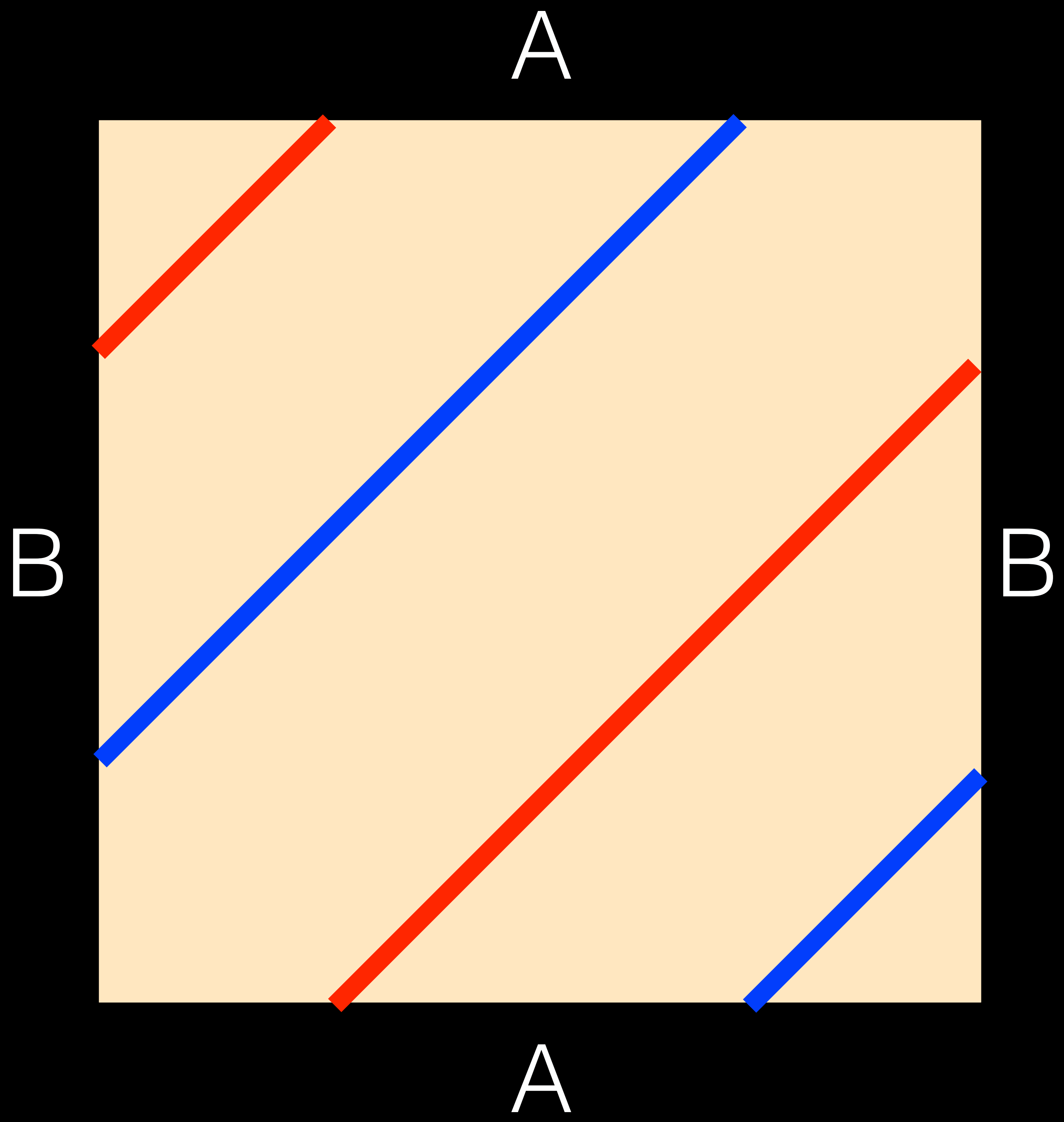
Billiards & the double pentagon: A Frank Morgan talk

