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Department of Mathematics and Statistics

MATH 394 : GALOIS THEORY

Final Exam – to be taken Wednesday, May 16th—Monday, May 21st

INSTRUCTIONS

The final exam will consist of four questions, to be discussed orally. The duration of the exam will be one hour. You will have access to a blackboard; *no other aids are permitted.*

The exam will begin with Question A, which will be asked of every student. The other three questions will be selected by coin flip from Lists B, C, and D, one question from each. (See next page for questions.)

I would like you to understand each topic as deeply as possible. To this end, I reserve the right to follow up on anything you mention during your discussion. For example, if you use the phrase 'maximal ideal' I may ask you to define what that is; if you quote the result that the quotient of a commutative ring by a maximal ideal is a field, I may ask you to prove it. If you use the cubic formula, I may ask you to derive it. In short, as you study the material, I want you to continually ask yourself the question: *can I define / prove this without looking it up*?

Often, it is during an exam that you realize for the first time that you don't properly understand something. This is not only natural, it is totally OK; I will give you as many hints as you need to get back on track. Although part of the exam is to see how far you can go on your own, the more valuable aspect of an oral exam is that it's a chance for some individualized learning.

I strongly encourage you to practice for the exam with someone (e.g. your tutorial partner).

Best of luck!

Problems on other page...

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PROBLEMS

Question A (The Theorem)
State and prove the Galois correspondence for any finite Galois extension L/K (i.e. part (1) of the FTGT).

List B (Theory)

B.0 Prove it is impossible to construct a regular heptagon using only compass and straightedge.

B.1 Give Geck's proof that any finite Galois extension is simple. (See Propositions 3.1 and 3.3, as well as Corollary 3.4, in lecture 15.)

B.2 Given $\zeta_n, \alpha^n \in K/\mathbb{Q}$. Prove that $\mathbb{Q}(\zeta_n)/\mathbb{Q}$ is abelian and that $K(\alpha)/K$ is cyclic.

B.3 Give the Galois-theoretic proof of the Fundamental Theorem of Algebra.

List C (Computations)

C.0 I will give you a polynomial and ask you to determine whether its roots can be expressed in radicals.

C.1 I will give you a Galois extension and ask you to illustrate the Fundamental Theorem of Galois Theory by drawing the lattice of all intermediate fields and all subgroups of the Galois group.

C.2 I will give you a polynomial and ask you to determine whether or not it's separable.

C.3 I will give you a field extension. List all four equivalent conditions for Galois-ity, and then use any one of them to decide whether or not the extension is Galois.

List D (Nostalgia)

D.0 Sketch Arnold's proof that there does not exist any finite quintic formula built out of the coefficients of a given quintic, the field operations $+, -, \times, \div$, arbitrary continuous functions on $\mathbb{C}$, and radicals.

D.1 Sketch the proof of Eisenstein's criterion.

D.2 Sketch the derivation of the cubic formula.

D.3 Sketch the proof of Kronecker's theorem.

Instructions on other page...