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MATC01: GROUPS AND SYMMETRY

Problem Set 1 – due Friday, September 20th, during the first 5 minutes of lecture

INSTRUCTIONS: Please print and attach this page as the first page of your submitted problem set.

PROBLEM	MARK
1.1	
1.2	
1.3	
1.4	
1.5	
Total	

Please read the following statement and sign below:

I understand that I am not allowed to use the internet to assist (in any way) with this assignment. I also understand that I must write down the final version of my assignment in isolation from any other person.

SIGNATURE:______________________________

Problem Set 1

1.1 In this problem, you will explore complex numbers and their properties.

(a) Prove that for all $z \in \mathbb{C}$, we have $|z|^2 = z\bar{z}$.

(b) Prove that for any $z, w \in \mathbb{C}$, we have $\overline{zw} = \bar{z}\bar{w}$.

(c) Simplify $\overline{re^{i\theta}}$. (Give your answer in complex polar coordinates.)

(d) Evaluate $(1 + 2i)^2$. (Give your answer in the form $a + bi$.)

(e) Determine all compex solutions z to the equation $z^2 = i$.

(f) Rewrite $3e^{7\pi i/6}$ in the form $a + bi$.

(g) Rewrite $-3\sqrt{2} + 3i\sqrt{2}$ in complex polar coordinates.

(h) Rewrite -5 in complex polar coordinates.

(i) Evaluate $R_{2\pi/3}(3 - 2i)$. Give your answer in the form $a + bi$. No approximations: your answer must be exact! [*Hint: you do <u>not</u> need to calculate* $\arctan(-2/3)$ *at any time to solve this problem.*]

1.2 Prove that the composition of two isometries is an isometry.

1.3 Prove that an isometry is a bijection. [Recall that this is usually accomplished in two steps: first, prove that an isometry is an injection (i.e. that if $f(X) = f(Y)$ then $X = Y$); then prove that an isometry is a surjection (i.e. that for all $Y \in \mathbb{R}^2$ there exists an $X \in \mathbb{R}^2$ such that $f(X) = Y$).]

1.4 Prove that the inverse of an isometry is an isometry.

1.5 We say that a function $f : \mathbb{R}^2 \to \mathbb{R}^2$ is *linear* iff for all $\alpha, \beta \in \mathbb{R}$ and all $X, Y \in \mathbb{R}^2$ we have

$$f(\alpha X + \beta Y) = \alpha f(X) + \beta f(Y).$$

Prove that $f : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear map iff there exist $a, b, c, d \in \mathbb{R}$ such that f acts like multiplication by $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. [COMMENT*: This problem shows that linear algebra is actually the study of linear maps.*]