Diana Davis

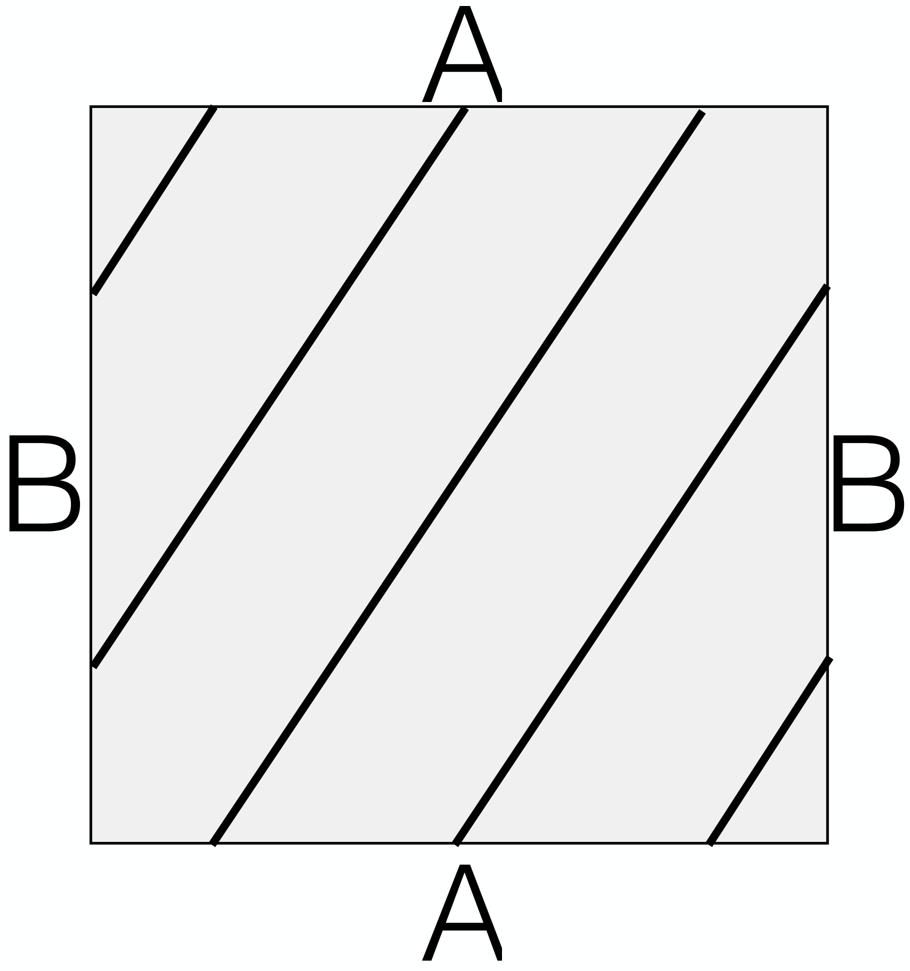
Current affiliation: Northwestern University
Next year: Williams College

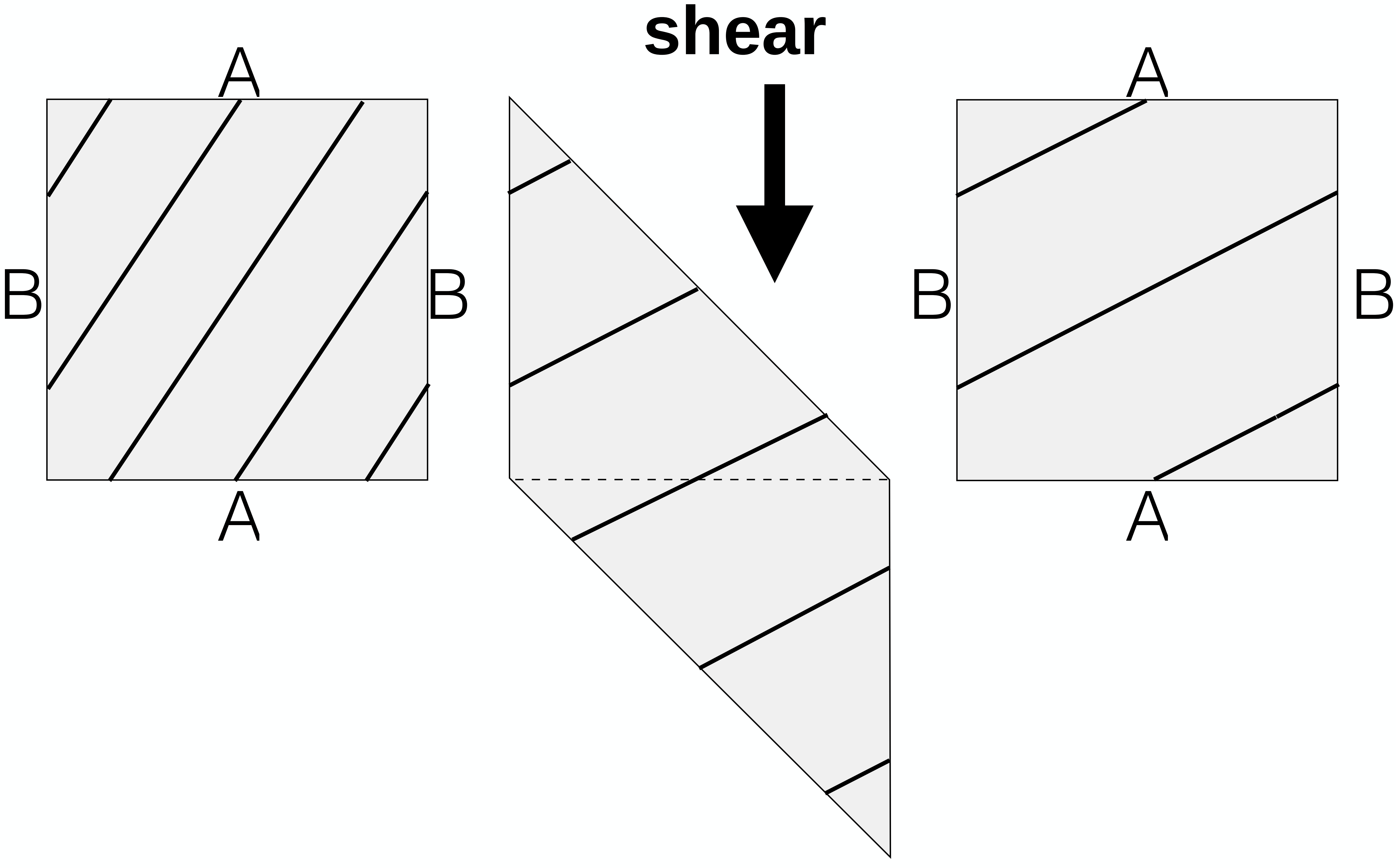
seeking senior thesis students for next year!

