

UNIVERSITY OF TORONTO SCARBOROUGH

MARCH 2010 MIDTERM EXAMINATION

MATA32H3 S

Calculus for Management I

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Duration—2 hours

Aids: calculator

NAME (PRINT): _____
Last/Surname First/Given Name

STUDENT NO: _____ **Name of TA:**_____

Please read the following statement and sign below:

I understand that any breach of academic integrity is a violation of The Code of Behaviour on Academic Matters. By signing below, I pledge to abide by the Code.

SIGNATURE:_____

Qn. #	Value	Score
1	10	
2	20	
3	10	
4	12	
5	18	
6	15	
Total	85	

You may find the following formulas helpful:

$$a + ar + ar^2 + \cdots + ar^{N-1} = a \times \frac{1 - r^N}{1 - r}$$

$$P(x) = P_0 \left(1 + \frac{r}{N}\right)^{Nx}$$

$$P_{\text{cont}}(x) = P_0 e^{rx}$$

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- (1) Given a constant k , let

$$f(x) = \begin{cases} 3x + 2k^2 & \text{for } x < 5 \\ x^2 + 2k + 2 & \text{for } x \geq 5 \end{cases}$$

Find all values of k for which $f(x)$ is a continuous function. Show all relevant work.

10 points

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20 points

- (2) You wish to buy a car which is currently selling for \$20,000. Since you don't have that much money to spend, you make the following arrangement with the seller:
- (i) You pay \$5000 upfront.
 - (ii) You pay off the balance in equal monthly installments, paid at the end of every month for the next two years, at an interest rate of 12% APR compounded monthly.

a) How much is the first installment? (Show all work.)

b) What is the finance charge? (Recall that the finance charge is the total interest paid.)

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- (3) Determine the least number of months required for a principal of P dollars to increase by 50% at an 8% APR compounded semiannually. (Show all work.)

10 points

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- (4) Suppose you owe someone \$8,000. You agree to pay off the debt according to the following plan:
- (i) you pay \$1000 at the end of the first year;
 - (ii) you pay \$3000 at the end of the 3rd year;
 - (iii) you pay the rest at the end of the 6th year.
- Assume an interest of 10% APR compounded semiannually. What is the amount of the final payment? (Show all work.)

12 points

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3 points each

- (5) Let $g(x) = \sqrt{\frac{2^x - 2}{x - 2}}$. For each of the following, choose the most appropriate answer. You do not need to justify your choice.

i) $\lim_{x \rightarrow 0} g(x) = \underline{\hspace{2cm}}$

- (a) 0 (b) 1 (c) 2 (d) $\sqrt{\frac{1}{2}}$ (e) $\sqrt{2}$ (f) ∞ (g) $-\infty$
(h) none of the above.

ii) $\lim_{x \rightarrow 1} g(x) = \underline{\hspace{2cm}}$

- (a) 0 (b) 1 (c) 2 (d) $\sqrt{\frac{1}{2}}$ (e) $\sqrt{2}$ (f) ∞ (g) $-\infty$
(h) none of the above.

iii) $\lim_{x \rightarrow 2^-} g(x) = \underline{\hspace{2cm}}$

- (a) 0 (b) 1 (c) 2 (d) $\sqrt{\frac{1}{2}}$ (e) $\sqrt{2}$ (f) ∞ (g) $-\infty$
(h) none of the above.

iv) $\lim_{x \rightarrow 2^+} g(x) = \underline{\hspace{2cm}}$

- (a) 0 (b) 1 (c) 2 (d) $\sqrt{\frac{1}{2}}$ (e) $\sqrt{2}$ (f) ∞ (g) $-\infty$
(h) none of the above.

v) $\lim_{x \rightarrow -\infty} g(x) = \underline{\hspace{2cm}}$

- (a) 0 (b) 1 (c) 2 (d) $\sqrt{\frac{1}{2}}$ (e) $\sqrt{2}$ (f) ∞ (g) $-\infty$
(h) none of the above.

vi) $\lim_{x \rightarrow \infty} g(x) = \underline{\hspace{2cm}}$

- (a) 0 (b) 1 (c) 2 (d) $\sqrt{\frac{1}{2}}$ (e) $\sqrt{2}$ (f) ∞ (g) $-\infty$
(h) none of the above.

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- (6) Suppose that the amount of Carbon-14 present in an old wooden tool is 20% of the amount the tool originally contained (i.e. when it was made). Given that the amount of Carbon-14 in inanimate objects decays exponentially and has a half life of 5730 years, estimate the age of the tool. Show all work.

15 points

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Scratch paper.

Total Marks = 85 points