Instructor: Leo Goldmakher

Name (Last, Nickname): ______________________

Section # (10am = 1, 11am = 2): ______________

Williams College
Department of Mathematics and Statistics

MATH 200 : DISCRETE MATH

Problem Set 4 – due Thursday, February 28th

INSTRUCTIONS:

This assignment must be turned in to my mailbox (on the right as you enter Bascom) by **4pm** sharp. Late assignments may be submitted at the beginning of Friday's class to me in person (i.e. don't leave them in my mailbox or ask someone else to submit on your behalf); however, 5% will be deducted for late submission. *Assignments submitted later than start of class on Friday will not be graded.*

Please print and attach this page as the first page of your submitted problem set.

PROBLEM	**GRADE**
4.1	
4.2	
4.3	
4.4	
4.5	
4.6	
4.7	
4.8	
Total	

Please read the following statement and sign **before writing the final version of this problem set:**

I understand that I am not allowed to use the internet to assist with this assignment, apart from accessing the course website or looking up definitions. I also understand that I must write down the final version of my assignment in isolation from any other person, and to not copy from any set of written notes created when another person was present. I pledge to abide by the Williams honor code.

SIGNATURE:______________________________

Problem Set 4

4.1 In class we found an explicit way to translate the boolean operators $\wedge$ and $\neg$ into arithmetic:

$$\#(P \wedge Q) = \#P \cdot \#Q \qquad\qquad \#(\neg P) = 1 - \#P.$$

Find arithmetic formulae for each of the following.

(a) $\#(P \vee Q)$

(b) $\#(P \implies Q)$

(c) $\#(P \iff Q)$

4.2 Prove that $n^2 + 2$ isn't a perfect square for any integer n.

4.3 Prove that n is divisible by 3 if and only if $n + 3$ is divisible by 3.

4.4 Given a real number x, we can form two associated integers:

- the *floor* of x, denoted $\lfloor x \rfloor$, is the largest integer that is $\leq x$, and
- the *ceiling* of x, denoted $\lceil x \rceil$, is the smallest integer that is $\geq x$.

Prove that x is an integer if and only if $\lfloor x \rfloor = \lceil x \rceil$. [*Hint: you may use that* $a = b$ *iff* ($a \leq b$ *and* $b \leq a$).]

4.5 Rewrite each of the following in formal predicate form:

(a) $x^2 - 1 = (x + 1)(x - 1)$

(b) Every integer that's one less than a multiple of four has a prime factor that's one less than a multiple of four.

(c) If $p \geq 5$ is prime, then $p^2 - 1$ is divisible by 24.

(d) If a prime is one less than a multiple of 4, then it cannot be written as the sum of two perfect squares.

(e) There are infinitely many primes p such that $p + 2$ is also prime.

4.6 Give an example of a predicate that's *not* a proposition.

4.7 For each of the following predicates P, express $\neg P$ using quantifier and relation symbols, without using 'not' anywhere in your statement (implicitly or explicitly).

(a) Every even number ≥ 4 is the sum of two primes.

(b) $\forall x \in \{\text{primes}\}$, $x \geq 11$ or $x < 8$.

(c) Every integer that's one less than a multiple of four has a prime factor that's one less than a multiple of four.

(d) There are infinitely many primes p such that $p + 2$ is also prime.

4.8 In class I mentioned that naïve set theory can lead you to paradoxes; the goal of this problem is to exhibit one of these. Consider the set S consisting of all sets that aren't elements of themselves. Carefully describe why this leads to a paradox.