COURSE INFORMATION

MATH 350 – Real Analysis

Course homepage: `https://web.williams.edu/Mathematics/lg5/350/`

Instructor: Leo Goldmakher
(*I prefer 'Leo', but 'Professor Goldmakher' or 'Dr. Goldmakher' are fine. Please don't call me just 'Professor'.*)
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Office hours: Mondays, 8:30**pm**-10:00pm at Goodrich; Wednesdays, 2pm-3:30pm in my office; other times by appointment. Generally, if you want to meet outside my office hours, it's best to email me in advance and arrange a time. That said, whenever my office door is open you're welcome to come in! I'm devoting most of every Friday to research activities, so I will generally be unavailable that day.

Lectures: Mondays & Thursdays 2:35–3:50 in Wachenheim 113.

Syllabus: In calculus and linear algebra you worked with spaces like $\mathbb{R}^2$, the set of all ordered pairs of real numbers. But what exactly is a real number? This is a surprisingly difficult question to answer. In this course we will develop a completely explicit and rigorous theory that subsumes everything you already know and love about $\mathbb{R}$, but which also goes much deeper. We'll discuss topics such as **set theory** (e.g. the Generalized Continuum Hypothesis), **sequences** (e.g. the Bolzano-Weierstrass theorem), **infinite series** (e.g. Riemann's rearrangement theorem), **functions on** $\mathbb{R}$ (e.g. when can f be expanded as a Taylor series), the **topology of** $\mathbb{R}$ (e.g. the Heine-Borel theorem), **metric spaces** (e.g. the Baire category theorem), **differentiation** (e.g. the mean value theorem), **integration** (e.g. Lebesgue's criterion for Riemann integrability), and **convergence of sequences and series of functions** (e.g. the Weierstrass approximation theorem).

Textbook: *Foundations of Mathematical Analysis* by Johnsonbaugh and Pfaffenberger (Dover edition). See course website for other resources.

Teaching Assistants: We're fortunate to have five fantastic TAs for this course:
- Emily Axelrod (eda2)
- Tommy Clarke (tdc2)
- Zoe Kane (zek2)
- Michael Keyes (mak5)
- Jenna Shuffelton (jms13)

Precept meetings: Like any other worthwhile activity, real analysis is *hard.* Some of the concepts and problems are going to be manifestly difficult, and you will need to persist in the face of feeling totally lost, in particular re-reading the text and your notes and reaching out to me, the TAs, and your peers for inspiration.

But the far greater challenge in this course is dealing with those difficulties and subtleties you **aren't** aware of. How can you fix a problem you don't know is a problem? The single most effective method is:

Try to explain the material to someone else.

One way I'm going to encourage this is to split the class up into *precept groups*, each consisting of three students. Once per week, your precept will meet for an hour with a TA. The precept will start with a 10 minute written quiz, which will consist of a single question: to prove one of the theorems we've covered in lecture (usually from the previous week). Following the quiz, each member of the precept will be asked to present their solution to a problem from the previous week's assignment. One of the goals is to discuss and understand the nuances of both the proof from the quiz and the homework problems. My hope is that you will **(a)** learn how to present difficult, technical material clearly and concisely, **(b)** learn from your peers' and TA's perspectives on the material, and **(c)** learn to think critically and constructively about other people's approaches to mathematics.

TA-hosted problem sessions. Every Tuesday and Wednesday, 8pm–10pm (tentative). Location TBA.

Assessment: Your grade will be calculated based on several components:

1. Problem sets – 5% total

   This course will have weekly problem sets. Some of the problems will be straightforward once you master the concepts from class / reading, while others will require additional creativity. **The problem sets are intended to be challenging**. The goal is to struggle with every question; it's OK not to solve every problem on the assignment, so long as you make a serious attempt at all of them. However, I have one hard rule: **please do not search for problems, solutions, or examples online.**

   Some problems from each problem set will be designated as typed, and for these you must submit solutions in LaTeX, the text editor of choice for anyone working in a technical field. *You are not expected to have prior experience with LaTeX, and guidance on how to write in LaTeX will be provided.* One of the TeXed problems will be graded thoroughly by your TA; the other problems will be graded on the basis of serious effort.

