# Commentary

## Another Opinion

# Pleasures vs. Problems

*"Can you give us more problems?"*
—A rarely heard sentiment

We mathematicians have many creative talents, but sharing our excitement and intellectual passion with nonmathematicians is not one of them. When the chalk dust settles at the end of the day, the mathematical community pays for this deficiency. Shrinking enrollments and suboptimal research funding are tangible consequences of the way mathematics is viewed by the public. Failed courses for nonscience students have a serious impact on the mathematics community.

Acknowledging this teaching issue is easy, but fixing the problem is much more difficult. Over the last ten years, the authors have experimented with various techniques for reaching liberal arts students. Some early attempts can best be described as disastrous, but gradually the following themes emerged for a style of presentation that brings mathematics to life for these students.

*Aim at great ideas.* In their other courses, students study the great works of literature, philosophy, art, and music. But frequently students' mathematics courses do not present correspondingly significant ideas. *Precalculus or college algebra should not be the final mathematics course for any student.* Precalculus and algebra provide essential techniques for a student who requires those quantitative skills, but we should not have students struggle up the first two rungs of a 100-rung ladder that they will never climb.

Mathematics courses for humanities students can present fascinating triumphs of thought. Embracing the ideas of infinity, facing quantitatively the uncertain and unknown, exploring geometrical reflections of our world and the geometry of the abstract worlds of the fourth dimension and topology, appreciating the nuances of number, and discovering the implications and beauty of chaos and fractals are just a few imaginative journeys that resonate with the human spirit. After taking such a liberal arts mathematics course, students will view mathematics as containing some of the greatest ideas of human history.

*Teach powerful methods of thinking.* Nonscience students will forget technical terminology, notation, and details of proofs. What can we realistically hope these students will retain from their mathematics course after ten years? Answer: Powerful methods of analysis, reasoning, and thought that transcend mathematics.

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Mathematicians have developed powerful techniques for exploring the unknown and analyzing complex situations. *Teaching effective thinking is an essential feature of this course.* Mathematics clearly illustrates potent methods of thought, and teaching those methods can be an overt feature of the everyday content of the course.

*Emphasize discovery.* To illustrate techniques for analyzing the unknown, we can demonstrate how various habits of thought lead to rich intellectual discoveries. Definitions and formal statements are not where the quest for discovery begins. They evolve through a process of exploration and analysis. Students can learn methods for developing insight such as: first formulating questions arising from observations; then looking for patterns, analogies, generalizations, examples, and beauty; and, finally, making and verifying conjectures.

These habits of discovery help students understand how abstract concepts are created and how such techniques are of value in their everyday lives. To assimilate these modes of thought, students need to experience them repeatedly throughout the course rather than just meeting them once in an introductory section.

*Foster positive attitudes.* Knowledge comes and goes, but hatred lasts forever. Many students enter their required mathematics courses with apathy, dislike, fear, and low expectations. For many, mathematics is *problems*. A critical goal for this course is for students to appreciate and enjoy mathematics more at the end of the course than at its beginning. If students enjoy mathematics and enjoy thinking in a mathematical way, they will do so for the rest of their lives.

To make this course enjoyable and intelligible to real students, a whole suite of enticements can be used. Physical experiences bring mathematics to life. Students can build the Platonic solids, physically try topological contortions and puzzles, experiment with strange dice, and create their own fractals. We must use textbooks that entice students to read on. The mathematical tendency to use cryptic notation and sparse prose does not encourage student readers. Humor is no joke. Mathematical standards of proof and elegance are not diminished by an entertaining treatment. Attractive art, humor, lively writing, intriguing questions, and surprising discoveries all make the substance more inviting for skeptical students.

