Friday,
March 1

Note: Math is hard! Not only is it a creative discipline, but it's also a creative discipline where ~~you can be~~ your creativity can be wrong! Don't worry too much about grades at this point... instead, use grades as a guide to what you should be studying. Talk to classmates! Talk to TAs! Talk to Professor Goldmakher!

A brief aside about a common quiz error: there were two common mistakes on proving the following claim:

Claim: a^2 is divisible by $3 \Rightarrow a$ is divisible by 3.

Incorrect justification 1: whenever 3 divides a number multiplied by itself, 3 must divide that number. (This is just the claim!)

Incorrect justification 2: if a is a multiple of 3, so is a^2. (This is the converse of the claim. Proving it doesn't tell us whether or not the claim is true.)

Recall the set $(0,1)$, which we described in words (the set of all real numbers between 0 and 1, ~~excluding~~ excluding 0 and 1). Let's describe this in symbols: $(0,1) = \{x \in \mathbb{R} : 0 < x < 1\}$. In general, the format is $\{\text{things} : \text{constraints on things}\}$. (Technically, we could write $(0,1)$ as $\{0 < x < 1 : x \in \mathbb{R}\}$, but it kind of makes a little more sense to start general and go specific, as opposed to saying, "We're on the interval from 0 to 1... and by the way, we're talking about real numbers." It's an aesthetic choice, though!)

Examples:

① $\mathbb{Z}_{>0} = \{x \in \mathbb{Z} : x > 0\}$

② $4\mathbb{Z} - 1 = \{x \in \mathbb{Z} : \exists a \in \mathbb{Z} \text{ s.t. } x = 4a - 1\} = \{4x - 1 : x \in \mathbb{Z}\}$.

(Important side notes: first, sets ignore repeats and order. For example, $\{3, 5, 1, 5\} = \{1, 3, 5\}$. Also, consider $\{x \in \mathbb{Z} : x = 4x - 1\}$. No integers satisfy $x = 4x - 1$, so the set is empty. We denote this $\{x \in \mathbb{Z} : x = 4x - 1\} = \emptyset$.

$\emptyset$ denotes the empty set.)

③ $\mathbb{Q}$? $\mathbb{Q} = \{ \frac{a}{b} : (a \in \mathbb{Z}) \wedge (b \in \mathbb{Z}_{\neq 0}) \}$.

Can we write $\mathbb{Z}_{\neq 0}$ more explicitly?

Definition: Given two sets A and B, we write $A \setminus B = \{ x \in A : x \notin B \}$

(in words, $A \setminus B$ is everything in A that isn't also in B).

Examples:

① $\mathbb{Z}_{\neq 0} = \mathbb{Z} \setminus \{0\}$.

② $\{1, 3, 5, 6\} \setminus \{2, 3\} = \{1, 5, 6\}$

③ $\mathbb{Z} \setminus 2\mathbb{Z}$ is the set of all odd numbers. (Note: $2\mathbb{Z} = \{2n : n \in \mathbb{Z}\}$.)

④ $\mathbb{Z} \setminus (0, 1) = \mathbb{Z}$

In addition to $\setminus$, there are other ways to combine two sets A & B:

① The <u>union</u> of A and B: $A \cup B = \{ x : x \in A \vee x \in B \}$.

② The <u>intersection</u> of A and B: $A \cap B = \{ x : x \in A \wedge x \in B \}$.

③ The <u>Cartesian product</u> of A and B: $A \times B = \{ (x, y) : (x \in A) \wedge (y \in B) \}$.

Example: $A = \{1, 3\}$, $B = \{2, 3, 17\}$.

$A \cup B = \{1, 2, 3, 17\}$.

$A \cap B = \{3\}$.

$A \times B = \{ (1,2), (1,3), (1,17), (3,2), (3,3), (3,17) \}$.

$(A \times B) \cup A = \{ (1,2), (1,3), (1,17), (3,2), (3,3), (3,17), 1, 3 \}$.

$(A \times B) \cap A = \emptyset$.

Definition: The <u>power set</u> of A is $P(A) = \{ X : X \subseteq A \}$.

Definition: $A \subseteq B \Leftrightarrow (x \in A \Rightarrow x \in B)$.

Example: $P(\{1, 3\}) = \{ \{1, 3\}, \{1\}, \{3\}, \emptyset \}$.