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MATH 140 : CALCULUS II

Problem Set 5 – due Monday, February 24th

INSTRUCTIONS:
Please submit this at the *start* of Monday's class. Don't worry if you don't manage the get an answer for any particular question, but please give each problem an honest try (and record what you were able to accomplish, even if you didn't solve it). Eventually you should make sure to understand the problems, as some of them may appear on next week's in-class quiz. You are encouraged to collaborate with other students on these problems. However, please write up your solutions in isolation from one another.

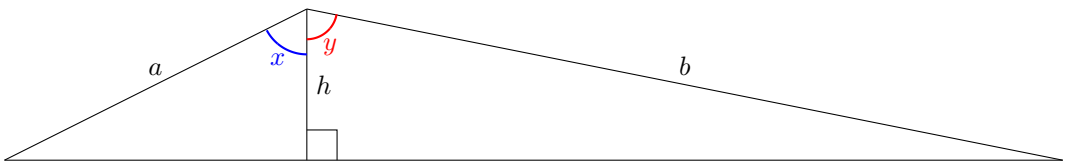
5.1 Determine exactly (no approximation!) each of the following values. You must explain how to find these values without using a calculator or computer. All angles are in radians.

(a) $\cos \frac{\pi}{6}$ [*Answer:* $\frac{\sqrt{3}}{2}$]
(b) $\sin \frac{7\pi}{6}$ [*Answer:* $-\frac{1}{2}$]
(c) $\tan \frac{5\pi}{4}$ [*Answer:* 1]

5.2 Explain why $\sin^2 \theta + \cos^2 \theta = 1$. [*Draw a picture!*]

5.3 In class I asserted that $\frac{d}{d\theta} \sin \theta = \cos \theta$ when θ is measured in radians. (We'll prove this soon.) Taking this as a fact, deduce from this that $\frac{d}{d\theta} \cos \theta = -\sin \theta$. [*Hint: make use of the previous problem.*]

5.4 Consider the following picture:



(a) Without adding any other labels to the picture, explain why $\sin(x + y) = \sin x \cos y + \cos x \sin y$. [*Hint: What's the area of the big triangle?*]
(b) Explain why $\cos(x + y) = \cos x \cos y - \sin x \sin y$. [*Hint: Use the identity* $\sin^2 x + \cos^2 x = 1$.]

5.5 Take as a given the formula $\frac{d}{d\theta} \sin \theta = \cos \theta$. Evaluate the following. (All angles are measured in radians.)

(a) $\frac{d}{d\theta} \sin(\theta^2)$
(b) $\frac{d}{d\theta} \sin(\sin \theta)$
(c) $\frac{d}{d\theta} \sin(\sin^2 \theta)$