Math 200

Friday, February 1

Leo Goldmakher

Leo	Prof. Goldmakher	Professor

Read for Monday: Chapter 1, sections 1–3

Problems for Monday: Exercises 1.1, 2.1, 3.11

Course website (live Saturday!): web.williams.edu/Mathematics/lg5/200/

What's the point of it all?

Well, what does a mathematician do?

A: Explain the world with equations. (More what a physicist does. Applied mathematicians do this as well, though this isn't an applied math course.)

A: Prove the validity of expressions (equations and numbers). (Proving and disproving certain theories.)

How often is a new theorem discovered and proved? Once a year? Once a decade? Twice a decade? A bit more frequently than that... . 100 theorems discovered a day, and that's a conservative estimate. Professors here discover four a year.

This course functions as art history and studio art, in a way — we're studying old math and learning how to create new math.

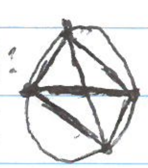
Q: Given a circle, place n points on its circumference. Connect each pair of points with a line segment. How many regions does this split the circle into?

Computations:

$n=1$: $R=1$ 

$n=2$: $R=2$ 

$n=3$: $R=4$ 

$n=4$: $R=8$ 

Conjecture: $R=2^{n-1}$. Why? We're not certain. Let's test $n=5$, then.

$n=5$: $R=16$ 

Maybe when we add a new point and draw in all the lines, the number of regions should double. Is it geometrically clear why?

Let's consider $n=4$: 

From this, it's not geometrically clear why adding a new point ~~[illegible]~~ would double the number of regions. Let's move on to $n=6$: 

$R=30$ or 31 (This drawing isn't great, but there are 31 here.)

The conjecture has failed!

$R=30$ is a special case if three lines meet at a point in the center of the circle. $R=31$ is more common if we're given six random points.

We're leaving this problem here, but we'll come back to it when we discuss graph theory. (Side note: when $n=7$, $R=57$.)

Proof distinguishes pattern from coincidence.

Example: $\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}$. $\int_0^\infty \frac{\sin t}{t}\cdot\frac{\sin t/101}{t/101}\,dt = \frac{\pi}{2}$. $\int_0^\infty \frac{\sin t}{t}\cdot\frac{\sin t/101}{t/101}\cdot\frac{\sin t/201}{t/201}\,dt = \frac{\pi}{2}$.

$\int_0^\infty \frac{\sin t}{t}\cdot\frac{\sin t/101}{t/101}\cdot\ldots\cdot\frac{\sin t/100n+1}{t/100n+1}\,dt = \frac{\pi}{2}$. This is true if $n \le 10^{42}$ and false if $n \ge 10^{43}$. (The number of elementary particles in the universe is 10^{70}.)

If we were to just work off of examples starting at $n=1$, we would never hit a counterexample, but there are infinitely many!

Example: Let $\pi(x)$ be the number of primes $\le x$ (e.g. $\pi(10.2)=4$).

Conjecture (Gauss): $\pi(x) \approx \int_2^x \frac{dt}{\log t} = \mathrm{li}(x)$. This is true!

Conjecture: $\pi(x) < \mathrm{li}(x)$. This is false for infinitely many values of x, but we don't know any yet. There is a counterexample by the time we hit 10^{316}, but that value is enormous!