COURSE INFORMATION

MATH 374 – Topology

Course homepage: `https://web.williams.edu/Mathematics/lg5/374/`

Instructor: Leo Goldmakher
(*I prefer 'Leo', but 'Professor Goldmakher' or 'Dr. Goldmakher' are fine. Please don't call me just 'Professor'.*)
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Office hours: Tuesdays & Wednesdays 3pm–4:30pm; also by appointment. You do not need to make an appointment for the Tuesday / Wednesday times listed above. *You are also welcome to come in any time my office door is open, but if my office door is shut I ask you not to knock.* I'm devoting most of every Friday to my research, so I will generally be unavailable that day.

Lectures: Tuesdays and Thursdays, 9:55–11:10 in Wachenheim 017.

Syllabus: In Real Analysis you developed tools to study convergence and continuity in $\mathbb{R}$. These notions have straightforward generalizations to arbitrary metric spaces (i.e. any set equipped with a reasonable notion of distance). This allows us to study the *topological properties* of sets, like openness, closedness, boundedness, compactness, etc. What if we start at the other end? In other words, what if we're given a set in which we know something about the topology, but don't have any way to measure distances between elements? This immediately makes it difficult to discuss basic analysis like convergence and continuity. Nonetheless, we'll be able to develop a version of analysis in this topological setting (called *point-set topology*). We'll then turn to applications of topology to the study of geometry, developing the basics of algebraic topology along the way. Our aim is to cover as many of the following topics as time allows: the basics of point-set topology (bases, convergence, continuity, compactness, connectedness, subspaces), metrizability and Urysohn's lemma, manifolds and the classification of surfaces, the fundamental group and the Brouwer fixed point theorem, and Betti numbers and Poincaré duality.

Textbook: None required. However, the references below form a superset of the material we'll cover (links and pdf files posted on the course webpage):

- A clear and comprehensive set of lecture notes on point-set topology by Ivan Khatchatourian, available free online.
- A canonical introductory text on the subject is *Topology* by Munkres. It's full of interesting material and examples, but is rather expensive.

Teaching Assistant: We're fortunate to have alum Reuben Kaufman (rfk1) as a TA.

Precept meetings: Like any other worthwhile activity, topology is *hard.* Some of the concepts and problems are going to be manifestly difficult, and you will need to persist in the face of feeling totally lost, in particular re-visiting the notes and videos from class and reaching out to me, the TAs, and your peers for inspiration. But the far greater challenge in this course is dealing with those difficulties and subtleties you *aren't* aware of. How can you fix a problem you don't know is a problem? The single most effective method is:

Try to explain the material to someone else.

One way I'm going to encourage this is to split the class up into *precept groups*, each consisting of three students. Every other week, rather than writing up your problem set, your precept will meet with me for an hour. Each member of the precept will be asked to present their solution to a problem from the previous week's assignment, and then this solution will be discussed by the group. My hope is that you will **(a)** learn how to present difficult, technical material clearly and concisely, **(b)** learn from your peers' perspectives on the material, and **(c)** learn to think critically and constructively about other people's approaches to mathematics.

Assessment: Your grade will be calculated based on several components:

1. Lecture summaries – 10% total

   After each class, a pair of students will be responsible for writing up a lecture summary in LaTeX. If you don't know what LaTeX is, don't worry about it—I will provide resources (and your TAs will help, too).

   Your lecture summary is expected to be a rigorous, polished, and readable account of what took place during the lecture. It will be graded on a ✓+ / ✓/ ✓– scale, based on how much editing is required post-submission.

   Your lecture summary is due within 48 hours of the lecture. *Failure to submit a summary will result in your overall grade for the course to decrease by* 2/3 *of a letter grade.*

2. Weekly problem sets – 15% for written, 25% for precept presentations

   This course will have weekly problem sets. Every other week you will submit solutions, written in LaTeX, by 4pm Friday on Glow. (As above, if you're unfamiliar with LaTeX, don't worry—I'll help you.) In the alternate weeks, you will not be responsible for submitting a written assignment, instead gathering with me and your precept group to take turns presenting solutions orally to one another.

