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Department of Mathematics and Statistics

MATH 350 : REAL ANALYSIS

Problem Set 3 – due Thursday, September 28th

INSTRUCTIONS:

You should aim to submit this assignment to me in person at the start of Thursday's class; if you cannot make it to class, email me by 11am on Thursday and we can discuss alternative ways to submit your assignment. Late assignments can be left in the mailbox outside my office until 4pm on Friday (incurring a small penalty, as described in the course syllabus). Assignments will not be accepted after 4pm on Friday.

- (0) Read Chapters 4-5 (pages 12-16).
- (1) Book problems 4.2, 4.3, 4.6
- (2) (Meta-analytic) Recall that $\mathbb{C} := \{a + bi : a, b \in \mathbb{R}\}$, the collection of complex numbers. Prove that \mathbb{C} isn't an ordered field, i.e. that it doesn't satisfy (A1)-(A12).
- (3) (Meta-analytic) Let $\mathbb{F}_7 := \{0, 1, 2, 3, 4, 5, 6\}$, endowed with two operations $+$ (mod 7) and \cdot (mod 7). Prove that \mathbb{F}_7 isn't an ordered field.
- (4) Prove that $1 + 1 \neq 0$. Must this still be true if we only required that \mathbb{R} satisfy (A1)-(A11)? Justify your answer.
- (5) There are real numbers between real numbers!
 - (i) Prove that $x^{-1} > 0$ for all positive x .
 - (ii) Suppose $a < b$. Prove $\exists x \in \mathbb{R}$ such that $a < x < b$.
- (6) Suppose $x, y \in \mathbb{R}$ and satisfy the inequality $x \leq y + \epsilon$ for every real number $\epsilon > 0$. Prove that $x \leq y$.