Instructor: Leo Goldmakher

NAME (LAST, NICKNAME): ______________________

SECTION # (10AM = 1, 11AM = 2): ______________

Williams College
Department of Mathematics and Statistics

MATH 200 : DISCRETE MATH

Problem Set 2 – due Thursday, February 14th ♡

INSTRUCTIONS:

This assignment must be turned in to my mailbox (on the right as you enter Bascom) by **4pm** sharp. Late assignments may be submitted at the beginning of Friday's class to me in person (i.e. don't leave them in my mailbox or ask someone else to submit on your behalf); however, 5% will be deducted for late submission. *Assignments submitted later than start of class on Friday will not be graded.*

Please print and attach this page as the first page of your submitted problem set.

PROBLEM	GRADE
2.1	
2.2	
2.3	
2.4	
2.5	
2.6	
Total	

Please read the following statement and sign **before writing the final version of this problem set:**

I understand that I am not allowed to use the internet to assist with this assignment, apart from accessing the course website or looking up definitions. I also understand that I must write down the final version of my assignment in isolation from any other person, and to not copy from any set of written notes created when another person was present. I pledge to abide by the Williams honor code.

SIGNATURE:______________________________

Problem Set 2

2.1 The goal of this exercise is to build up your intuition for rational and irrational numbers.

(a) Prove that $\frac{\sqrt{2}}{2}$ is irrational. [*Hint: you shouldn't have to work too hard!*]

(b) Suppose a and b are rational. Prove that $a + b$ and ab are both rational.

(c) Suppose a and b are rational. Show (by example) that a^b can be rational or irrational.

(d) Suppose a is rational and b is irrational. Must $a + b$ be irrational? What about ab? In each case, either prove that it must be irrational or give an example where it isn't.

(e) Suppose a and b are irrational. Must $a + b$ be irrational? What about ab? In each case, either prove that the outcome must be irrational or give an example where it is rational.

2.2 Prove that there exist irrational numbers a and b such that a^b is rational. [*Hint: start by considering* $\sqrt{2}^{\sqrt{2}}$*... but then keep thinking about what choices you could make for* a *and* b.]

2.3 Recall from class that we invented an algorithm for generating a new prime from a given list. Following our algorithm in class starting with the single prime 2 we generated the sequence $2, 3, 7, 43, \ldots$, and starting with the single prime 7 we generated $7, 2, 3, \ldots$ For each of the following choices of initial prime p, determine the first six primes generated by the algorithm (the starting prime p counts as one of the six). You are allowed to access the website

`https://primes.utm.edu/lists/small/100000.txt`

for a list of primes, and you may use a calculator (including google's free calculator), but—as always—do **not** use any other websites.

(a) $p = 2$

(b) $p = 3$

(c) $p = 5$

(d) $p = 7$

(e) $p = 11$

(f) $p = 13$

[*Hint: In all the parts of this problem, almost all the primes you generate should have one or two digits; the only exceptions are one three-digit prime, one four digit prime, and one five-digit prime.*]

2.4 In this exercise you'll explore a few basic properties of primes.

(a) Prove that every prime larger than 2 is odd.

(b) Prove that for any prime $p > 2$ there exists an integer k such that $p = 4k + 1$ or $p = 4k - 1$.

(c) Prove that for any prime $p > 3$ there exists an integer k such that $p = 6k + 1$ or $p = 6k - 1$.

2.5 We define a *prime triple* to be a triple of numbers $n, n + 2, n + 4$ such that all three are prime. (For example, $3, 5, 7$ is a prime triple.) Find all prime triples, and prove that your list is complete.

2.6 Prove that there are infinitely many primes of the form $4k-1$. [*Hint: Given any finite list of such primes* $p_1, p_2, \ldots, p_k$*, consider the number* $4p_1p_2 \cdots p_k - 1$.]