square torus with non-intersecting paths

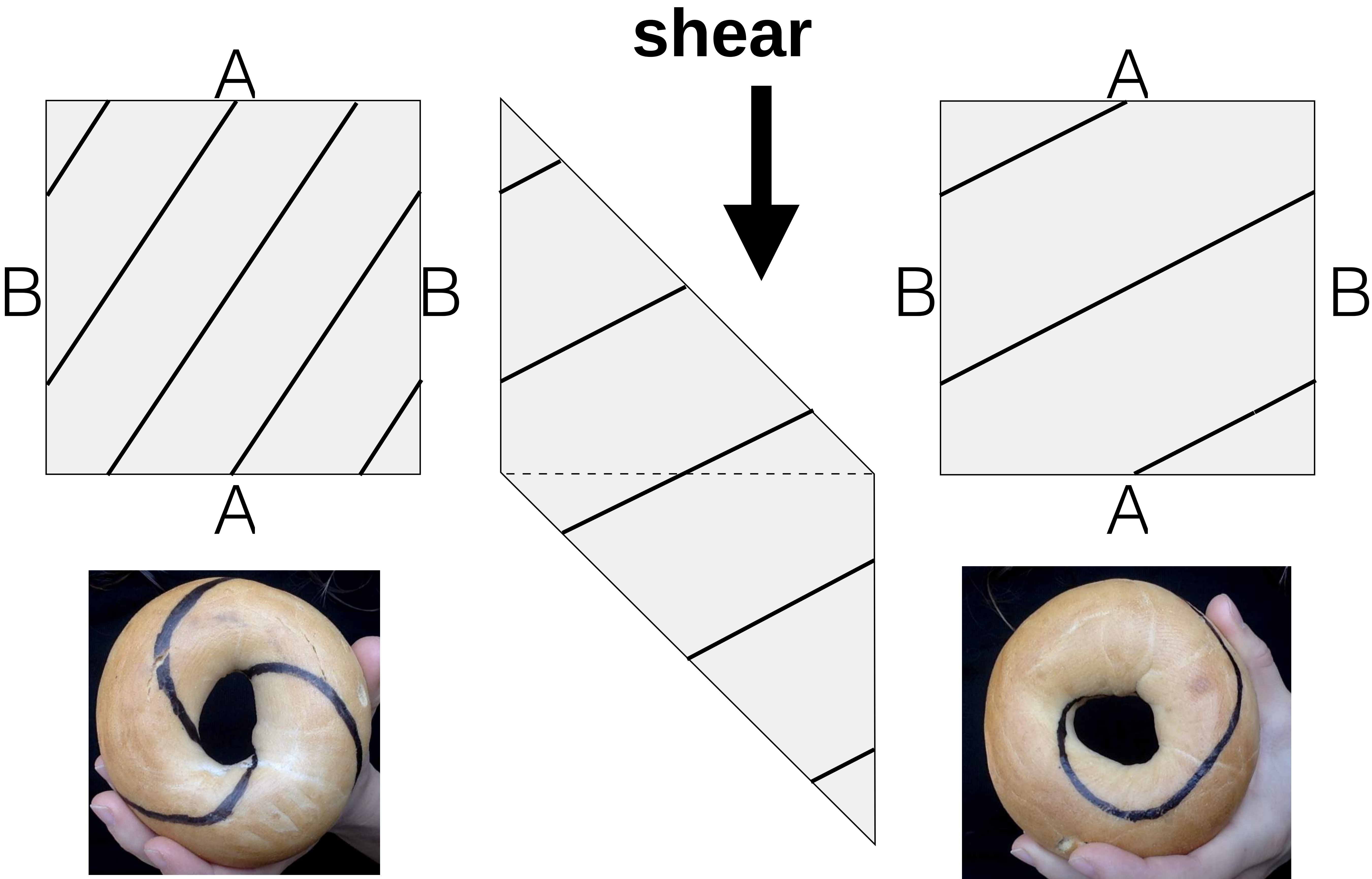


square torus with path





new path on square torus



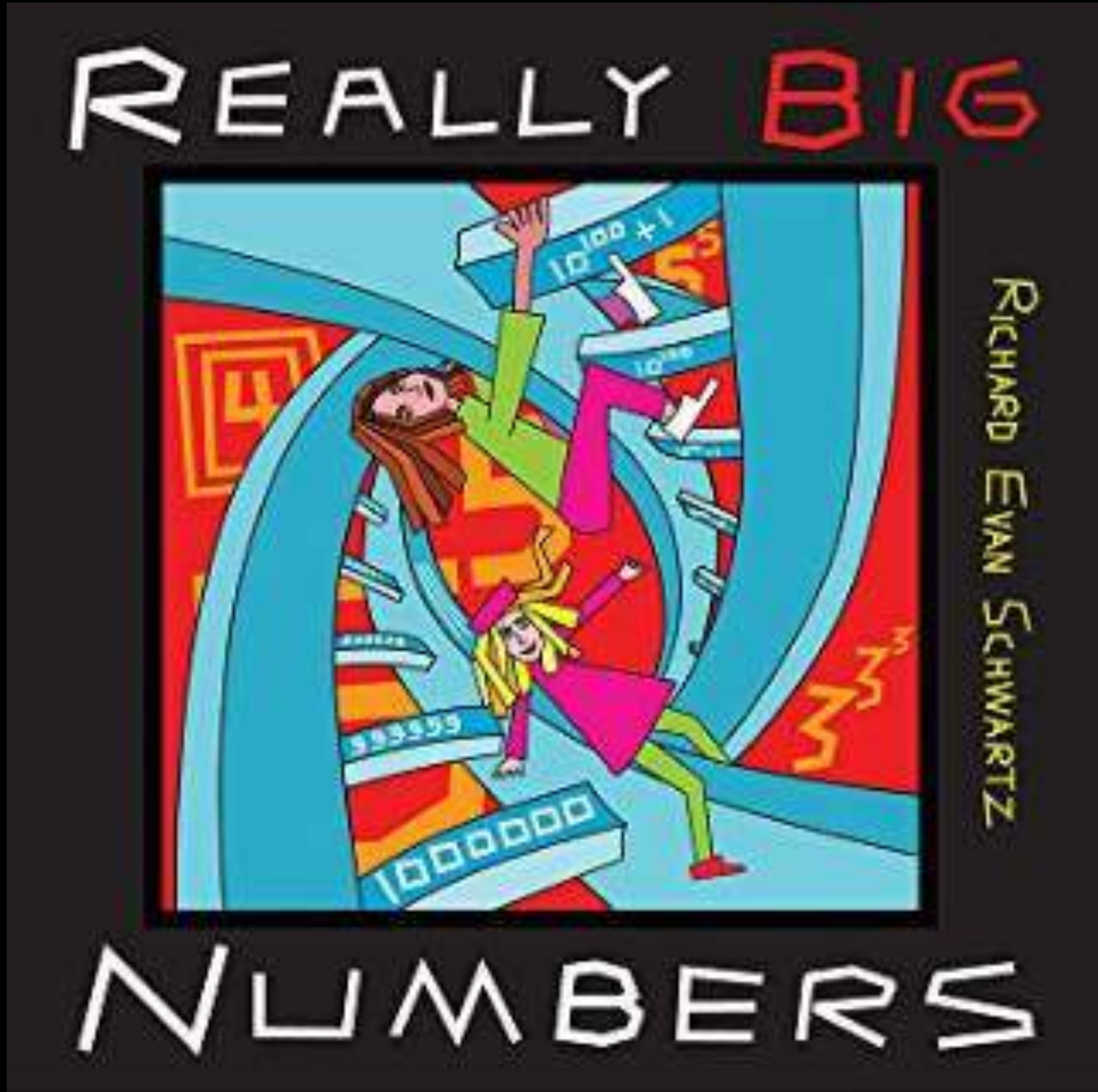
new path on square torus

Goal: describe all straight paths on a given surface

Strategy: understand symmetries of the surface to generate new paths from known paths


Guy
Fawkes
Rich
Schwartz

RICHARD EVAN SCHWARTZ


REALLY BIG
NUMBERS
RICHARD EVAN SCHWARTZ

Symbolic coding for linear trajectories in the regular octagon
Dynamique symbolique pour le flot linéaire sur l'octagone régulier

John Smillie* Corinna Ulcigrai†

Abstract

We consider a symbolic coding of linear trajectories in the regular octagon with opposite sides identified (and more generally in regular 2n-gons). Each infinite trajectory gives a cutting sequence corresponding to the sequence of sides hit. We give an explicit characterization of these cutting sequences. The cutting sequences for the square are the well studied Sturmian sequences which can be analyzed in terms of the continued fraction expansion of the slope. We introduce an analogous continued fraction algorithm which we use to connect the cutting sequence of a trajectory with its slope. Our continued fraction expansion of the slope gives an explicit sequence of substitution operations which generate the cutting sequences of trajectories with that slope. Our algorithm can be understood in terms of renormalization of the octagon translation surface by elements of the Veech group.

regular octagon by

Cutting sequences, regular polygons, and the Veech group

Diana Davis*

January 12, 2012

Abstract

We describe the cutting sequences associated to geodesic flow on regular polygons, in terms of a combinatorial process called *derivation*. This work is an extension of some of the ideas and results in Smillie and Ulcigrai' recent paper, where the analysis was made for the regular octagon. It turns out that the main structural properties of the octagon generalize in a natural way.

