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MATH 394 : GALOIS THEORY

Midterm Exam – to be taken on Thursday, March 15th or Friday, March 16th

INSTRUCTIONS:

The midterm exam will consist of three questions, to be discussed orally. The duration of the exam will be between 20 and 30 minutes. You will have access to a blackboard; *no other aids are permitted*.

The exam will begin with questions A and B below, which will be asked of every student. The third question will be selected from List C by coin flip.

Although I am deliberately keeping the number of topics on this exam small, I would like you to understand them as deeply as possible. To this end, I reserve the right to follow up on anything you mention during your discussion. For example, if you use the phrase ‘maximal ideal’ I may ask you to define what that is; if you quote the result that the quotient of a commutative ring by a maximal ideal is a field, I may ask you to prove it. If you use the cubic formula, I may ask you to derive it. In short, as you study the material, I want you to continually ask yourself the question: *can I define / prove this without looking it up?*

Often, it is during an exam that you realize for the first time that you don’t properly understand something. This is not only natural, it is totally OK; I will give you as many hints as you need to get back on track. Although part of the exam is to see how far you can go on your own, the more valuable aspect of an oral exam is that it’s a chance for some individualized learning.

I strongly encourage you to practice for the exam with someone (e.g. your tutorial partner). Best of luck!

Question A:

State and prove Kronecker’s theorem on the existence of a root of a polynomial inside a suitable field extension.

Question B:

Sketch Arnold’s proof that any general cubic formula built out of the coefficients of the cubic, the field operations $+$, $-$, \times , \div , continuous functions on \mathbb{C} , and radicals must involve at least two levels of nested radicals. [*Your sketch should take **no longer than five minutes**. Please practice with someone and time yourself pre-exam.*]

List C

- C.1** I will give you a polynomial and ask you to determine whether or not it is irreducible. [*I may also ask you to prove any theorems you employ!*]
- C.2** I will give you a cubic polynomial and ask you to determine its roots. [*Note: I expect you to be able to derive the roots from scratch; do not simply memorize the cubic formula!*]
- C.3** I will give you a polynomial $f(x) \in \mathbb{Q}[x]$ and ask you to determine its Galois group (using the method presented in Lecture 4), as well as the shape of the roots of f .
- C.4** I will give you a field extension L/K and ask you to determine the degree of the extension (with proof).