

Monday,  
April 15

It's time for modular arithmetic!

You've probably done modular arithmetic without knowing it.

Today is Monday.

Q: What day will it be 3 days from now?

A: Thursday.

Q: What day will it be 10 days from now?

A: Thursday.

Q: What day will it be 28 days from now?

A: Monday.

Q: What day will it be 81 days from now?

A: Friday.

How did we get that last one? Well, 81 days is 77 days and 4 days. 77 days is 11 weeks, and adding a week doesn't change what day it is, so we get that we should go four days forwards from Monday, which is Friday.

Let's format this!

Day	Mnemonic	Number
Sunday	None day	0
Monday	One day	1
Tuesday	Two day	2
Wednesday	~~~~~	3
Thursday	Four day	4
Friday	Five day	5
Saturday	~~~~~	6

Now we have an arithmetic of days of the week!

From our examples, we can rewrite "10 days past Monday is Thursday" as  $10+1=4$  and "81 days past Monday is Friday"

as  $81 + 1 = 5$ .

(Wait a second...  $10 + 1 \neq 4$  and  $81 + 1 \neq 5$ . We'll introduce new notation to address this in a bit!)

With this in mind, we can discuss John Conway's Doomsday Algorithm.

Examples: 9/16/1999 ~~is~~<sup>was</sup> a Thursday, 3/12/1999 was a Friday.

The Doomsday Algorithm:

Step 1: Doomsday difference.

Step 2: Century day.

Step 3: Year divided by 12.

Step 4: Remainder from step 3.

Step 5: Value from step 4 divided by 4.

If we add these together and get a value between 0 and 6, we can find the day of the week!

Before we describe these steps, what's a doomsday?

Doomsdays: 4/4, 6/6, 8/8, 10/10, 12/12, 9/5, 7/11, 5/9, 11/7, 3/0.

In any given calendar year, all of these days fall on the same day of the week.

There are no dates from January or February because leap days can mess things up. (3/0 indicates the last day of February, which is either 2/28 or 2/29.)

Let's go through the steps!

Step 1: We compute how far ahead ~~of a doomsday~~ of a doomsday the given date is. (We can also go backwards from a doomsday.)

Step 2: We compute the day of the year for the first doomsday of the century. This may sound frustrating,

but there's a pattern:

17xx    18xx    19xx    20xx    21xx    22xx    23xx    24xx    ...  
7        5        3        2        7        5        3        2        ...

This is the primes going down from 7 and the pattern repeats!

Step 3: When we say ~~"year divided by 12"~~ "year divided by 12" we actually mean  $\lfloor \frac{\text{last two digits of year}}{12} \rfloor$ . (For ~~an~~ example, if the year were 1993, we'd get ~~19~~  $\lfloor \frac{93}{12} \rfloor = 7$ .)

Step 4: We take the remainder from the above calculation. (For example,  $93 = 7 \cdot 12 + 9$ , so we'd get 9 as the remainder.)

Step 5: Like in step 3, we take  $\lfloor \frac{\text{remainder from step 4}}{4} \rfloor$ . (For example,  $\lfloor \frac{9}{4} \rfloor = 2$ .)

Example: 7/4/1776

Step 1 yields -7 (we can go backwards!), step 2 yields 7, step 3 yields 6, step 4 yields 4 and step 5 yields 1. The sum is 11, but we can cast out sevens, meaning this is equivalent to 4, so it's a Thursday.

(Leap year note: Years divisible by 4 are leap years, except years divisible by 100 aren't, except years divisible by 400 are.)

This is called arithmetic modulo 7. We can rewrite our earlier examples as:

$10 + 1 \equiv 4 \pmod{7}$  (which is read "10 plus 1 is congruent to 4 mod 7") and  $81 + 1 \equiv 5 \pmod{7}$ .