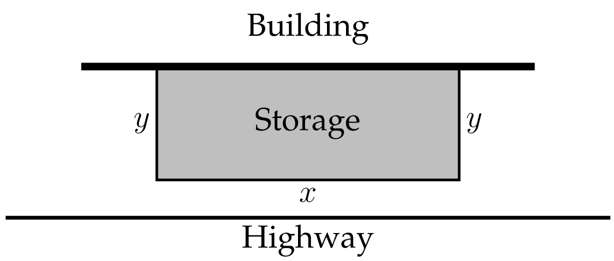
MATA32 – Winter 2010
Quiz 9 Solutions

1. A manufacturer plans to fence in a 500 m^2 rectangular storage area adjacent to a building by using the building as one side of the enclosed area (see the picture below). The fencing parallel to the building faces a highway and will cost \$5 per meter installed, whereas the fencing for the other two sides costs \$2 per meter installed. What is the minimum the fence will cost?



The area is 500, which means that $xy = 500$. The cost of the project, which we wish to minimize, is $C = 2y + 5x + 2y = 5x + 4y$. From the area condition, we have that $y = \frac{500}{x}$; plugging this in we rewrite the cost function as a function of x:

$$C(x) = 5x + \frac{2000}{x}$$

To minimize this, we differentiate and set equal to 0:

$$C'(x) = 5 - \frac{2000}{x^2} = 0.$$

This has two solutions, $x = \pm 20$, but of course x cannot be negative. So, $C'(20) = 0$. Is this a minimum? We use the second derivative test:

$$C''(x) = \frac{4000}{x^3}$$

so in particular, $C''(20) > 0$ which means that $C(x)$ has a relative minimum at $x = 20$. Moreover, $x = 20$ is the *only* positive value of x for which $C'(x) = 0$. This means $C(x)$ has an *absolute* minimum at $x = 20$, and this minimal value is $C(20) = 200$. So, the minimal cost is \$200.

Continued on reverse...

2. A cable company currently has 100,000 subscribers who are each paying a monthly rate of \$40. A survey reveals that for each \$0.25 increase to the rate, the company will lose 250 subscribers. What monthly rate should the company charge to maximize their revenue?

Let n be the number of \$0.25 increases the company will make. The company's revenue is $R = pq$, where p is the monthly rate and q is the number of customers. We can write all of these in terms of n:

$$p = 40 + 0.25n, \qquad q = 100,000 - 250n$$

so $R(n) = (40 + 0.25n)(100,000 - 250n)$. To maximize this, we differentiate and set equal to 0: by product rule,

$$R'(n) = 0.25(100,000 - 250n) - 250(40 + 0.25n) = 0$$

Solving for n gives the unique solution $n = 120$. Is $R(n)$ maximized or minimized at this value of n? We use the second derivative test:

$$R''(n) = -0.25 \times 250 - 0.25 \times 250 < 0$$

so in particular, $R''(120) < 0$ and $R(n)$ has a relative maximum at $n = 120$. However, since $R'(n) \neq 0$ except at $n = 120$, the maximum at $n = 120$ must be *absolute*. When $n = 120$, we have $p = 40 + 0.25n = 70$, so the rate which maximizes the revenue is \$70 per month.