   During precept meeting weeks, grading will be based purely on meaningful participation, both in presenting your own work and discussing others'. Note that *your grade will not depend on the correctness of your presented solution.* Even if you don't know how to approach a problem, you will be expected to present whatever ideas you did come up with.

   Your written assignments will be graded by Reuben in a more traditional way. He will also meet with you early in the week (on Zoom) to give individual feedback on your previous assignment.

   The problem sets are intended to be challenging. The goal is to struggle with every question; it's OK not to solve every problem on the assignment, so long as you make a serious attempt at all of them. However, I have one hard rule: **please do not search for problems, solutions, or examples online.**

3. Oral midterm exam – 20%

   There will be a short oral midterm exam. Format will be discussed in class.

4. Final project – 15%

   There will be a final project (in lieu of a final exam). Details will be announced closer to the end of the semester.

5. Expository essay – 15%

   In the second half of the semester I'll assign an expository essay, to be written in LaTeX, explaining a theorem or area related to topology that we haven't covered in this class. More details forthcoming.

Classroom culture: The goal of this class, apart from learning about point set and algebraic topology, is to learn how to *think* like a practicing research mathematician (specifically, like a set theorist or topologist). During our class meetings we will endeavor to collaboratively discover the material as a class. *Efficiency of information transfer will not be our paramount goal*; instead, free discussion will drive the class. Right ideas, wrong ideas, pre-ideas, and all questions are encouraged, as all these are vital to mathematical discovery. For example, we will sometimes make several incorrect attempts at definitions before arriving at the official one. While this might seem inefficient, it's the only way I know to explain *why* people arrived at the official definition, and will (I hope!) give you the confidence to start inventing some of your own. In this collaborative spirit, it is imperative that (a) we are not afraid to argue with one another's mathematical ideas, and (b) that we keep the conversation respectful and open at all times. *Respectful criticism of ideas is welcome, but ad hominem statements will not be tolerated.*

Problem sessions: I highly recommend getting together with your peers (for example, from your precept) to collaborate on the problem sets. That said, to maximize your understanding of the material, I urge you to work on the problems on your own first, and only afterwards brainstorm with others. Note that there will not be any dedicated TA-led problem sessions; I encourage you to take initiative and organize student-run sessions.

Team work and plagiarism: I strongly encourage you to brainstorm with other students as you work on your problem sets. However, *you must write up the solutions on your own without copying from any text (written or spoken).* For example, if you take notes during a meeting based on a solution explained to you by another student, *do not copy from these notes* when writing up your assignment! To avoid a slippery slope, I encourage you to write up your problems sets in physical isolation from any other student and from any notes you've taken while with other students.

Internet usage: The internet is an amazing resource, but I urge you to use it wisely. In particular, I request that you do not search for problems or examples. *Looking up definitions is OK, looking up (or asking about) problems online is not.* It is better to struggle on your own and *not* solve the problem than to simply copy a solution. When it comes to exams, please don't use the internet for any class-related reason apart from email or accessing the official course website.

Covid policy: We will follow the protocols set by the college. For the time being, masks are optional in the classroom, but of course everyone is welcome to wear one. Please behave responsibly—if you feel sick, or if you test positive for covid, please don't come to class. Just reach out to me and we will work out how to best handle your absence / work.

Anonymous feedback: On the website (under the "Feedback" tab) there is a form for submitting anonymous feedback. Although I strongly prefer face-to-face conversations, I understand that this is not always possible or comfortable on some sensitive subjects, in which case please submit via the form. Negative comments, positive comments, confusions, and suggestions are all welcome! Please try to keep your feedback respectful and succinct. *Note that I will post anonymous feedback, along with my response, publicly on the course webpage.*