

Instructor: Leo Goldmakher

Williams College
Department of Mathematics and Statistics

MATH 140 : Calculus II

Problem Set 21 – due Friday, May 15th

INSTRUCTIONS: This assignment must be turned in by Friday at **6pm** EDT (that's early evening in Williamstown) by going to

<https://bit.ly/2RRu2aV>

You may submit photos or scans of your written work; please make sure your name appears on each page. (You can also try using the **scratchwork** app: <https://app.scratchwork.io/> to write up your HW.)

First: Watch this video introducing the idea of improper integration, and then this second video. You should try these problems to make sure the ideas make sense (you don't need to turn these in).

Second: Watch the first 6 minutes of this video introducing the idea of differential equations. Then watch this video applying differential equations to population growth. Try out these problems for practice (you don't have to turn these in). Finally, watch this video to learn about Newton's Law of Cooling.

Third: Submit your solutions to the following problems.

21.1 Evaluate the integral $\int_{-\infty}^{-2} \frac{dx}{(x+1)^3}$. [Ans. $-\frac{1}{2}$]

21.2 Evaluate the integral $\int_0^{\infty} xe^{-x^2} dx$. [Ans. $\frac{1}{2}$]

21.3 Evaluate the integral $\int_2^{\infty} \frac{dx}{x \ln x}$. [Ans. *Diverges.*]

21.4 Solve the differential equation $\frac{dy}{dx} = -\frac{x}{y}$, given that $y(12) = -5$. [Ans. $y = -\sqrt{169 - x^2}$]

21.5 Suppose Williamstown has a fixed population of 5000. A rumor starts to spread about the College's plans for the Fall; within a week, 500 people have heard the rumor. Assuming the rate of increase of the number of people who've heard the rumor is proportional to the number who have not yet heard it, how long will it be until half the population of Williamstown has heard the rumor? [Ans. *About 46 days after the rumor began.*]

21.6 You are a detective called in to investigate a murder scene in a hotel room. You arrive at 9am and examine the body; its temperature is 80° . (All temperature measurements in this problem are in Fahrenheit.) At 10am you measure the temperature once more and discover that it's gone down to 75° . The temperature in the hotel room is a constant 70° . Assume the victim's body temperature while alive was 98.6° .

(a) What was the temperature of the victim one hour after the murder? [Ans: 84.3]

(b) At what time did the crime take place? [Ans: $\approx 7:30am$.]