1 Introduction

In this paper, we will consider a surface obtained by identifying parallel edges of a pair of regular polygons. This creates a cone surface: a surface that is flat everywhere except possibly at a finite number of *cone points*. For example, identifying opposite parallel edges of a regular octagon yields a cone surface that is flat everywhere except at the single vertex, where the cone angle is $8 \cdot \frac{3\pi}{4} = 6\pi$.

Cone surfaces arise in billiards on rational polygons: Instead of considering a billiard path on the polygon, we use the Zemlyakov-Katok construction to reflect the polygon

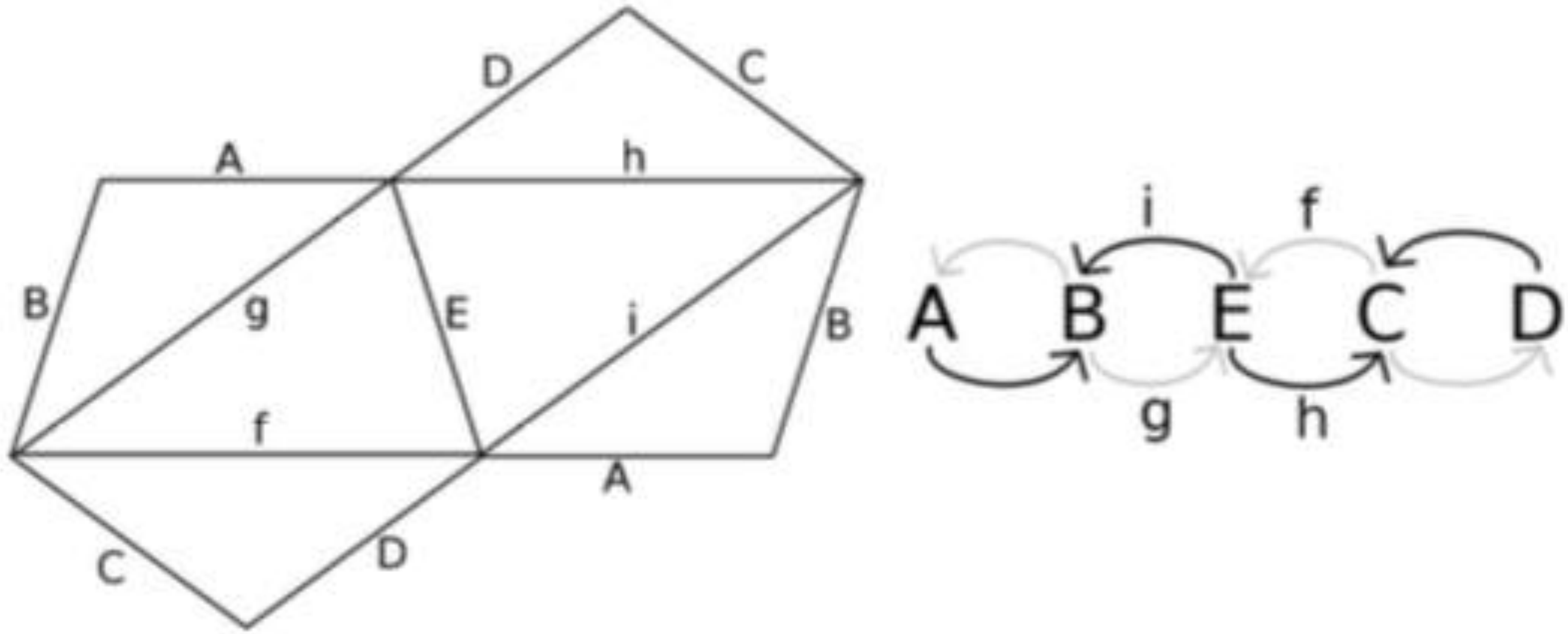


Figure 22: The double pentagon with auxiliary diagonals, and the "augmented arrows" transition diagram

