Williams College Department of Mathematics and Statistics

MATH 140: CALCULUS II

Problem Set 7 - due Friday, February 28th

INSTRUCTIONS:

Please submit this at the *start* of Friday's class. Don't worry if you don't manage the get an answer for any particular question, but please give each problem an honest try (and record what you were able to accomplish, even if you didn't solve it). Eventually you should make sure to understand the problems, as some of them may appear on next week's in-class quiz. You are encouraged to collaborate with other students on these problems. However, please write up your solutions in isolation from one another.

7.1 In class we came up with a number

$$e = \lim_{h \to 0} (1+h)^{1/h}$$

that has the interesting property that $\frac{d}{dx}e^x = e^x$. Use a calculator to plug in smaller and smaller values of h to find some approximations to e. (Explicitly show which value of h you're plugging in, and which value of e that gives.)

- **7.2** Which functions are their own derivatives? We saw in class that e^x is one such a function. Are there others? Suppose f(x) is its own derivative, i.e. $\frac{d}{dx}f(x) = f(x)$.
 - (a) What's $\frac{d}{dx}e^{-x}$?
 - (b) What's $\frac{d}{dx}(f(x)e^{-x})$?
 - (c) What does the previous part tell you about f(x)?
- **7.3** Evaluate the following.
 - (a) $\sin^{-1}(1/2)$
 - (b) $\sin^{-1}(3)$
 - (c) $\frac{d}{dx}\sin(x^2)$
 - $(d) \frac{d}{dx} \sin^{-1}(x^2)$
 - (e) $\frac{d}{dx}e^{\sin x}$
 - (f) $\frac{d}{dx}e^{\sin^2 x}$