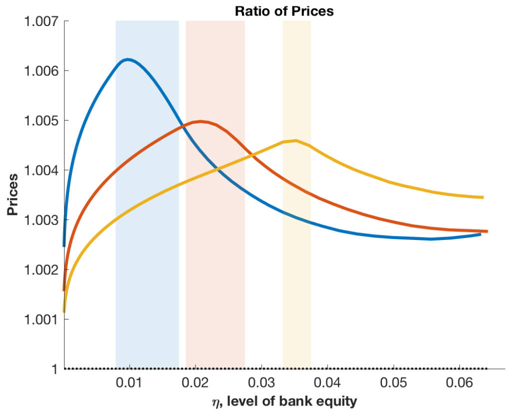
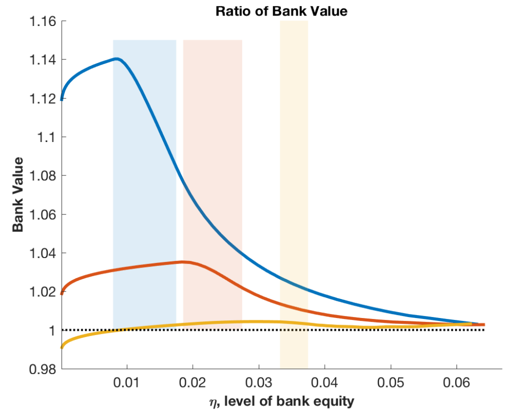


Figure 16: Welfare and inflation costs. Solid lines are high inflation cost ($\varepsilon_\pi = 40$) and dashed lines are low inflation cost ($\varepsilon_\pi = 20$).



(a) Change in Q



(b) Change in bank value

Figure 17: Marginal impacts of extending low interest rates on asset price and bank value.

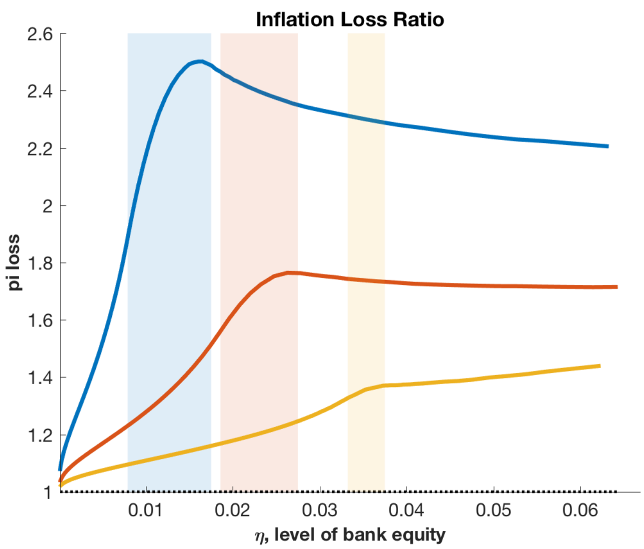


Figure 18: Marginal impact on inflation costs of extending low rates.