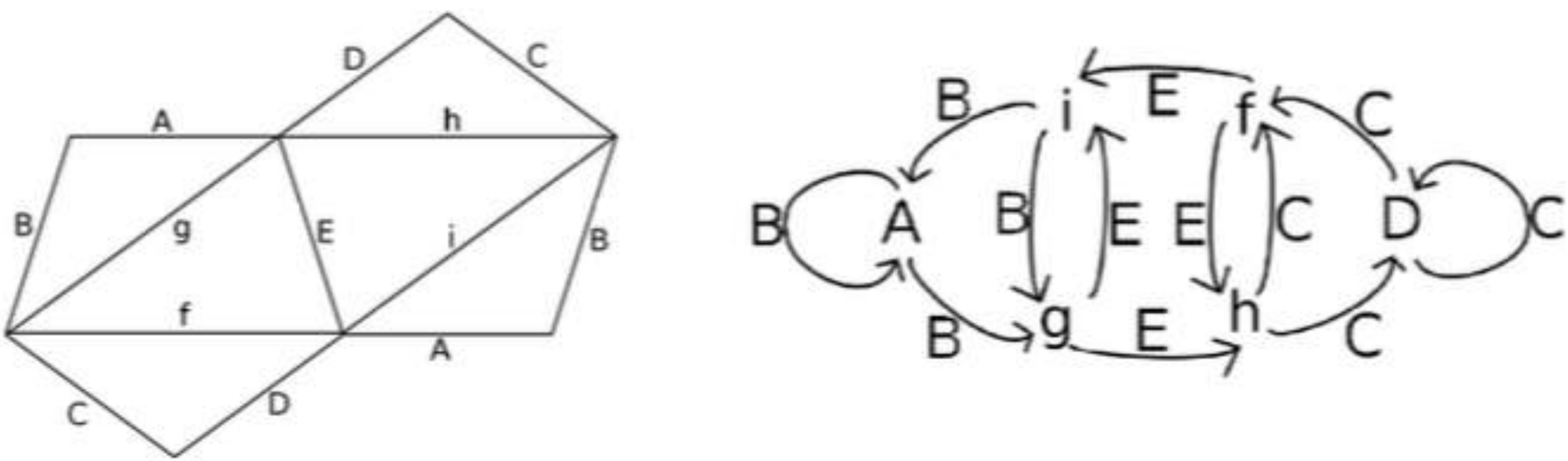


Figure 23: The "dual" transition diagram, with the sequence unchanged

Bouw-Möller surfaces

Bouw-Möller surfaces

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FROM RATIONAL BILLIARDS TO DYNAMICS ON MODULI SPACES

ALEX WRIGHT

ABSTRACT. This short expository note gives an elementary introduction to the study of dynamics on certain moduli spaces and, in particular, the recent breakthrough result of Eskin, Mirzakhani, and Mohammadi. We also discuss the context and applications of this result, and its connections to other areas of mathematics, such as algebraic geometry, Teichmüller theory, and ergodic theory on homogeneous spaces.

CONTENTS

New York Journal of Mathematics

New York J. Math. **20** (2014) 399–429.

