Wednesday,
February 23

Last time, we studied if... then... propositions. (Note that the book calls what we call propositions "statements.") We concluded:

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

(Note: the book uses $\rightarrow$ instead of $\Rightarrow$, which Professor Goldmakher strongly disagrees with.) (If it's not clear, T is short for true and F is short for false.)

This "truth table" defines the logical connective $\Rightarrow$.

A closely related logical connective is $P \Leftarrow Q$, which we can say as "if Q then P" or "P is implied by Q." This may look similar to $P \Rightarrow Q$, but it's not the same! To see this, consider the case when $P =$ "a figure is a square" and $Q =$ "a figure is a rectangle." $P \Rightarrow Q$ is true, but $Q \Rightarrow P$ is false. Thus, we say $P \Rightarrow Q$ and $Q \Rightarrow P$ are "logically independent." (On an upcoming assignment, a bonus question will be to find an example of when a news story messed this up.)

We can visualize the above statements with a truth table:

P	Q	$P \Rightarrow Q$	$P \Leftarrow Q$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

$P \Rightarrow Q$ is a conditional statement.
$P \Leftarrow Q$ is the converse of $P \Rightarrow Q$.

Let's discuss some other logical connectives:

$\neg$	not
$\wedge$	and ("conjunction")
$\vee$	or

That's almost it! (There's one more symbol, but we'll get to it later.)

(A brief side note on the ~~TFF~~ FTT row of the $P \Rightarrow Q$ truth table - consider the phrase, "if my grandmother had wheels, she would have been a truck.")

Let's use some logical operators! What is $(\neg Q) \Rightarrow (\neg P)$?

P	Q	$\neg P$	$\neg Q$	$(\neg Q) \Rightarrow (\neg P)$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	F

Wait a second... we've seen this before! This is the truth table for $P \Rightarrow Q$!

Because of this, we say $(\neg Q) \Rightarrow (\neg P)$ is logically equivalent to $P \Rightarrow Q$. This is denoted $(\neg Q) \Rightarrow (\neg P) \equiv P \Rightarrow Q$.

This may look abstract, but we've used this logical equivalence before! It came up in our proof that $\sqrt{2}$ is irrational... but where? It came up implicitly a few times ("if $\sqrt{2}$ is rational, $\sqrt{2}$ can be written as $\frac{a}{b}$" $\equiv$ "if $\sqrt{2}$ can't be written as $\frac{a}{b}$, $\sqrt{2}$ is irrational" and "if $\sqrt{2} = \frac{a}{b}$, $\frac{a^2}{b^2} = 2$" $\equiv$ "if $\frac{a^2}{b^2} \neq 2$, $\frac{a}{b} \neq \sqrt{2}$"), but it came up explicitly in the step:

$$a^2 \text{ is even} \Rightarrow a \text{ is even},$$

which we proved by showing that if a is odd, a^2 is odd.

$(\neg Q) \Rightarrow (\neg P)$ is super useful. It's called the "contrapositive" of $P \Rightarrow Q$.