Instructor: Leo Goldmakher

NAME (LAST, NICKNAME): ____________________

SECTION # (10AM = 1, 11AM = 2): ____________

**Williams College**
**Department of Mathematics and Statistics**

## MATH 200 : DISCRETE MATH

**Problem Set 5 – due Thursday, March 7th**

**INSTRUCTIONS:**

This assignment must be turned in to my mailbox (on the right as you enter Bascom) by **4pm** sharp. Late assignments may be submitted at the beginning of Friday's class to me in person (i.e. don't leave them in my mailbox or ask someone else to submit on your behalf); however, 5% will be deducted for late submission. *Assignments submitted later than start of class on Friday will not be graded.*

Please print and attach this page as the first page of your submitted problem set.

| PROBLEM | GRADE |
|---|---|
| 5.1 | |
| 5.2 | |
| 5.3 | |
| 5.4 | |
| 5.5 | |
| 5.6 | |

| PROBLEM | GRADE |
|---|---|
| 5.7 | |
| 5.8 | |
| 5.9 | |
| 5.10 | |
| 5.11 | |

| **Total** | |
|---|---|

Please read the following statement and sign **before writing the final version of this problem set:**

*I understand that I am not allowed to use the internet to assist with this assignment, apart from accessing the course website or looking up definitions. I also understand that I must write down the final version of my assignment in isolation from any other person, and to not copy from any set of written notes created when another person was present. I pledge to abide by the Williams honor code.*

**SIGNATURE:**______________________________________

## Problem Set 5

**5.1** For each of the following sets, give the simplest description of the set that you can. Try to use as few mathematical symbols as possible.

(a) $\{x, y \in \mathbb{Q} : x + y = 1\}$

(b) $\{4n - 1 : n \in \mathbb{Q}\}$

(c) $\{4n - 1 : n \in \mathbb{R}\}$

(d) $\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + 2 = y^2\}$

(e) $\{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 1\}$

(f) $\big\{x \in \mathbb{R} : \{bx : b \in \mathbb{Z}_{>0}\} \cap \mathbb{Z} \neq \emptyset\big\}$

(g) $\{(x, y) \in \mathbb{R} \times \mathbb{R} : |x| + |y| = 1\}$ [*Don't use a calculator or computer for this one!*]

**5.2** For each of the following sets described informally, write it in set notation using only mathematical symbols (i.e. without using any words). Please do not use ellipses (...) in your answer.

(a) The set of all even numbers between 3.4 and 9.2

(b) The set of all even numbers between 2.4 and 3.5

(c) The set of all even numbers between 2.4 and 184573.2

(d) The set of all real numbers greater than or equal to $-2$.

(e) The set of all integers that aren't multiples of 3. [*Don't use the divisibility symbol* $|$ *on this one.*]

**5.3** For each of the following, identify whether or not it's a function. If it is, explain why. If not, give an example that shows it's not a function.

(a) $f : \mathbb{Q} \to \mathbb{Z}$, where $f(a/b) = |a| + |b|$ for any rational number $a/b$ with $a \in \mathbb{Z}$ and $b \in \mathbb{Z} \setminus \{0\}$.

(b) $g : \mathbb{Q} \to \mathbb{Z}$, where $g(\alpha)$ is the number of 3's appearing in the decimal expansion of $\alpha$. For example, $g(0.273) = 1$.

(c) $h : \mathbb{R} \to \mathbb{Z}_{\geq 0}$, where $h(x)$ is the number of prime numbers $\leq x$. For example, $h(6) = 3$.

(d) $F : \mathbb{R}_{\geq 0} \to \mathbb{R}$ where $F(x) = \log\big(\log(\sin x)\big)$. [*The logarithm here is base 10. You may look up the definitions of logarithm and sine, but don't use any calculator on this problem.*]

(e) $G$ : {words in the English language} $\to \mathbb{Z}_{>0}$ where $G$ applied to a word outputs the number of vowels in that word. For example, $G(\text{horse}) = 2$.

**5.4** Recall from class that $\mathcal{P}(A)$ denotes the *power set* of $A$, the set of all subsets of $A$.

(a) Write out $\mathcal{P}(A)$ for $A = \{1, 2, 3\}$.

(b) By working out a few examples, make a guess about how many elements $\mathcal{P}(A)$ has when $A$ is the set of the integers from 1 to $n$. [*Recall that an element of* $\mathcal{P}(A)$ *is a set.*]

**5.5** In class we proved that $(0, 1)$ is uncountable by showing that any enumeration of its elements always misses (at least one) number in $(0, 1)$. Why doesn't this proof also show that $\mathbb{Q} \cap (0, 1)$ is uncountable? To be more precise: if you try to apply the same proof to the set $\mathbb{Q} \cap (0, 1)$, where precisely does it fail?

**5.6** Prove that any infinite set contains a countable subset.

**5.7** Prove that any infinite subset of a countable set must be countable.

**5.8** Prove that if $A$ is countable and $B$ is countable, then $A \cup B$ is countable.

**5.9** Prove that $\mathbb{Q}$ is countable.

**5.10** Prove that the union of countably many countable sets is countable.

**5.11** Suppose $A$ is countable and $B$ is uncountable. Prove that $B \setminus A$ is uncountable.