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MATH 374 : TOPOLOGY

Problem Set 8 - due Friday, November 1st

INSTRUCTIONS:

This assignment is due at 4pm on Friday, to be submitted to the mailbox outside my office. (This week there will be no late penalty for submitting Friday.) However, assignments will not be accepted after 4pm on Friday.

- 8.1 In class, Jake originally (very reasonably!) proposed that a homeomorphism should simply be a continuous bijection between topological spaces; we ended up imposing a stronger condition (that both the bijection and its inverse are continuous). Is our definition actually different from Jake's? In other words, do there exist continuous bijections that aren't homeomorphisms? Either give an example of one (with justification), or prove that there doesn't exist one.
- 8.2 Here we explore connectedness and path-connectedness.
 - (a) Prove that if X is path-connected, then X must be connected.
 - (b) Suppose $A \subseteq \mathbb{R}^2$ is a countable set. Prove that $\mathbb{R}^2 \setminus A$ is path-connected. (From part (a), it follows it must also be connected!)
 - (c) In class we mentioned the 'infinite broom' (often called the 'deleted infinite broom') as an example of a connected space that is not path connected. Prove this assertion.



Note that there is no line segment connecting (1,0) to any other point. (Illustration by Keith Conrad.)

- **8.3** Suppose $f : (X, \mathcal{T}) \to (Y, \mathcal{S})$ is a homeomorphism, and that \mathcal{B} is a basis for \mathcal{T} . Prove that $\{f(A) : A \in \mathcal{B}\}$ is a basis for \mathcal{S} .
- 8.4 For each of the pairs of spaces below, decide whether or not they are homeomorphic. Provide justification.
 - (a) \mathbb{R} and \mathbb{R}^2 (both with respect to the usual topologies).
 - (b) \mathbb{Q} and \mathbb{Q}^2 (both with respect to the usual topologies).
 - (c) $\mathbb{R} \times \mathbb{R}_{\geq 0}$ and \mathbb{R}^2 (both with respect to the usual / subspace topologies).