Monday
April 8

A quick note on the problem sets: don't assume what you're trying to prove! For instance, if we're supposing $k \in A$ and we want to show $k+1 \in A$ in induction, if we suppose $k+1 \in A$, we've lost, as we've already supposed that what we're trying to prove is true, so we can no longer prove that it's true.

Last time(s), we used the well-ordering principle to prove that induction is true and induction to prove that strong induction is true. On the homework, we'll use strong induction to prove that the well-ordering principle is true.
(Recall that the well-ordering principle states that a nonempty subset of the positive integers has a least element.)
Thus, the well-ordering principle, induction, and strong induction are all equivalent. Either they're all true or they're all false. As a result, we can effectively think of proofs using the well-ordering principle as proofs by induction.
Well... have we proved anything using the well-ordering principle?
Yes! In fact, BOTH proofs that $\sqrt{2}$ is irrational use the well-ordering principle! Let's quickly go over those proofs.
Let $A = \{n \in \mathbb{Z}_{>0} : n\sqrt{2} \in \mathbb{Z}\}$ (this is "the set of all denominators of $\sqrt{2}$").

First proof $\sqrt{2}$ is irrational:
$b \in A \Rightarrow \frac{b}{2} \in A$. This implies A has no least element. By the well-ordering principle, $A = \emptyset$. Q.E.D.

Second proof $\sqrt{2}$ is irrational:
$b \in A \Rightarrow (\sqrt{2}-1)b \in A$. This implies A has no least element. By the well-ordering principle, $A = \emptyset$. Q.E.D.
(There will be another proof $\sqrt{2}$ is irrational on the homework.)

Let's talk about recursive sequences!
Example (likely the most famous one):

$$\underset{f_1}{1}, \underset{f_2}{2}, \underset{f_3}{3}, \underset{f_4}{5}, \underset{f_5}{8}, \underset{f_6}{13}, \underset{f_7}{21}, \underset{f_8}{34}, \underset{f_9}{55}, \underset{f_{10}}{89}, \ldots$$

This is known as the Fibonacci sequence. (Brief aside: his name was actually Leonardo da Pisa, but he was the son of a merchant named Bonacci. That eventually got corrupted into "Fibonacci." Also, he introduced Hindu-Arabic numerals to Europe, meaning people could stop doing mathematics in Roman numerals, which is horribly inconvenient.)

What is f_{1000}? Can we figure it out without first computing all the preceding terms?

We define the $(n+1)^{th}$ term in the Fibonacci sequence as:

$f_{n+1} = f_n + f_{n-1}\ \forall n \geq 2,\ f_1 = 1,\ f_2 = 2.$

Where does it come from? A model of rabbit population growth! That model is based on the following stipulations:

1. It takes one month for rabbits to reach maturity.
2. Mature rabbits conceive once per month.
3. It takes 1 month from conception to birth.
4. Each litter consists of one male and one female.
5. The brother and sister become a monogamous couple.
6. Rabbits are immortal.

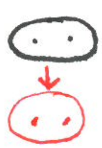
(Note: these are all false.)

Q: Starting with a couple of baby bunnies, how does the population grow?

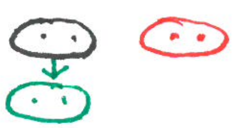
 = original bunnies

Month 0:  immature bunnies = 1 pair

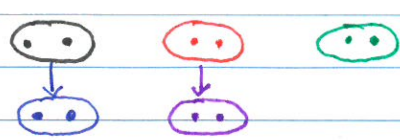
Month 1:  mature bunnies = 1 pair

Month 2:  mature rabbits = 2 pairs

immature bunnies

Month 3:  mature rabbits = 3 pairs

immature bunnies

Month 4:  mature rabbits = 5 pairs

immature bunnies

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