Cutting sequences on translation surfaces

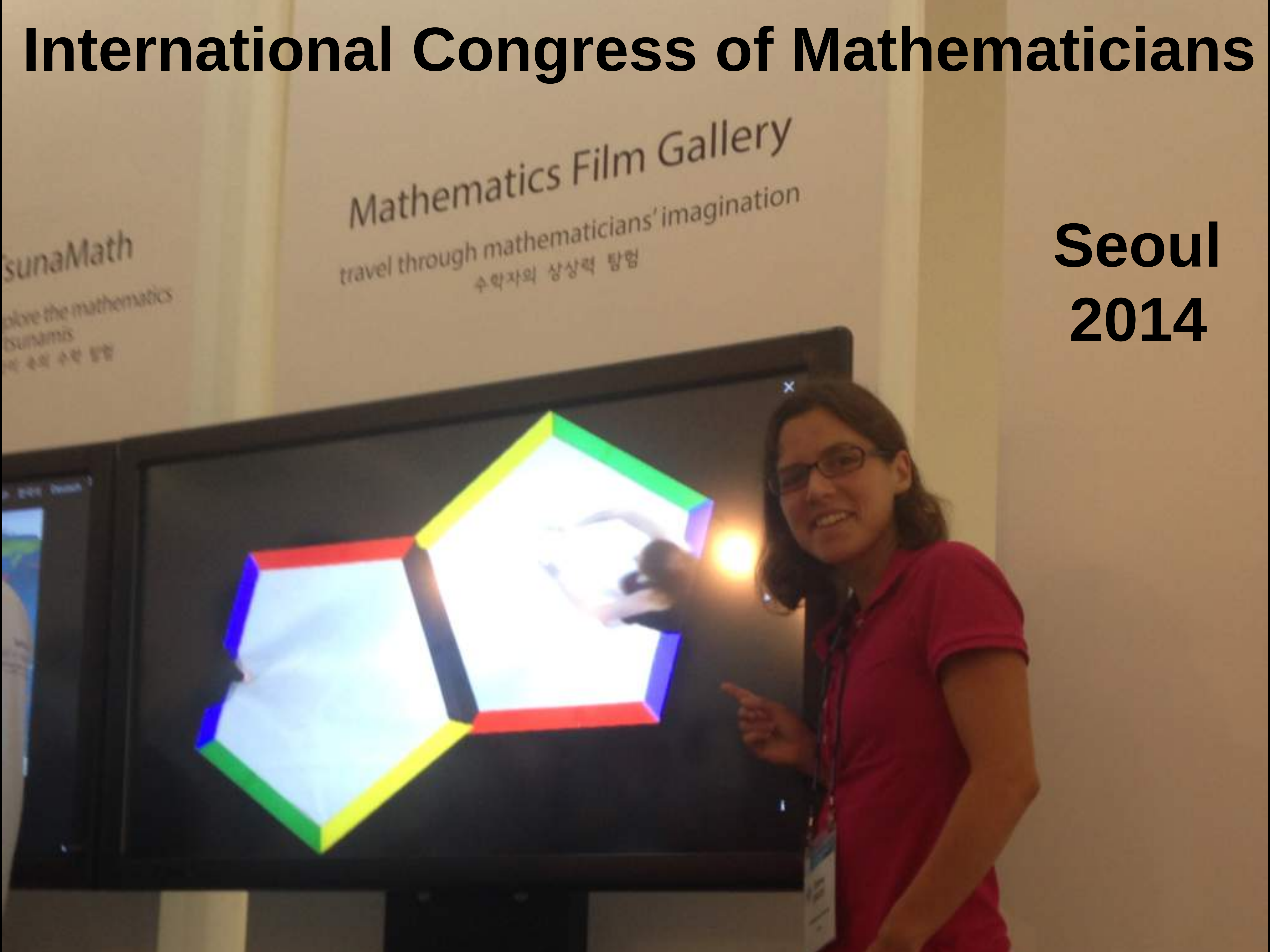
Diana Davis

Abstract. We analyze the cutting sequences associated to geodesic flow on a large class of translation surfaces, including Bouw–Möller surfaces. We give a combinatorial rule that relates a cutting sequence corresponding to a given trajectory, to the cutting sequence corresponding to the image of that trajectory under the parabolic element of the Veech group. This extends previous work for regular polygon surfaces to a larger class of translation surfaces. We find that the combinatorial rule is the same as for regular polygon surfaces in about half of the cases, and different in the other half.