Making liberal arts mathematics courses fun, important, and satisfying has far-reaching benefits. Future societal leaders will view mathematics as a living source of powerful ideas. Whether or not research leaders teach these courses, they can advocate courses that present the heart of mathematics to a broad audience. We can improve the lives of *every* student. However, we must meet people where they live and take that reality as the starting place. We mathematicians have a great story to tell. Improving our ability to tell that story will have an enormous impact well beyond our field.

*—Edward B. Burger and*
*Michael Starbird*

# Letters to the Editor

## References for a Constrained Volume Capillary Problem

My survey article "Capillary Surface Interfaces" that appeared in the August 1999 issue of the *Notices* contained, as I originally submitted it, a rather long list of all relevant references. This was not acceptable to the editors, who limited me to a few articles that could serve as leads for those who wish to pursue the subject. I could understand the reason for the policy, and I accepted the change. Regrettably, it led in this instance to deletion of all reference to some papers concerning the existence of a continuum of solutions of a constrained volume capillary problem and the resulting consequences of this remarkable behavior concerning stability and symmetry breaking.

The *Notices* policy did leave me the option of identifying contributing authors within the text of the article. That is done with regard to some of the contributors, but there is no mention of the paper by Gulliver and Hildebrandt in 1986, nor of my own 1988 paper, nor of later results by Concus and me, and by Wente. The Gulliver-Hildebrandt paper contains (as one part of a larger study) the first proof of existence of the continuum of solutions; my own paper provides a somewhat stronger existence result obtained in later independent work and presents the first (although limited) proof of instability. Concus and I improved the instability result, and Wente gave a complete proof. The properties of the family of solutions as indicated in these papers provided part of the conceptual basis for experiments that are described in my article and illustrated on the cover of that *Notices* issue.

At the time of writing I took the view that clarification of the contributions in these papers was available in the literature and would be distracting for the substantive text. In retrospect, it now seems to me that the needs of clear citation could better have taken precedence.

—*Robert Finn*
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**Editor's Note:** The *Notices* policy in question appears in the September 1998 issue on page 948 and again in the January 2000 issue on page 70.

## Research Mailing Lists

Mailing lists are one way that researchers in a field can better keep in touch with each other. For some time I had felt that a mailing list on universal algebra would be useful, but it seemed difficult to marshall the hardware and software to start such a list. Then I found out about `http://www.onelist.com`. This Web site provides free mailing list facilities in return for the opportunity to append brief advertisements to each message, and for a small fee they will omit the advertisements entirely. The facility has mailing lists in a number of languages. Currently, mathematics research mailing lists in English include ones on chaos theory (named caostheory [*sic*]), matroid theory (compumath), mathematical computation (compute), group theory (grouptheory), quasigroups and loops (loopforum), number theory (numbertheory), and universal algebra (univalg). To subscribe to one of these lists, visit the Web site `http://www.onelist.com`, or simply send an e-mail to the appropriate address, formed from the name of the list. For the univalg list on universal algebra, this would be `univalg-subscribe@onelist.com`.

—*William H. Rowan*
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## Kerosinka

I will always welcome the opportunity to read in the *Notices* any article on the history of mathematics, earlier or more recent. But it is unacceptable for the professional society AMS to publish historical articles containing political and demagogical statements. I am referring to the article "Kerosinka: An Episode in the History of Soviet Mathematics" (*Notices*, November 1999).

My only objection is to the introductory paragraph stating that "[i]n the Western world, access to a mathematical education is not difficult for an eager and talented student. This was not the case in the former Soviet Union. Young people pursuing mathematical careers faced numerous obstacles." What are we to conclude from these sentences and the story that follows? That there is no bias, discrimination, or other obstacles in math education in the Western world, whereas those were main characteristics of math education in the former Soviet Union? We all know that this cannot be true. As mathematicians, knowing what precision is, we should do better than publish such generalized, misleading statements. Concluding the article, the author of "Kerosinka" says that "[w]e must be careful in taking lessons from experiences in other countries." Well, one of the lessons should be that we remain critical about our own country or part of the world we live in.

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