Instructor: Leo Goldmakher

NAME: ________________________________

Williams College
Department of Mathematics and Statistics

MATH 394 : GALOIS THEORY

Problem Set 7 – due Thursday, April 12th

INSTRUCTIONS:
This assignment must be turned in to my mailbox (on the right as you enter Bascom) by **4pm** sharp. Assignments may be submitted later than this by email to Alyssa, but no later than 4pm on Friday; in this case, the grade will be reduced by 5%.
Assignments submitted later than Friday at 4pm will not be graded.

Please print and attach this page as the first page of your submitted problem set.

PROBLEM	GRADE
7.1	
7.2	
7.3	
7.4	
7.5	
Total	

Please read the following statement and sign below:

I understand that I am not allowed to use the internet to assist with this assignment. I also understand that I must write down the final version of my assignment in isolation from any other person. I pledge to abide by the Williams honor code.

SIGNATURE:________________________________

Problem Set 7

7.1 Prove that $\mathbb{Q}(\omega\sqrt[3]{2}) \simeq \mathbb{Q}(\omega^2\sqrt[3]{2})$, but $\mathbb{Q}(\omega\sqrt[3]{2}) \neq \mathbb{Q}(\omega^2\sqrt[3]{2})$.

7.2 In class we found four fields lying between $\mathbb{Q}$ and $\mathbb{Q}(\omega, \sqrt[3]{2})$. Prove that there are no others.

7.3 Carefully check that our lattice of all subgroups of D_3 (the dihedral group of order 6) is correct and complete. Also identify all those subgroups which are normal.

7.4 We imitate the construction of the Galois correspondence from class, but this time with the polynomial $f(x) := x^4 - 4x^2 + 2$. Let $\alpha := \sqrt{2+\sqrt{2}}$ denote one of the roots of f.

(a) Prove that $\mathbb{Q}(\alpha)$ is a splitting field of f.

(b) Draw a lattice of all intermediate fields between $\mathbb{Q}$ and $\mathbb{Q}(\alpha)$, along with the degrees of each extension.

(c) Determine $\mathrm{Aut}\big(\mathbb{Q}(\alpha)\big)$. What familiar group is it isomorphic to?

(d) Draw a lattice of all subgroups of $\mathrm{Aut}\big(\mathbb{Q}(\alpha)\big)$, labelling all the connecting edges by the index of one group inside the other.

7.5 Another Galois correspondence, this time for the polynomial $g(x) := x^4 - 12x^2 + 35$.

(a) Determine a splitting field K of g. (Write it in the form $\mathbb{Q}(\beta_1, \beta_2)$.)

(b) Draw a lattice of all intermediate fields between $\mathbb{Q}$ and K, along with the degrees of each extension.

(c) Determine $\mathrm{Aut}(K)$. What familiar group is it isomorphic to?

(d) Draw a lattice of all subgroups of $\mathrm{Aut}(K)$, labelling all the connecting edges by the index of one group inside the other.