LINEAR ALGEBRA: LECTURE 1

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We started by dealing with the usual administrivia. The single most important piece of information is the address of the course homepage, since it contains all of the other information needed about the course:

`web.williams.edu/Mathematics/lg5/250/`

In particular, make sure you check out the syllabus.

After some general discussions, we began exploring the following:

Question. *Suppose* $f : \mathbb{R} \to \mathbb{R}$ *is a function which satisfies*

$$f(x+y) = f(x) + f(y) \tag{1}$$

for every real number x *and* y*. What can you say about* f*?*

After some starts and stops, we figured out a few things. First of all, we observed that

$$f(0) = 0.$$

This is because $f(0) = f(0+0) = f(0) + f(0)$. Next, we realized that we can write many different values of $f(n)$ in terms of $f(1)$. For example,

$$f(2) = 2f(1),$$

since $f(2) = f(1+1) = f(1) + f(1) = 2f(1)$. Similarly,

$$f(3) = 3f(1).$$

If we keep using this approach, we get similar results, like $f(17) = 17f(1)$.

Another observation was that we can 'factor out' negatives:

$$f(-x) = -f(x).$$

This follows from $0 = f(0) = f\Big(x + (-x)\Big) = f(x) + f(-x)$. Thus, for example,

$$f(-3) = -3f(1).$$

A succinct way to summarize everything we've discovered so far is:

$$f(n) = nf(1) \tag{2}$$

for any integer n. What about other inputs? After some playing around, we found that

$$f\Big(\frac{1}{2}\Big) = \frac{1}{2}f(1);$$

this follows from the relation $f(1) = f(1/2) + f(1/2)$. Similarly,

$$f\Big(\frac{1}{3}\Big) = \frac{1}{3}f(1),$$

since $f(1) = f(1/3) + f(2/3) = f(1/3) + f(1/3) + f(1/3)$. Thus, it is tempting to conclude that equation (2) holds for every real number n, not simply integers. We will return to this question more carefully next class.

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