Contents

International Congress of Mathematicians

Seoul 2014

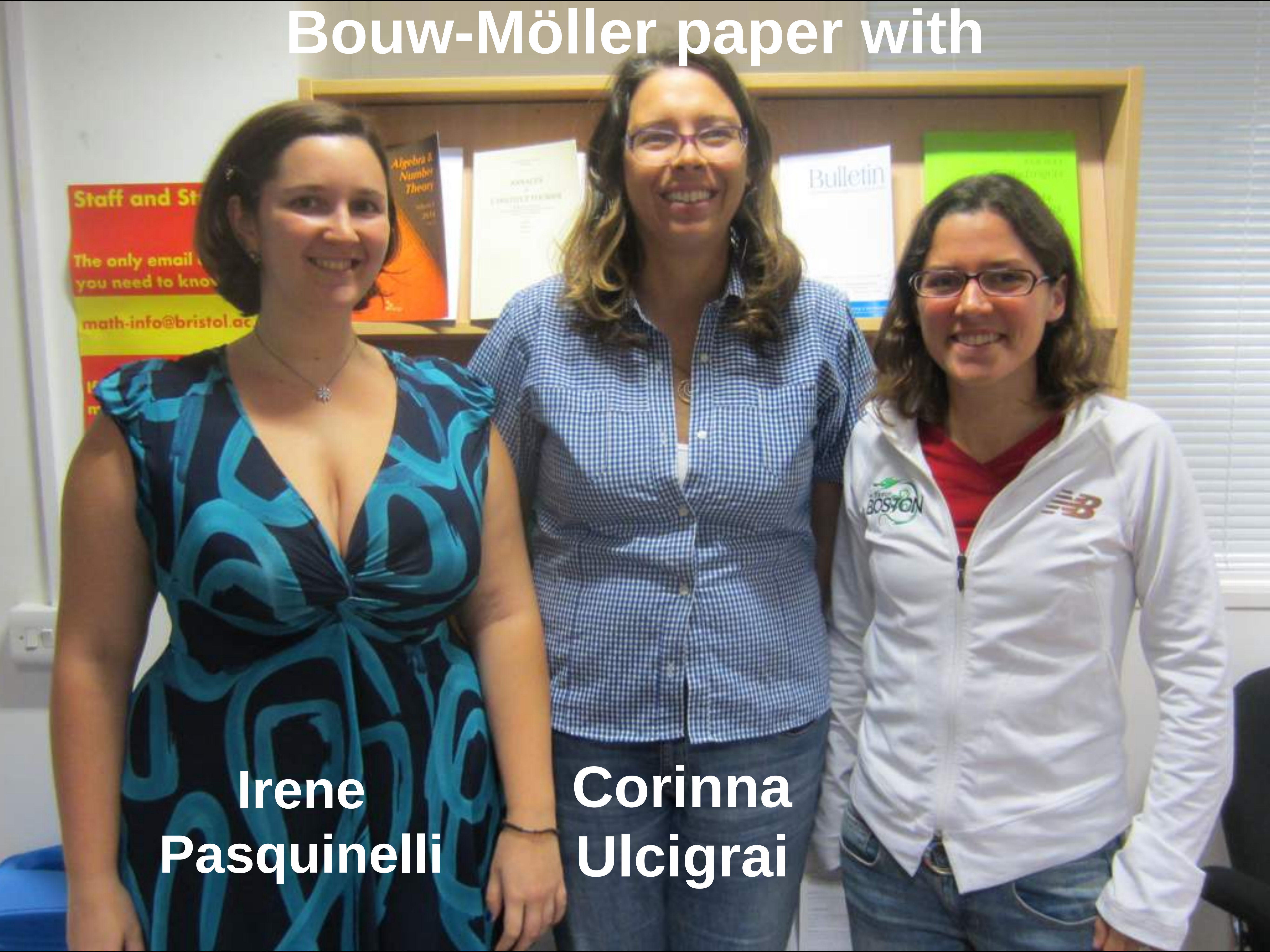



First, we shear the pentagons.
BRIDGES SEOUL 2014

together in Seoul

DD

Frank
Morgan


Bouw-Möller paper with
Irene
Pasquinelli
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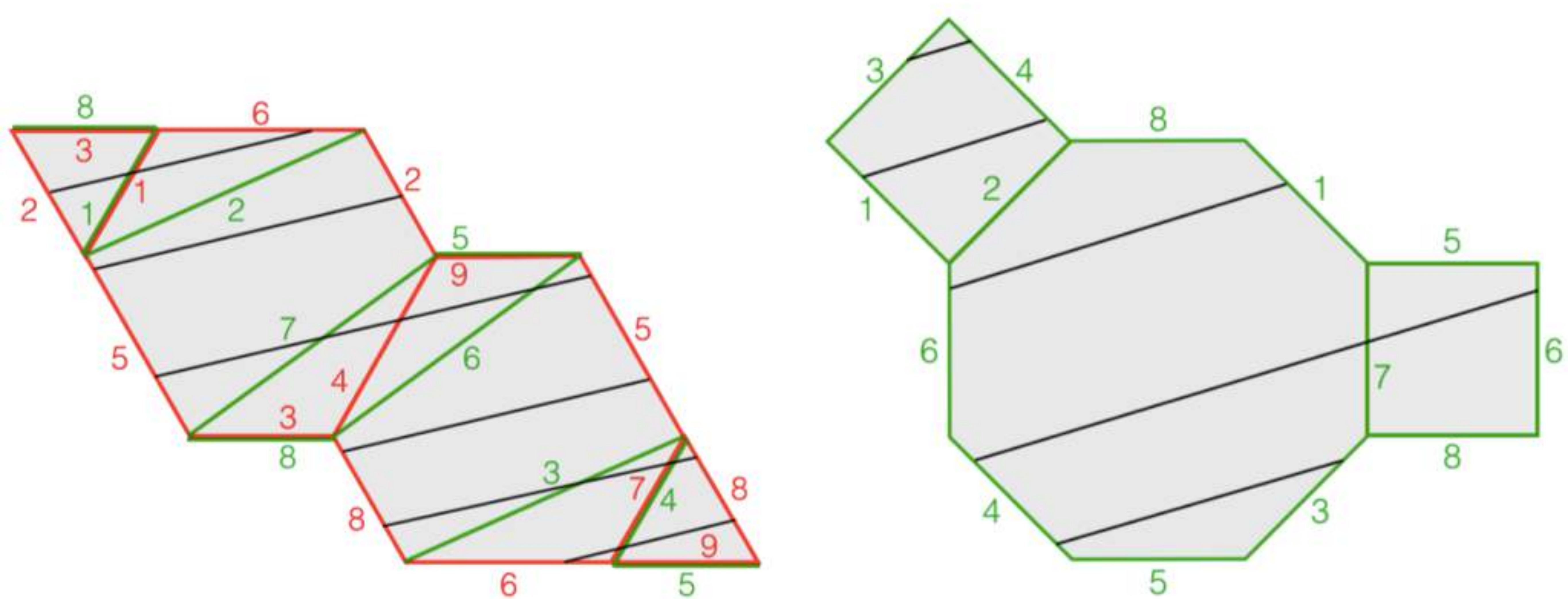
The only email
you need to know
math-info@bristol.ac
Algebra & Number Theory
Bulletin
BOSTON

Cutting sequences on Bouw-Möller surfaces: an $\mathcal{S}$-adic characterization

Diana Davis, Irene Pasquinelli and Corinna Ulcigrai

Abstract. We consider a symbolic coding for geodesics on the family of *Veech surfaces* (translation surfaces rich with affine symmetries) recently discovered by *Bouw* and *Möller*. These surfaces, as noticed by *Hooper*, can be realized by cutting and pasting a collection of *semi-regular polygons*. We characterize the set of symbolic sequences (*cutting sequences*) that arise by coding linear trajectories of the sequence of polygon sides crossed. We provide a full characterization for the closure of the set of cutting sequences, in the spirit of the classical characterization of Sturmian sequences and the recent characterization of Smillie-Ulcigrai of cutting sequences of linear trajectories on regular polygons. The characterization is in terms of a system of finitely many substitutions (also known as an $\mathcal{S}$-*adic presentation*), governed by a one-dimensional continued fraction-like map. As in the Sturmian and regular polygon case, the characterization is based on *renormalization* and the definition of a suitable combinatorial *derivation* operator. One of the novelties is that derivation is done in two steps, without directly using Veech group elements, but by exploiting an affine diffeomorphism that maps a Bouw-Möller surface to the *dual* Bouw-Möller surface in the same Teichmüller disk. As a technical tool, we crucially exploit the presentation of Bouw-Möller surfaces via *Hooper diagrams*.

Bouw-Möller surfaces



1678785452

1167483748574652

143476


my academic grandfather
Bill
Thurston

Mathematics is an art of human understanding...

Mathematical concepts are abstract, so it ends up that there are many different ways they can sit in our brains.

A given mathematical concept might be primarily a symbolic equation, a picture, a rhythmic pattern,
a short movie --

or best of all, an integrated combination of several different representations.

- Bill Thurston