FrankFest 2016

Conference on Isoperimetric Problems

Williams College

Friday and Saturday, February 5-6, 2016

Fred Brasz (Exa Corporation)

Title: *Voronoi Games*

Abstract: Given a set of sites in a plane, the Voronoi diagram divides the plane into nearest-neighbor regions surrounding each site. That is, all points in a given site's region are closer to that site than any other. After introducing Voronoi diagrams and how they are calculated, I will discuss a simple game in which opposing players take turns placing sites in a plane, trying to maximize the total area of their regions. Only for the one-dimensional version of the game is an optimal strategy known. A demo of the full game will be shown, along with a possible extension using weighted Voronoi diagrams.

Thomas Coleman (University of Missouri-Columbia)

Title: *Exotic Cohomology Products on Toric Stacks*

Abstract: The cohomology ring of a toplogical space uses the cup product to produce a useful algebraic structure. Taking an algebro-geometric viewpoint, I will discuss some interesting variations on this concept which originate in Gromov-Witten theory.

Mark Conger (University of Michigan)

Title: *The Logistic Equation: A Multimedia Experience*

Abstract: The discrete Logistic model of population growth is a great example of how a deterministic system can sometimes be predictable, and sometimes not. It often serves as an introduction to chaotic behavior. This talk will present a tool for viewing it in several ways, with the aim of communicating the immense strangeness of the situation. We will ask some deep questions and generate some very weird music.

Ivan Corwin (Clay Mathematics Institute, Columbia University)

Title: *Integrable probability and the KPZ universality class.*

Abstract: The Gaussian central limit theorem says that for a wide class of stochastic systems, the bell curve (Gaussian distribution) describes the statistics for random fluctuations of important observables. In this talk I will look beyond this class of systems to a collection of probabilistic models which include random growth models, particle systems, optimization problems, matrices and stochastic PDEs. I will explain how through certain "integrable" or "exactly solvable" examples we are able to see how these different examples all fall into a single universality class – the KPZ class – with a much richer mathematical structure than that of the Gaussian. This talk is expository and meant for a wide audience without any background in these areas.

Ellie Cowen (Edutopia)

Title: *The Area of Autumn Leaves, or How to Teach Calculus to Sixth Graders*

Abstract: Increasing student engagement, especially among reluctant learners, is a hot topic of discussion among math educators. A selection of case studies at the elementary level debunk some education myths and suggest that teachers can heighten engagement and participation by communicating more refined academic values, encouraging students to struggle and fail regularly, allowing them to talk more often, and posing high-level questions about topics in which they express interest.

Trubee Davidson (University of Colorado-Boulder)

Title: *A Kantorovich metric for projection-valued measures*

Abstract: Given a compact metric space X, the collection of Borel probability measures on X can be made into a complete metric space via the Kantorovich metric. We generalize this well known result to projection-valued measure from X onto the projections on a fixed Hilbert space $\mathcal{H}$. We develop properties of this generalized metric space, and look at an application to iterated function systems on X.

Diana Davis (Northwestern University)

Title: *Billiards and the double pentagon*

Abstract: Identifying opposite parallel edges of a square creates a torus, and in a similar way we can create a surface from many other polygons as well. I'll explain my favorite such surface, the double pentagon, with lots of pictures and one video.

Alexander Diaz-Lopez (University of Notre Dame)

Title: *Isoperimetric problems in sectors with density*

Abstract: Isoperimetric problems have been a topic of discussion for many years. In this talk we consider the isoperimetric problem in planar sectors with density. We will focus in sectors with density r^p, and with density $a > 1$ inside the unit disk and 1 outside. We characterize solutions as a function of sector angle.

Herbert Edelsbrunner (IST Austria)

Title: *Approximation and convergence of the intrinsic volume*

Abstract: We study the computation of intrinsic volumes of a solid body from a sequence of binary images of progressively finer resolution. While the intrinsic volumes of the binary images do not necessarily converge to the correct value, we show that the formula can be rigged to give the correct limit for the first intrinsic volume, which in R^3 relates to the total mean curvature of the boundary of the body.

Work with Florian Pausinger.

Max Engelstein (University of Chicago)

Title: *Geometric Measure Theory and the Singular Set of a Two-Phase Free Boundary Problem*

Abstract: We will combine tools from geometric measure theory and harmonic analysis with more "traditional" methods from minimal surface theory to analyze the singular set of a two-phase free boundary problem for harmonic measure. This talk is based on joint work with Matthew Badger (UConn) and Tatiana Toro (UW-Seattle).

Joel Foisy (SUNY Potsdam)

Title: *A survey of intrinsically linked and intrinsically knotted graphs*

Abstract: Take 6 points in space, and connect every possible pair of points by non-intersecting arcs. In the 1980s, Conway-Gordon and Sachs proved that no matter how the points are connected, two non-splittably linked loops will form.

