Instructor: Leo Goldmakher

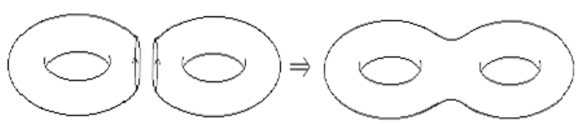
Williams College
Department of Mathematics and Statistics

Problem Set 9 – due Friday, November 18th

INSTRUCTIONS:

This week I'd like everyone to submit written assignments; in particular, there will not be any meetings this Friday. Please submit your assignment via Glow by 4pm on Friday. If you have any questions—either about math or about LaTeX—please don't hesitate to reach out to me and we can figure it out.

We've seen many examples of *surfaces* (2-dimensional objects—we'll discuss a formal definition of this on Monday). For example, the sphere S^2, the torus T^2, the real projective plane $\mathbb{RP}^2$, the Klein bottle K, and the Möbius strip M are all surfaces. Given two surfaces X and Y, there's a natural way to combine them: their *connected sum* (denoted $X\#Y$) is the surface you get by deleting the interior of a disk from each surface, and then gluing the boundaries of these two holes together. Here's an illustration of $T^2\#T^2$:



$T^2\#T^2$ *is a two-holed torus*

9.1 Above we saw an illustration that $T^2\#T^2$ is a two-holed torus. Use gluing diagrams to show this. [*Hint: You might wish to make use of Problem* **8.4** *from the last problem set.*]

9.2 Suppose X is a surface. What's a simple description of $S^2\#X$?

9.3 Show that $M\#M \approx K$.

9.4 Show that $\mathbb{RP}^2\#T^2 \approx \mathbb{RP}^2\#K$.

9.5 Show that $\mathbb{RP}^2\#T^2 \approx \mathbb{RP}^2\#\mathbb{RP}^2\#\mathbb{RP}^2$.

9.6 Compute $\chi(T^2\#T^2)$ and $\chi(T^2\#T^2\#T^2)$. Make a conjecture about $\chi(\underbrace{T^2\#T^2\#\cdots\#T^2}_{n \text{ copies}})$.

9.7 Make a conjecture about $\chi(X\#Y)$ in terms of $\chi(X)$ and $\chi(Y)$.