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MATA31 – Calculus I for Mathematical Sciences

Problem Set 2 (due the week of October 8th – 12th)

At the top of your assignment, please write your full name and student number. Also, please copy (by hand) the following statement onto the top of your assignment, and sign it:

I understand that I am not allowed to use the internet to assist (in any way) with this assignment. I also understand that I must write down the final version of my assignment in isolation from any other person.
[signature]

A. Carefully explain what's wrong with the following proof. (The claim is wrong, but that's not what's wrong *with the proof.*)

Claim: For all integers $n \geq 0$, $2^n = 1$.

'Proof': By Strong Induction. Let $\mathcal{A} = \{n \in \mathbb{N} : 2^{n-1} = 1\}$. We clearly have $1 \in \mathcal{A}$. Next, suppose $\{1, 2, \ldots, n\} \subseteq \mathcal{A}$; in particular, both $n-1 \in \mathcal{A}$ and $n-2 \in \mathcal{A}$. Thus we have

$$2^n = \frac{2^{n-1} \cdot 2^{n-1}}{2^{n-2}} = \frac{1 \cdot 1}{1} = 1.$$

Thus, $\{1, 2, \ldots, n\} \subseteq \mathcal{A} \Longrightarrow n+1 \in \mathcal{A}$. By Strong Induction, $\mathcal{A} = \mathbb{N}$. 'QED'

B. In the sovereign territory of Strong Badia, there are 4-‡ stamps and 7-‡ stamps (the ‡ is the unit of currency, the International Strong Badia Numeration). What are all the possible postages one can create with these two types of stamps? Prove your answer. [*Hint: use strong induction*]

C. Bartle & Sherbert, 1.2 # 1, 2, 5, 6, 8, 14, 16, 18