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MATH 140 : CALCULUS II

Problem Set 9 – due Wednesday, March 4th

INSTRUCTIONS:
Please submit this at the *start* of Wednesday's class. Don't worry if you don't manage the get an answer for any particular question, but please give each problem an honest try (and record what you were able to accomplish, even if you didn't solve it). Eventually you should make sure to understand the problems, as some of them may appear on next week's in-class quiz. You are encouraged to collaborate with other students on these problems. However, please write up your solutions in isolation from one another.

9.1 Write down an algebraic expression (involving a limit) for the average value of a function $f(x)$ on the interval $[3, 10]$. [*We did this type of computation in class.*]

9.2 Consider the region of the plane above the x-axis and below the function $y = 4 - x^2$; note that the base of this graph (along the x-axis) ranges from $x = -2$ to $x = 2$. (Draw a picture: the region should look like the entrance to a train tunnel.) The goal of this problem is to approximate the area of this region.

(a) By splitting the interval $[-2, 2]$ into four equal intervals of length $\Delta = 1$, draw rectangles based on the x-axis (as in we did in class) to approximate the area of the region.

(b) Do the same thing, but split the interval into 8 equal intervals.

(c) Do the same thing, but split into equal intervals of length $\Delta = \frac{1}{4}$.

9.3 Consider the region of the plane above the x-axis and below the function $y = \sqrt{4 - x^2}$; note that the base of this graph (along the x-axis) ranges from $x = -2$ to $x = 2$. (Draw a picture!) The goal of this problem is to approximate the area of this region.

(a) By splitting the interval $[-2, 2]$ into four equal intervals of length $\Delta = 1$, draw rectangles based on the x-axis (as in we did in class) to approximate the area of the region.

(b) Do the same thing, but split the interval into 8 equal intervals.

(c) Do the same thing, but split into equal intervals of length $\Delta = \frac{1}{4}$.

(d) What is the exact area of this region?