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MATH 350 : REAL ANALYSIS

Problem Set 4 – due Thursday, October 5th

INSTRUCTIONS:

You should aim to submit this assignment to me in person at the start of Thursday's class; if you cannot make it to class, email me by 11am on Thursday and we can discuss alternative ways to submit your assignment. Late assignments can be left in the mailbox outside my office until 4pm on Friday (incurring a small penalty, as described in the course syllabus). Assignments will not be accepted after 4pm on Friday.

(0) Read Chapter 6 (pages 17-20).

(1) Textbook problems 5.1, 5.2, 5.3, 5.5, 5.6, 5.7, 6.2, 6.3, 6.4, 6.5, 6.6.

A few comments on the problems:

- I ask that you use the notation from class (rather than the book): sup, inf, and $\mathbb{Z}_{\text{pos}}$.
- **5.1** admits the following interpretation, which is important to remember:

  You can't move left from the supremum without hitting an element of A.
- In **6.6**, you can't use that $\mathbb{Z}_{\text{pos}}$ is well-ordered in your proof, since our proof of this *used* Strong Induction! Here's a hint for how to approach this: Given a sequence of assertions $S(n)$ that satisfy the two hypotheses of Strong Induction, construct a different sequence of assertions $R(n)$ **about** the original assertions such that (i) $R(n)$ satisfies the two hypotheses of (ordinary) Induction, and (ii) if all the $R(n)$ are true, then all the $S(n)$ must also be true.

(2) A well-known puzzle game works as follows. (See figure below.) There are three vertical posts, numbered 1, 2, and 3. At the start of the game, there's a stack of n concentric rings of decreasing radius stacked on post 1, and the other two posts are empty. The goal is to end up with the entire stack of rings on post 3, again in order of decreasing radius. A move consists of transporting a ring at the top of any stack to another post; however, no ring can be placed on top of a smaller ring. Prove that the entire stack of n rings can be moved onto post 3 in $2^n - 1$ moves, and that this cannot be done in fewer than $2^n - 1$ moves.