Instructor: Leo Goldmakher

NAME: ______________________________

Williams College
Department of Mathematics and Statistics

MATH 394 : GALOIS THEORY

Problem Set 6 – due Thursday, April 5th

INSTRUCTIONS:
This assignment must be turned in to my mailbox (on the right as you enter Bascom) by **4pm** sharp. Assignments may be submitted later than this by email to Alyssa, but no later than 4pm on Friday; in this case, the grade will be reduced by 5%.
Assignments submitted later than Friday at 4pm will not be graded.

Please print and attach this page as the first page of your submitted problem set.

PROBLEM	GRADE
6.1	
6.2	
6.3	
6.4	
6.5	
6.6	
Total	

Please read the following statement and sign below:

I understand that I am not allowed to use the internet to assist with this assignment. I also understand that I must write down the final version of my assignment in isolation from any other person. I pledge to abide by the Williams honor code.

SIGNATURE:______________________________

Problem Set 6

6.0 Play Euclid the Game! `http://eu.jugregator.org`

6.1 Prove that z is constructible iff both Re z and Im z are constructible.

6.2 In class we asserted that constructing a regular heptagon boiled down to constructing the seventh root of unity. Verify this rigorously by proving that a regular heptagon is constructible iff the point $e^{2\pi i/7}$ is constructible.

6.3 Determine the minimal polynomial over $\mathbb{Q}$ of $\zeta_{18} := e^{\pi i/9}$.

6.4 Solve the following problems from Chapter 7 of the book.

(a) Problem 7.3 (trisecting an angle via a marked ruler)

(b) Problem 7.5 (impossibility of constructing a regular nonagon) [*Hint: your proof should be very short!*]

(c) Problem 7.6 (construct the regular pentagon) [*Hint: Write $e^{i(5\theta)}$ in two different ways.*]

(d) Problem 7.13 (doubling the cube via a marked ruler)

(e) Problem 7.15 (impossibility of constructing a regular hendecagon)

(f) Problem 7.19 (sufficient condition for an angle to be constructible)

6.5 Prove that if $2^k + 1$ is prime, then $k = 2^m$.

6.6 Let $S := \{\sqrt{p} : p \text{ is prime}\}$. Prove that $\mathbb{Q}(S)/\mathbb{Q}$ is algebraic but infinite.