

Monday,
May 6

Let's talk more about graph theory!

Graphs appear often in modeling social networks, like Facebook:

- Each person is a vertex.
- Connect two people with an edge if and only if they're "friends."

Now we have a graph...now what?

We could study the following:

- ① "Six degrees of separation" — how far apart are the two vertices that are furthest apart?

(Note: to do this, we have to ~~be~~ ignore disconnected vertices for this to work.)

(Also, we have to define a notion of distance, because if we don't, calling two things "furthest apart" doesn't make sense. By distance between a and b , we mean the minimal number of edges needed to walk from a to b .)

The distance between the two vertices that are furthest apart is called the diameter of the graph.

- ② How many connected components are in the graph? (A connected component is a maximal piece of a graph on which you can walk from any vertex to any other component. Example:



This has 8 vertices, 7 edges, and 2 connected components.)

What does this measure? In a way, it measures how fractured society is — the number of connected components is the number of social groups that don't communicate with others.

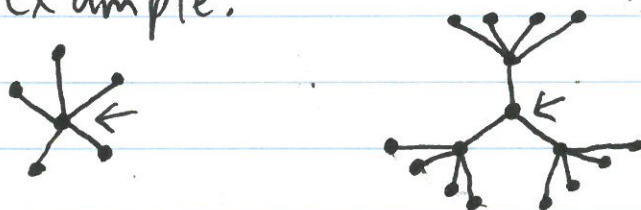
③ We could measure the friend-distance between two ~~people~~ people, i.e. the shortest path between two given vertices.

④ Which vertices are the most influential?

One approach is to measure how many edges come out of a given vertex. (This is called the degree of a vertex.)

This doesn't tell the ~~whole~~ whole story, though.

For example:



~~The~~ The central vertex on the left has a higher degree, but the central vertex on the right appears more influential (the vertices it ~~is~~ is connected to are connected to more vertices).

This goes towards the connectivity of a graph, but we'll get back to that later.

Last time, we proved:

Proposition: If there exists an Eulerian path (a walk on G crossing each edge exactly once) on a connected graph G , then there are at most two vertices of odd degree.

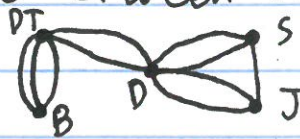
The language of degrees makes our definition a bit neater!

Question: What if we don't care about Eulerian paths? What about in general? How many vertices are there of odd degree?

To approach this, we shook hands! Each group had an even number of people who shook an odd number of hands. This is interesting, but how does it relate to graphs?

We can make a graph by making each person a vertex with an edge between two ~~vertices~~ vertices for every handshake between the two.

Example:



We can capture all of the information present in the graph in a table (numbers indicate number of handshakes between two people):

	DT	B	D	S	J
DT	0	3	2	0	0
B	3	0	0	0	0
D	2	0	0	2	2
S	0	0	2	0	1
J	0	0	2	1	0

(This is called the adjacency matrix.)

What are some patterns?

① The matrix is symmetric about the main diagonal.

(Proof: being connected by an edge is symmetric.)

② The diagonal is all 0's.

(Proof: ~~the~~ graphs don't have loops.)

③ The sum of all numbers in the matrix is $2|E|$.

④ The sum of all entries in row v is $\deg(v)$.

So, we've proved:

Proposition:

$$\sum_{v \in V} \deg(v) = 2|E|.$$