Class # 29: Multiple Comparisons

Reading: Section 10.2 of Devore

Let us assume that you test \( H_0 : \mu_1 = \mu_2 = \cdots = \mu_I \) vs. \( H_a : \) not \( H_0 \). If you reject \( H_0 \) in favor of \( H_a \) and then look at the means to see which one (or which ones) is different from the others, you need to account for the fact that you are doing multiple comparisons. To account for the fact that you are doing multiple comparisons, I recommend that you use Tukey’s procedure. In Tukey’s procedure, a confidence interval can be calculated between group \( i \) and group \( j \) using the formula

\[
\bar{X}_i - \bar{X}_j \pm Q_{\alpha, I(J-1)} \sqrt{\frac{MSE}{I}},
\]

where \( Q_{\alpha, I(J-1)} \) is the \( 1 - \alpha \) quantile of Tukey’s studentized range distribution. This can be calculated in R using \( \text{qtukey}(1-\alpha, I, I(J-1)) \). It is important to understand that the formula given above is not a 100(1 - \( \alpha \))% confidence interval for \( \mu_i - \mu_j \). This interval guarantees that all possible comparisons made this way will contain the truth 100(1 - \( \alpha \))% of the time.

Homework: These problems are due at the beginning of class on 5/5/10.

1. Show that the total sums of squares, \( SST = \sum_{i=1}^{I} \sum_{j=1}^{J} (x_{ij} - \bar{x}_i)^2 \), can be calculated as

\[
SST = \sum_{i=1}^{I} \sum_{j=1}^{J} x_{ij}^2 - \frac{1}{IJ} \left( \sum_{i=1}^{I} \sum_{j=1}^{J} x_{ij} \right)^2.
\]

2. Show that \( MSE = \frac{s_i^2 + s_j^2 + \cdots + s_k^2}{I} \).


5. Devore, page 395, #36.

Homework Solutions:

1. (a) \( H_0 : \mu_P - \mu_L = \mu_D = 25 \) vs. \( H_a : \mu_P - \mu_L = \mu_D > 25 \). In this case, \( t = (\bar{d} - 25)/\frac{s_d}{\sqrt{n}} = \frac{(105.7 - 25)}{103.85/\sqrt{19}} = 2.45 \), and the \( p \)-value is \( P(T > 2.45) \) where \( T \sim t_9 \). This is calculated as 1 - \( pt(2.45,9) = .018 \). Since .018 < .05, we reject \( H_0 \) in favor of \( H_a \).

(b) \( \bar{d} + t_{.05,9} \frac{s_d}{\sqrt{n}} = 105.7 + qt(.95,9) \frac{103.85}{\sqrt{19}} = 165.9 \)

(c) The “incorrect” use of the two-sample t-test gives a \( t \)-score of \( t = (\bar{X}_P - \bar{X}_L - 25)/\sqrt{\frac{s_P^2}{16} + \frac{s_L^2}{16}} = 0.44 \) and the corresponding \( p \)-value is \( P(T > 0.44) \) where \( T \sim t_9 \). This probability is 0.65, so using the incorrect two-sample \( t \)-test would make us fail to reject \( H_a \).

2. \( H_0 : \mu_{\text{Exposed}} - \mu_{\text{Unexposed}} = 0 \) vs. \( H_a : \mu_{\text{Exposed}} - \mu_{\text{Unexposed}} < 0 \). In this case \( t = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}} = -1.1/\sqrt{5} = -2.2 \). The \( p \)-value is \( P(T < -2.2) \) where \( T \sim t_9 \), and this is calculated in R as \( pt(-2.2, 9) = .027 \). Since this is less than \( \alpha = .05 \), we reject \( H_0 \) in favor of \( H_a \).

3. \( H_0 : \mu_B - \mu_A = 0 \) vs. \( H_a : \mu_B - \mu_A < 0 \). In this case, \( t = (\bar{X}_B - \bar{X}_A)/\sqrt{\frac{s_B^2}{14} + \frac{s_A^2}{16}} = (61.0 - 74)/\sqrt{\frac{12.57}{14} + \frac{14.82}{16}} = -2.61 \). The \( p \)-value is \( P(T < -2.61) \) where \( T \sim t_{13} \), and this is calculated in R as \( pt(-2.61,13) = .0107 \). Since this is less than \( \alpha \), we reject \( H_0 \) in favor of \( H_a \).

4. \( H_0 : \mu_d = 0 \) vs. \( H_a : \mu_d \neq 0 \). In this case, \( t = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}} = 1.757/\frac{1.10}{\sqrt{19}} = 3.88 \). The \( p \)-value is calculated as \( P(T > 3.88) + P(T < -3.88) \) where \( T \sim t_6 \), and this is calculated in R as \( 2pt(-3.88,6) = .0082 \). So we reject \( H_0 \) in favor of \( H_a \) at the .05, .01, but not at the .001 level.

5. We would always fail to reject \( H_0 \) since the \( p \)-value would always be greater than .5. Recall that \( p \)-value = \( P(T < .23/\frac{s_d}{\sqrt{n}}) \). Regardless of what \( s_d \) and \( n \) are, the probability is greater than .5.