Instructor: Leo Goldmakher

Williams College Department of Mathematics and Statistics

MATH 374 : TOPOLOGY

Problem Set 7 - due Thursday, October 24th

INSTRUCTIONS:

You should aim to submit this assignment to me in person at the start of Thursday's class; if you cannot make it to class, email me by 11am on Thursday and we can discuss alternative ways to submit your assignment. Late assignments can be left in the mailbox outside my office until 4pm on Friday (incurring a small penalty, as described in the course syllabus). Assignments will not be accepted after 4pm on Friday.

- 7.1 Problem # 3 on page 17 of Ivan's Big List.
- **7.2** Give an explicit example of a Hausdorff topology \mathcal{T} and a non-Hausdorff topology \mathcal{T}' , both on \mathbb{R} , and a continuous bijection $f : (\mathbb{R}, \mathcal{T}) \to (\mathbb{R}, \mathcal{T}')$. (Thus, Hausdorffness is not preserved under bijective continuous maps.)
- **7.3** Is \mathbb{R}^2 with respect to the Zariski topology connected? Prove your assertion. (Your answer can be quite short!)
- **7.4** Consider \mathbb{R}_{20} , the reals under the particular point topology \mathcal{T}_{20} . When is a subspace of \mathbb{R}_{20} connected? When is it disconnected? Prove your assertions. [As always, if an explicit topology isn't specified on a subset, you should assume it's the subspace topology.]
- 7.5 Recall that a space is *totally disconnected* if and only if its only connected subsets are singletons and Ø.
 - (a) In class we saw that $\mathbb{R}_{sorgenfrey}$ is disconnected. Prove that it's totally disconnected.
 - (b) In class we saw that \mathbb{Q}_{usual} is disconnected. Is it totally disconnected? Prove your assertion.
 - (c) In class we saw $\mathbb{Z}_{\text{furstenberg}}$ is disconnected. Is it totally disconnected? Prove your assertion.
- **7.6** Prove that continuous image of connected set is connected. In other words, if $f: X \to Y$ is continuous and X is connected, prove that f(X) is connected.
- **7.7** Prove that if A is a connected subset of X and $A \subseteq C \subseteq \overline{A}$, then C is connected.
- **7.8** Prove that a topological space X is disconnected if and only if $X = A \sqcup B$ with A, B nonempty and $\overline{A} \cap B = \emptyset = A \cap \overline{B}$.