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Problem Set 6 – due Friday, October 28th

INSTRUCTIONS:

If this is your week to write, please submit this assignment via Glow by 4pm on Friday; the solutions to at least four of the problems should be written in LaTeX. If this is your oral week, please be prepared by Friday to present your solutions orally (but you do not have to write them up in any form; during our meetings you won't be using any notes). If you have any questions—either about math or about LaTeX—please don't hesitate to reach out to me and we can figure it out.

6.1 Consider $\mathbb{R}_7$ (the reals under the particular point topology $\mathcal{T}_7$). When is a subspace of $\mathbb{R}_7$ connected? When is it disconnected? Prove your assertions.

6.2 Prove that continuous image of connected set is connected. In other words, if $f : X \to Y$ is continuous and X is connected, prove that $f(X)$ is connected.

6.3 Exploring path-connectedness

(a) Prove that if X is path-connected, then X is connected.

(b) Prove that $\mathbb{R}^2 \setminus \mathbb{Q}^2$ (under the subspace topology of $\mathbb{R}^2_{\text{usual}}$) is path-connected, hence connected.

6.4 Prove that if A is a connected subset of X and $A \subseteq C \subseteq \overline{A}$, then C is connected.

6.5 Must any continuous bijection be a homeomorphism? In other words, is the condition that the inverse be continuous redundant?

6.6 Give an explicit example of a Hausdorff topology $\mathcal{T}_1$ and a non-Hausdorff topology $\mathcal{T}_2$, both on $\mathbb{R}$, and a continuous bijection $f : (\mathbb{R}, \mathcal{T}_1) \to (\mathbb{R}, \mathcal{T}_2)$. (Thus, Hausdorffness is not preserved under bijective continuous maps.)

6.7 Is $\mathbb{R}^2$ with respect to the Zariski topology connected?