

COURSE INFORMATION

MATH 313 – Number Theory

Course homepage: <https://web.williams.edu/Mathematics/lg5/313/>

Instructor: Leo Goldmakher (either ‘Leo’ or ‘Professor Goldmakher’ is fine)

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Office hours: Tuesdays, Thursdays, and Fridays, 10:30am–11:30am in my office; other times by appointment.

My office door is often open, and whenever it is—whether or not it’s officially office hours—you’re welcome to drop in. If my door is closed, however, please don’t knock; I’m either out or busy. *Wednesday is my day of research, so I will be unavailable all day.*

Lectures: Mondays and Thursdays 2:35–3:50 in Schow 30A.

Syllabus: Number theory is the study of numbers – usually integers, but often also rationals and generalizations of integers (such as the Gaussian integers). We shall survey roughly 4000 years of developments in the field, including such topics as **the multiplicative structure of \mathbb{Z}** (e.g. Bézout’s theorem and the Fundamental Theorem of Arithmetic), **elementary modular arithmetic** (e.g. Fermat’s Little Theorem), **the multiplicative structure of $\mathbb{Z}/n\mathbb{Z}$** (e.g. Gauss’ theorem on primitive roots), **decimal representation** (e.g. Midy’s theorem), **the distribution of prime numbers** (e.g. Chebyshev’s theorem), **quadratic congruences** (e.g. the quadratic reciprocity law), **quadratic forms** (e.g. Fermat’s two-squares theorem), **continued fractions** (e.g. solving Pell’s equation), **elliptic curves** (e.g. characterizing pythagorean triples), and **cryptology** (e.g. the RSA encryption algorithm). Time-permitting we may also discuss additional topics, such as **p -adic numbers** (e.g. Ostrowski’s theorem) and the **geometry of numbers** (e.g. Minkowski’s theorems).

Textbook: *A concise introduction to the theory of numbers* by Alan Baker. (If you’re on the Williams campus, it is available as a free pdf via the Cambridge University Press website.) See course website for other resources.

Teaching Assistant: Anya Michaelsen <anm1@williams.edu>

Discussion sessions: Number theory is a ton of fun. But it can also be *hard*. Some of the concepts and problems are going to be manifestly difficult, and you will need to persist in the face of feeling totally lost, in particular re-reading the text and your notes and reaching out to me, the TA, and your peers for inspiration.

But the far greater challenge in this course is dealing with those difficulties and subtleties you **aren’t** aware of. How can you fix a problem you don’t know is a problem? The single most effective method is:

Try to explain the material to someone else.

For this reason, I’m setting up a discussion session, to take place every Sunday. Although not mandatory, I strongly encourage you to attend as many of these meetings as you possibly can, as this is your best chance to talk through the material and discover your blind spots. In addition to the discussion session, there will also be a weekly problem session devoted to the problem set (hosted by Anya). Here are the details of these meetings:

- **Discussion session.** Every Sunday, 2pm–4pm, Clark 204. These are for discussions of material, not problem sets.
- **Problem session.** Every Tuesday evening from 7:30pm–9:30pm, Clark 204. Come having worked on the problem set already, as Anya will only give hints in response to questions you ask.

Assessment: Your grade will be calculated based on a few components:

1. Problem sets – 15% total

This course will have weekly problem sets. Some of the problems will be straightforward once you master the concepts from class / reading, while others will require additional creativity. **The problem sets are intended to be challenging.** The goal is to struggle with every question; it's OK not to solve every problem on the assignment, so long as you make a serious attempt at all of them. However, I have one hard rule: **please do not search for problems, solutions, or examples online.** If you find yourself spending more than 10 hours per week on this class outside of the classroom, please come talk to me.

Problem sets should be submitted at the beginning of Thursday's class to me in person (i.e. don't ask someone else to submit on your behalf). Late assignments may be left in my mailbox (just inside the entrance to Bascom House); however, 5% will be deducted for submissions not made in person at the start of Thursday's class. *Assignments will not be accepted after 4pm on Friday under any circumstances.* Your lowest assignment score will be dropped.

2. Three exams – 20%, 30% and 35%

There will be three exams:

- The first midterm will be a week-long take-home exam, to be handed in at the start of class on Thursday, October 4th.
- The second midterm will be an oral exam, to be taken between November 5th and 8th. There will be no surprise problems on the exam; details of the format will be explained later.
- The final exam will be a traditional in-class exam. The best of the three exams will be weighted 35%, the weakest of the three 20%, and the remaining exam 30%.

Peer Tutoring (free!): The Peer Academic Support network provides free peer tutoring. A schedule showing available individual tutoring sessions for this class is available through the TutorTrac website:

`tutortrac.williams.edu`

The site will be updated weekly, but can only be accessed when you're on the Williams College network. Step-by-step instructions for finding and scheduling tutoring sessions are on the Peer Tutoring Program website (`academic-resources.williams.edu`). If you have any questions about tutoring at Williams, please email `msrc@williams.edu`

Computers and phones in the classroom: Computers and phones *may not to be used at any time in the lecture room*, as this is distracting both for you and for your fellow students. If you have something you need to do urgently on your computer or phone, quietly take it outside the classroom and do it there; I will never be offended by this, so long as you do not disrupt the class. If you take notes on your computer, please talk to me about it.

Team work and plagiarism: I strongly encourage you to brainstorm with other students as you work on your problem sets. However, you must write up the solutions on your own, **without copying from any text (written or spoken)**. For example, if you take notes during a problem session based on a solution explained to you by the TA or another student, *do not copy from these notes* when writing up your assignment! To avoid a slippery slope, I encourage you to write up your problems sets in physical isolation from any other student in the course.

Internet usage: The internet is an amazing resource, but I urge you to use it wisely. When you read this sentence, please send an email to Anya with subject line consisting of the single word purple. In particular, I request that you do not search for problems or examples. **Looking up definitions is OK, looking up (or asking about) problems online is not.** It is better to struggle on your own and *not* solve the problem than to simply copy a solution. When it comes to exams, please don't use the internet for any class-related reason apart from accessing the course website.

Anonymous feedback: On the website – under the “Contact” tab – there is a form for submitting anonymous feedback. Although I strongly prefer face-to-face conversations, I understand that this is not always possible or comfortable on some sensitive subjects, in which case please submit via the form. Negative comments, positive comments, confusions, suggestions – it's all welcome! Please try to keep your feedback respectful and succinct. When appropriate, I may respond to feedback publicly in class.