MATA32 – Winter 2010 Quiz 3: Solutions

Name: KEY

1. The demand equation for a product is $p = 8^{2-\frac{1}{2}q}$. Express q in terms of p, using only base 10 logarithms.

Take the logarithm (base 10) of both sides. We find: $\log p = \log 8^{2-\frac{1}{2}q} = \left(2 - \frac{1}{2}q\right) \log 8$

$$= 2\log 8 - \frac{1}{2}q\log 8$$

From this we deduce that

$$\frac{1}{2}q\log 8 = 2\log 8 - \log p$$

Multiplying both sides by 2 and dividing both sides by $\log 8$, we conclude that

 $q = \frac{2(2\log 8 - \log p)}{\log 8}.$

Continued on reverse...

2. If an investor has a choice of investing money at 6% compounded weekly, or at 6.25% compounded semiannually (i.e. twice per year), which is the more profitable choice? Justify your answer. You may use the fact that there are 52 weeks in a year.

Say the initial investment is P_0 .

6% compounded weekly:

In this case, after one year, the investor's money will have grown to

$$P_0 \left(1 + \frac{0.06}{52} \right)^{52} \approx 1.0618 P_0$$

which means the effective rate is approximately 6.18%.

6.25% compounded semiannually:

In this case, after one year the investor's money will have grown to

$$P_0 \left(1 + \frac{0.0625}{2} \right)^2 \approx 1.0635 P_0$$

which means the effective rate is approximately 6.35%. Since the second effective rate is larger than the first, the semiannually compounded investment is the better one.

NOTE: It was enough to compute the effective rate of the first investment to see that the second one is better. Why? The effective rate is higher than the A.P.R., so whatever the effective rate of the second investment is, it's larger than 6.25%... so it's certainly larger than 6.18%, the effective rate of the first investment scheme.