MATA32 – Winter 2010
Quiz 8 Solutions

Name: KEY

1. Differentiate $y = \dfrac{\ln x}{x^2}$.

This is quotient rule: bottom times derivative of the top, minus top times derivative of the bottom, over the bottom squared. In this case:

$$\frac{dy}{dx} = \frac{x^2 \times \frac{1}{x} - (\ln x) \times 2x}{x^4} = \frac{x - 2x \ln x}{x^4} = \frac{1 - 2 \ln x}{x^3}$$

Continued on reverse...

2. Find $f'(x)$ if $f(x) = \Big(\ln(4x-10)\Big)^3$.

Basically, this is the function $(\text{something})^3$. Differentiating this in the usual way would give $3(\text{something})^2$, but by chain rule we have to adjust this by multiplying by the derivative of the something. In our case, the something is $\ln(4x-10)$. This looks like $\ln(\text{blah})$, so we expect to get something like $\frac{1}{\text{blah}}$ after we differentiate. Again by chain rule, we need to adjust our expectation by multiplying by the derivative of blah. Our blah is $4x-10$, so the derivative is just 4. Putting all this together:

$$f'(x) = 3\Big(\ln(4x-10)\Big)^2 \times \frac{1}{4x-10} \times 4 = \frac{6\Big(\ln(4x-10)\Big)^2}{2x-5}$$

3. Find the slope of the curve $x^3 = (3y-5x^3)^4$ at $(1,2)$.

Differentiating both sides and using chain rule, we find:

$$3x^2 = 4\Big(3y-5x^3\Big)^3\left(3\,\frac{dy}{dx} - 15x^2\right)$$

Plugging in $x=1$ and $y=2$ this becomes

$$3 = 4\left(3\,\frac{dy}{dx}\bigg|_{(1,2)} - 15\right)$$

from which we conclude that

$$\frac{dy}{dx}\bigg|_{(1,2)} = 5.25$$