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MATH 140 : CALCULUS II

Problem Set 2 – due Monday, February 17th

INSTRUCTIONS:
Please submit this at the *start* of Monday's class. Don't worry if you don't manage the get an answer for any particular question, but please give each problem an honest try (and record what you were able to accomplish, even if you didn't solve it). Eventually you should make sure to understand the problems, as some of them may appear on next week's in-class quiz. You are encouraged to collaborate with other students on these problems. However, please write up your solutions in isolation from one another.

2.1 State the limit definition of $\frac{d}{dx}f(x)$. [*You don't have to explain anything – just literally write it down.*]

2.2 In class we saw that $\frac{d}{dx}x^2 = 2x$, and we also derived the product rule $\frac{d}{dx}\big(f(x)g(x)\big) = f'(x)g(x) + f(x)g'(x)$. Combine these facts to evaluate $\frac{d}{dx}x^4$.

2.3 Suppose $h(x)$ is a function with $h(3) = 5$ and $h'(3) = -1$. Approximate $h(3.01)$.

2.4 This question is intended as a review of continuity. Throughout this question, suppose f is a function which is defined everywhere (i.e. no matter which value of x you specify, $f(x)$ is defined).

(a) Explain intuitively (i.e. in words – using as few symbols as possible) what it means for $f(x)$ to be continuous at $x = 2$. [*There's no one right answer here – give your own, not one you see in the book or hear from someone else. If you're having trouble formulating your thoughts, you may find it helpful to start by thinking about what it means for f to be* discontinuous *at $x = 2$.*]

(b) State the formal (limit) definition of what it means for $f(x)$ to be continuous at $x = 2$. [*This is just equation (1) of section 2.4 of the book.*]

(c) Explain why the formal definition of continuity is equivalent to your informal (intuitive) explanation. [*No need for essays here – two or three sentences should suffice.*]

2.5 We say that a function $f(x)$ is *differentiable* at $x = 3$ if $f'(3)$ exists. More generally, we say $f(x)$ is *differentiable* at $x = a$ if $f'(a)$ exists.

(a) Give an example of a function which is continuous at 0 but is not differentiable at 0. [*Make sure to explain* why *it is continuous at 0, and* why *it's not differentiable at 0.*]

(b) Suppose $f(x)$ is differentiable at $x = 2$. Explain why $f(x)$ is continuous at $x = 2$. [*Explain the proof of Theorem 2 in section 3.4 of the book, but in the special case that $x = 2$.*]

2.6 We'll talk about the infamous chain rule in class on Monday. Here's a warm-up to it.

(a) Use the *product rule* to determine $\frac{d}{dx}\big(f(x)^2\big)$. [*Answer:* $2f(x)f'(x)$]

(b) Combine your answer from part (a) with the product rule to determine $\frac{d}{dx}\big(f(x)^3\big)$. [*Answer:* $3f(x)^2f'(x)$]

2.7 Suppose f is a function which is differentiable at $x = 3$. Carefully explain the difference between $\frac{d}{dx}f(3)$ and $f'(3)$.