   Your problem set should be submitted directly to me by you at the start of Thursday's class. Late assignments may be left in the mail slot outside my office door any time before 4pm on Friday; however, 5% will be deducted for submission past the start of Thursday's class. *Assignments will not be accepted after Friday at 4pm under any circumstances.*

   Sometimes you will be unable to complete an assignment by the Friday deadline due to circumstances beyond your control. To try to account for this, I'll drop your lowest assignment score at the end of the semester.

2. Quizzes – 5% total

   These will take place at the start of your precept meeting. They will consist of a single question: to prove one of the theorems covered in lecture.

3. Precept meetings – 5% total

   At the precept meetings, each student will present a solution to a problem from the most recent assignment. Presentations will be graded not on the basis of correctness, but on effort and preparation, on a ✓+ / ✓ / ✓− scale; you can get a ✓+ even if you don't solve a problem by describing in detail the progress you made on it, the approaches you tried (and why you decided they wouldn't work), and especially by presenting conditional solutions ("If only I could prove such-and-such, I would be able to solve the problem in the following way...").

4. Midterm exam – 30%

   There will be an in-class midterm exam; date to be announced soon.

5. Final exam – 35%

   There will be an oral final exam; format and date will be discussed in class.

6. Best of your two exam scores – 5%

7. Expository essay – 15%

Problem sessions: I highly recommend getting together with your peers (for example, from your precept) to collaborate on the problem sets. That said, to maximize your understanding of the material, I urge you to work on the problems on your own first, and only afterwards brainstorm with others. There will be dedicated TA-led problem sessions (to be scheduled soon), but I strongly encourage you to take initiative and organize some student-run sessions as well.

Team work and plagiarism: I urge you to brainstorm with other students as you work on your problem sets. However, **you must write up the solutions on your own without copying from any text (written or spoken)**. For example, if you take notes during a meeting based on a solution explained to you by another student, *do not copy from these notes* when writing up your assignment! When you read this sentence, please send an email to our TA Tommy with subject line consisting of the single word ephlat. To avoid a slippery slope, I encourage you to write up your problems sets in physical isolation from any other student and from any notes you've taken while with other students.

Internet usage: The internet is an amazing resource, but I urge you to use it wisely. In particular, I request that you do not search for problems or examples. **Looking up definitions is OK, looking up (or asking about) problems online is not**. It is better to struggle on your own and *not* solve the problem than to simply copy a solution. When it comes to exams, please don't use the internet for any class-related reason apart from email or accessing the official course website.

Peer Tutoring (free!): The Peer Academic Support network provides free peer tutoring. Step-by-step instructions for finding and scheduling tutoring sessions are on the Peer Tutoring Program website:

`academic-resources.williams.edu`

If you have any questions about tutoring at Williams, please email `msrc@williams.edu`

Classroom culture: The goal of this class, apart from learning about the subject of real analysis, is to learn how to *think* like a practicing research mathematician (specifically, like an analyst). During our class meetings we will endeavor to collaboratively discover the material as a class. *Efficiency of information transfer will not be our paramount goal*; instead, free discussion will drive the class. Right ideas, wrong ideas, pre-ideas, and all questions are encouraged, as all these are vital to mathematical discovery. In this collaborative spirit, it is imperative that (a) we are not afraid to argue with one another's mathematical ideas, and (b) that we keep the conversation respectful and open at all times. *Respectful criticism of ideas is welcome, but ad hominem statements will not be tolerated.*

Covid policy: We will follow the protocols set by the college. For the time being, masks are optional in the classroom, but of course everyone is welcome to wear one. Please behave responsibly—if you feel sick, or if you test positive for covid, please don't come to class. Just reach out to me and we will work out how to best handle your absence.

Anonymous feedback: On the website (under the "Feedback" tab) there is a form for submitting anonymous feedback. Although I strongly prefer face-to-face conversations, I understand that this is not always possible or comfortable on some sensitive subjects, in which case please submit via the form. Negative comments, positive comments, confusions, and suggestions are all welcome, but please try to keep your feedback respectful and succinct. *Note that I reserve the right to post anonymous feedback, along with my response, publicly on the course webpage.*