We say that the complete graph on six vertices is *intrinsically linked*. Conway and Gordon also proved that the complete graph on seven vertices is *intrinsically knotted*. Mathematicians have since attempted to classify all intrinsically linked and intrinsically knotted graphs. In the 1990s, Robertson, Seymour and Thomas classified the complete set of "minor-minimal" intrinsically linked graphs. Their proof is difficult, and intrinsically knotted graphs have not yet been classified.

In this talk, we will survey some known results and open questions about intrinsically linked and intrinsically knotted graphs. There will be a lot of pictures.

Joel Hass (University of California, Davis)

Title: *Isoperimetric Regions*

Abstract: It is known that the circle is the shortest curve in the flat plane enclosing a given area. But what about a non-flat plane? I will discuss some results on the shape of isoperimetric regions in a nonflat plane.

Sean Howe (University of Chicago)

Title: *Computers, monodromy, and the one line on a cubic surface*

Abstract: Computers, monodromy, and the one line on a cubic surface Abstract: Youve probably heard about the 27 lines on a cubic surface, but did you know theres actually just ONE?! Well, sort of. In this talk we explain the following result: The average number of rational lines on a smooth cubic surface over a finite field F_q is 1 (as $q \to \infty$ dodging certain primes). In fact, this is a reinterpreta- tion of a monodromy computation dating back to the 19th century. In this talk I'll explain this connection between geometric monodromy and arithmetic statistics, and show you how to compute monodromy groups using only a computer and some good luck.

Michael Hutchings (University of California, Berkeley)

Title: *Fun with symplectic embeddings*

Abstract: We give an introduction to the problem of determining when one domain in $\mathbb{R}^{2n}$ can be symplectically embedded into another. This problem is studied with the help of pseudoholomorphic curves, which are a kind of minimal surface. Many intriguing combinatorial structures arise, as well as some problems suitable for undergraduate research.

Jeff Jauregui (Union College)

Title: *Huiskens isoperimetric mass in general relativity*

Abstract: Consider a time snapshot M of a spacetime (i.e., a Riemannian 3-manifold asymptotic to Euclidean space). Huisken defined the isoperimetric mass of M in terms of the isoperimetric behavior of large balls in M. The isoperimetric mass has applications to problems involving metrics of low regularity, by virtue of being defined only in terms of volume and area (and not curvature). We will discuss some properties of this mass and describe, in joint work with Dan Lee, its behavior under local uniform convergence of the metrics.

Patricia Klein (University of Michigan)

Title: *What are Cohen-Macaulay Modules?*

Abstract: We will give a definition of 'Cohen-Macaulay' and then discuss through pictures and examples why Cohen-Macaulay is the right condition for doing intersection theory. Time permitting, we will also state some other desirable properties of Cohen-Macaulay modules.

This talk will be directed at finishing undergrads and first- or second-year grad students.

Jake Levinson (University of Michigan)

Title: *What is Schubert Calculus?*

Abstract: I will discuss a famous geometric problem: given four fixed lines in three-dimensional space, how many lines pass simultaneously through all four? This question, and others like it, led to the development of enumerative geometry, including the branch now known as Schubert calculus.

Interestingly, although Schubert knew the answer in the nineteenth century, it took decades before the theory was made truly rigorous. Even now, many similar questions remain unsolved.

Jonathan Lovett (Obama speech writer, creator of primetime TV show 1600 Penn and FacTrak)

Title: *Good at Math (for a writer)*

Abstract: Jon Lovett, '04, uses a series of absurd charts and graphs to reflect on his time at Williams, his appreciation for Professor Morgan and the math department, and how studying math has helped him in politics, comedy, and picking Powerball numbers.

Nicholas Neumann-Chun (Coding House)

Title: *Coding Bootcamp*

Abstract: How does a math background help you in entering the software industry? Answer: a lot! I will talk about the phenomenon of coding bootcamps in general and, more specifically, my experiences at the bootcamp I am currently attending. It is called Coding House and is located in Silicon Valley. Its purpose is to teach as much about web development focusing on JavaScript as possible in 14 weeks and help its students find a job in the industry afterwards.

Rohini Ramadas (University of Michigan)

Title: *The Mandelbrot set*

Abstract: We discuss the polynomial $p(z) = z^2 + c$.

Ben Steinhurst (McDaniel College)

Title: *Bond Percolation on the Hexacarpet and a Non-P.C.F. Sierpinski gasket*

Abstract: A Non-P.C.F. (post-critically finite) variation of the Sierpinski gasket is introduced as a limit of graphs embedded in R^2. The hexacarpet is the limit of the duals of "thinned out" non-p.c.f. Sierpinski gaskets. On these two fractals the question arises of the critical probability for bond percolation in the limit. It will be shown that the critical probability is for the non-p.c.f. Sierpinski gasket less than 0.282. Consequently on the hexacarpet the probability is greater than 0.718.

