Williams College
Department of Mathematics and Statistics

MATH 350 : REAL ANALYSIS

WRITING ASSIGNMENT

Outline due at 10pm on Friday, December 8th
Final draft due at 10pm on Tuesday, December 12th

The purpose of this assignment is to write a clear, polished, self-contained, and engaging introduction to the *Cantor set* (sometimes called the *Cantor ternary set*). Your final draft should read like a chapter from a textbook, written for an audience of your classmates in Real Analysis who don't yet know anything about the Cantor set.

On the next page I've listed a few basic topics that I ask you to describe in detail in your essay. However, there are plenty of cool topics I haven't listed, and I encourage you to touch on some of them in your essay; feel free to go into as much or as little detail as you feel serves the reader best. Perhaps most important is that you try to motivate the Cantor set—why should we care about its existence at all? In other words, what questions is it answering?

The length of the essay is up to you, but my guess is anything shorter than three pages is not telling enough of the story. On the other hand, including too many details in a proof can obscure its overall structure—the ideal is not to explain every step, but to explain just enough that the idea comes across clearly and that the interested reader can sit down and fill in all the missing details. One of the most important aspects of mathematical writing is to distinguish the creative steps in a proof from the technical steps. It usually takes considerable thought to identify which parts of a proof belong in which camp!

You may freely consult any references you find (textbooks, articles, course notes, blogs, wikipedia, online forums, etc); Schow library has a rich collection of math books. However, you must always explicitly cite sources, in particular including a thorough bibliography. Although you may occasionally employ a direct quote, this should be extremely rare—almost all descriptions and arguments should be paraphrased, and proofs should be digested and then presented in an order that makes the most sense to you.

The assignment should be written using LaTeX, the text editor used by the overwhelming majority of mathematicians. Instructions on how to set up and use LaTeX on your computer, as well as information on tutorials and example files, will be posted separately to the course website.

You should submit an outline (written in LaTeX) by email to me no later than **10pm on Friday, December 8th**. The outline should contain a preliminary list of sources, a very rough idea of the introduction, and a sketch of the sorts of topics you wish to cover and in which order they might appear. Of course, this is allowed to change as your essay evolves! The essay itself will receive a maximum of a B+, with the rest of the grade on the project determined by the outline. Late outlines will be accepted until 10pm on Saturday, December 9th, but will be worth a maximum of 1/3 of a letter grade (as opposed to 2/3).

The deadline for the final draft of the essay is **10pm on Tuesday, December 12th**, to be submitted by email to me. Late submissions will be accepted, but the overall grade will be reduced by 1/3 of a letter grade each subsequent day after the deadline.

Best wishes, and please don't hesitate to reach out with questions!

-Leo

Prompts. The following facts should all be addressed in some detail in your essay. The order in which you address these is up to you. Throughout, let $\mathcal{C}$ denote the Cantor (ternary) set.

(1) $\mathcal{C}$ contains no intervals.

(2) $\mathcal{C}$ is uncountable.

(3) $\mathcal{C}$ is closed.

(4) $\mathcal{C}$ has measure 0, i.e. can be covered by intervals whose total length is arbitrarily small.

(5) $\mathcal{C} + \mathcal{C} = [0, 2]$.

Things to consider. Here are some things that you don't have to address in detail in your essay, but are worth thinking about and potentially mentioning in your essay.

(i) Finite unions of closed sets are closed, but arbitrary unions of closed sets are not necessarily closed.

(ii) Every real number can be expressed in binary (base 2) notation; also in ternary (base 3) notation. For example, in binary we have

$$\frac{17}{6} = 10.1101010101010101\ldots$$

whereas in ternary we have

$$\frac{17}{6} = 2.2111111\ldots$$

(iii) $\mathcal{C}$ is *compact*, i.e. if $\mathcal{C}$ is contained in the union of a bunch of open intervals, then it's always possible to find a finite number of these intervals whose union still contains $\mathcal{C}$.