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MATH 374 : TOPOLOGY

Oral Midterm Exam

Friday, October 21st
(alternate days by special appointment)

INSTRUCTIONS

This oral exam will be quite short, around 15 minutes long, and will consist of describing the properties of two given topological spaces (see next page).

The exam will take place in my office, Wachenheim 341. You will have access to a blackboard, but apart from this I request that you not use any other aids during your exam: no notes, books, etc.

When explaining something during your exam, I ask that you present a high-level overview: no more than a few sentences, with as few technical details as possible. This will usually suffice, but I reserve the right to ask you to give more detail. This includes asking you to define any (mathematical) word that you use in your explanation.

Often, it is during an exam that you realize for the first time that you don't properly understand something. This is not only natural, it is totally OK; I will give you as many hints as you need to get back on track. Although part of the exam is to see how far you can go on your own, the more valuable aspect of an oral exam is that it's a chance for some individualized learning.

You may use any resources you like while preparing for the exam, but the only humans you can discuss the exam with are me, other students currently enrolled in the course, and the TAs for the course. (In particular, no posting questions on online forums!) Moreover, once you have taken your exam, I request that you not discuss any aspect of it with other students until our class meeting the following Tuesday, October 25th.

Best of luck, and please don't hesitate to reach out with any questions!

Leo

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Midterm Exam

PROBLEMS

Two topological spaces from the list below will be selected at random via coin flips. If not otherwise specified, assume the usual / subspace topology.

List of Topological Spaces.

- **L.0** $\mathbb{Z}$ under the Furstenberg topology
- **L.1** The letter X (viewed as a subspace of $\mathbb{R}^2$)
- **L.2** The letter O (viewed as a subspace of $\mathbb{R}^2$)
- **L.3** The closure of a non-trivial open ball in $\mathbb{R}^2$
- **L.4** $\mathbb{R}_7$ (the particular point topology)
- **L.5** $\mathbb{R}_{\text{usual}}$
- **L.6** $\mathbb{Q}_{\text{usual}}$
- **L.7** $\mathbb{R}_{\geq 0}$
- **L.8** $\mathbb{R}_{\text{sorgenfrey}}$
- **L.9** $\mathbb{R}_{\text{ray}}$
- **L.10** $\mathbb{R}^2 \setminus \{0\}$
- **L.11** $\mathbb{R}_{\text{discrete}}$
- **L.12** $\mathbb{R}_{\text{indiscrete}}$
- **L.13** $(0,1) \cup [2,3]$ (viewed as a subspace of $\mathbb{R}$)
- **L.14** $\mathbb{R}_{\text{cofinite}}$
- **L.15** $\mathbb{Q} \times \mathbb{Q}_{\geq 0}$ with respect to $\mathcal{T}_{\text{sf}}$, defined below.

The topology in **L.15** is defined as follows. Given $\mathbf{p} := (p_1, p_2) \in \mathbb{Q} \times \mathbb{Q}_{\geq 0}$, set

$$\ell(\mathbf{p}) := \left(p_1 - \frac{p_2}{\sqrt{3}}, 0\right) \qquad r(\mathbf{p}) := \left(p_1 + \frac{p_2}{\sqrt{3}}, 0\right).$$

This has a geometric interpretation: the three points $\mathbf{p}$, $\ell(\mathbf{p})$, and $r(\mathbf{p})$ are the vertices of a (possibly degenerate) equilateral triangle whose base lies on the horizontal axis. Now for any $\epsilon > 0$, define

$$B_\epsilon(\mathbf{p}) := \{\mathbf{p}\} \cup \Big\{\mathbf{x} \in \mathbb{Q} \times \{0\} : |\mathbf{x} - \ell(\mathbf{p})| < \epsilon \text{ or } |\mathbf{x} - r(\mathbf{p})| < \epsilon\Big\}.$$

(I highly recommend drawing a picture.) The collection of all such sets, i.e.

$$\mathcal{B} := \{B_\epsilon(\mathbf{p}) : \mathbf{p} \in \mathbb{Q} \times \mathbb{Q}_{\geq 0}, \epsilon > 0\}$$

is a basis on $\mathbb{Q} \times \mathbb{Q}_{\geq 0}$, hence generates a topology on $\mathbb{Q} \times \mathbb{Q}_{\geq 0}$. This is the topology I called $\mathcal{T}_{\text{sf}}$ in **L.15**.

For each of your two spaces, answer the following questions. You should be able to justify your answers with proof sketches, but keep details to a minimum.

- **(i)** Describe the topology of the space. You may do this by either giving a direct description of all the open sets, or by describing a basis.
- **(ii)** Is the space T_0? Is it T_1? Is it Hausdorff?
- **(iii)** Is the space connected or disconnected?
- **(iv)** I'll give you a sequence; does it converge or diverge?
- **(v)** I'll give you a function from one space to the other; is it continuous?