Matthew Simonson (Northeastern University)

Title: *Between Two Curves: The Isoperimetric Problem on Bounded Surfaces*

Abstract: In this talk, we will examine the isoperimetric problem, the least perimeter way to enclose a given area, on surfaces where the boundary is "free," meaning it does not contribute to total perimeter. A farmer wishing to enclose a pig pen against the side of a barn, for example, would do best to build a semicircular fence that meets the barn wall at right angles in order to save on fencing. We begin by examining an annulus, the region between two circles, and then

develop a theorem that applies to all regions of the plane, hyperbolic plane, and sphere bounded by one or two constant curvature curves. We will also explore the solution on bands, Mobius bands, twisted chimney spaces, concluding with a most surprising result about a certain type of hyperbolic band where the minimizer alternates back and forth between two types of curves as the area grows.

Rob Silversmith (University of Michigan)

Title: *The space of triangles*

Abstract: How do we classify plane triangles, up to similarity? This question leads us naturally to the notion of an orbifold, a quotient of a manifold by a group action.

Sarah Tammen (University of Georgia)

Title: *The isoperimetric problem in* $\mathbb{R}^n$ *with density* r^p

Abstract: In isoperimetric problems in spaces with density, the objective is to enclose a prescribed weighted volume with minimum weighted perimeter. According to Gregory Chambers' proof of the Log-Convex Density Conjecture, for a certain class of density functions on $\mathbb{R}^n$, isoperimetric regions are balls centered at the origin. We adapt Chambers methods to prove that if $\mathbb{R}^n$ has density r^p, where r is distance from the origin and $p > 0$, then isoperimetric regions are bounded by spheres that pass through the origin. The $n = 2$ case was proved by the 2008 SMALL Geometry Group. For $n \geq 3$, an isoperimetric hypersurface is a surface of revolution, and the generating curve satisfies a differential equation resulting from the fact that the surface has constant generalized mean curvature. We use the differential equation to deduce the behavior of the curve based on initial curvature and to prove that the curve is a circle through the origin. The proof is joint work with Wyatt Boyer, Bryan Brown, Gregory Chambers, and Alyssa Loving.

Hung Tran (University of California-Irvine)

Title: *Complete manifolds with bounded curvature and spectral gaps*

Abstract: We study the spectrum of complete noncompact manifolds with bounded curvature and positive injectivity radius. We give general conditions which imply that their essential spectrum has an arbitrarily large finite number of gaps. Some applications include showing spectral gaps for noncompact

covering of a compact manifold or a noncompact manifold with a metric uniformly equivalent to a given one. One ingredient in the proof is the usage of isoperimetric (Sobolev) inequalities. This is a joint work with Richard Schoen.

Robin Walters (Northeastern University)

Title: *The Bernstein-Sato Polynomial of a Hyperplane Arrangement*

Abstract: The Bernstein-Sato polynomial, or b-function, is an important invariant in singularity theory, which is difficult to compute in general. I will describe several related computations of the b-function for different hyperplane arrangements coming from Lie theory. I will also show how the b-function relates to other singularity invariants such as vanishing cycles, zeta functions, Milnor monodromy, and jumping coefficients.

William Wylie (Syracuse University)

Title: *Comparison Geometry for manifolds with density*

Abstract: We'll discuss the extension of the theory of weighted Ricci curvature to the range of "negative" synthetic dimension. In this regime a new natural geometric approach to manifolds with density arises focusing on a weighted torsion-free affine connection which is projectively equivalent to the usual Riemannian connection. This is joint work with Dmytro Yeroshkin of Syracuse.

Luis Sordo Vieira (University of Kentucky)

Title: *Artins Conjecture for Diagonal Forms*

Abstract: One of Artins famous conjectures states that a homogeneous polynomial of the type $a_1x_1^d + \cdots + a_sx_s^d$ over a p-adic field K has a nontrivial zero in K^s provided $s > d^2$. The conjecture is known to be true over $\mathbb{Q}_p$ and recently now by collaborative work with David Leep for all unramified extensions of $\mathbb{Q}_p$ with $p > 2$. We will talk about the history of Artins conjecture and explore some of the known results about Artins conjecture on local fields and other fields of interest.

Ted Welsh (Westfield State University)

Title: *Dividing Fractions Using Pictures and Words: an Elementary Approach*

Abstract: What the heck does it even mean to divide one fraction by another fraction? I'll bet we all know an algorithm or two we can use to find the

right answer, but why do they work? I have examined this problem with my future elementary school teachers, and we have come up with some visualization strategies I would like to share with you.

Jason Zimba (Student Achievement Partners)

Title: *Is Math for Everyone? Lessons I Learned from Frank Morgan at Williams and Beyond*

Abstract: As a Williams student, I took Linear Algebra with Frank Morgan as my professor. That experience led to my declaring a math major, which in turn set me on an unpredictable journey that, so far, has included co-authoring the Common Core State Standards for Mathematics. In this talk I'll discuss the standards and how they relate to the mission of WIlliams itself.
